ARE/ECN 240C Time Series Analysis — Professor Òscar Jordà
Winter 2010 — Economics, U.C. Davis

PROBLEM SET 1 – DUE: JANUARY 14, 2006

Instructions

This problem set is divided into two parts: (1) Analytical Questions, and (2) Applied Questions. All parts should be attempted by each student *individually*. Please try to answer the questions rigorously by stating any implied assumptions and ensuring all the steps to your conclusion have been properly verified.

Part I – Analytical Questions

1. Suppose

$$\sqrt{T}\left(\hat{\theta}_T - \theta_0\right) \xrightarrow{L} N(0,\sigma^2)$$

Does it follow then that $h(\hat{\theta}_T) \xrightarrow{p} h(\theta_0)$ where h is a continuous function? Explain your answer.

2. *Combining the delta method and Lindeberg-Levy.*
Let $\{z_t\}$ be an i.i.d. sequence of random variables with $E(z_t) = \mu \neq 0$ and $V(z_t) = \sigma^2$. Let $\bar{z}_T$ be the sample mean, derive the asymptotic distribution of $\ln(\bar{z}_T)$.

3. Calculate the variance of the process $y_t = y_{t-1} + \varepsilon_t$ where $\varepsilon_t \overset{iid}{\sim} (0,\sigma^2)$. Hence, show that it is not stationary.

4. Suppose $X_T \xrightarrow{p} \dfrac{1}{\sigma}$ and $Y_T \xrightarrow{p} Y$ where $Y \sim N(0, \sigma^2)$. Derive the limiting distribution of $X_T Y_T$. Hence derive the distribution of $(X_T Y_T)^2$. Be explicit about your assumptions.

5. Does a martingale difference sequence have to be covariance stationary? Explain.

Part II – Empirical Questions

GAUSS

The website for the course contains a GAUSS file labeled "clt.g" This file illustrates the law of large numbers and the central limit theorem. It generates draws for an exponentially distributed random variable and it shows how the sample mean from these draws are centered at the mean of the exponential random variable and how the distribution of the sample mean converges to a normal (even is the draws themselves are not normal but rather, exponential. The program generates graphical output but if you are running GAUSS from a unix environment, this will not work. The graph is saved instead to a postscript file that you can view with ghostview, transform to pdf and then view, or simply print.

Your job is to modify the program as follows. Instead of draws from an exponential random variable, you will need to generate draws from a chi-square with 2 degrees of freedom. Once you have accomplished this, rerun the program and verify that the LLN and CLT apply. You will need to turn in:

1. One page with the graphs
2. A copy of the code you used to modify the code. At the end, you can append the screen output on the mean and the standard errors.