

Path Forecast Evaluation

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Alan Greenspan, 2003

[...] a central bank seeking to maximize its probability of achieving its goals is driven, I believe, to a risk-management approach to policy. By this I mean that policymakers need to consider not only the most likely future path for the economy but also the distribution of possible outcomes about that path.

Motivation

- Path forecasts are of interest for decision making: often we care about $\{\hat{y}_{T+1}, \dots, \hat{y}_{T+H}\}$, not just $\{\hat{y}_{T+1}\}, \dots, \{\hat{y}_{T+H}\}$
- Path forecasts have a *joint predictive density* – there is correlation between $\{\hat{y}_{T+1}\}, \dots, \{\hat{y}_{T+H}\}$.
- This has implications for:
 - Construction of Simultaneous Confidence Regions
 - Path Forecasting conditional on alternative scenarios
 - Path Forecast Model Comparison
 - Path Forecast Predictive Ability Testing

Chart 5.3 CPI inflation projection based on market interest rate expectations

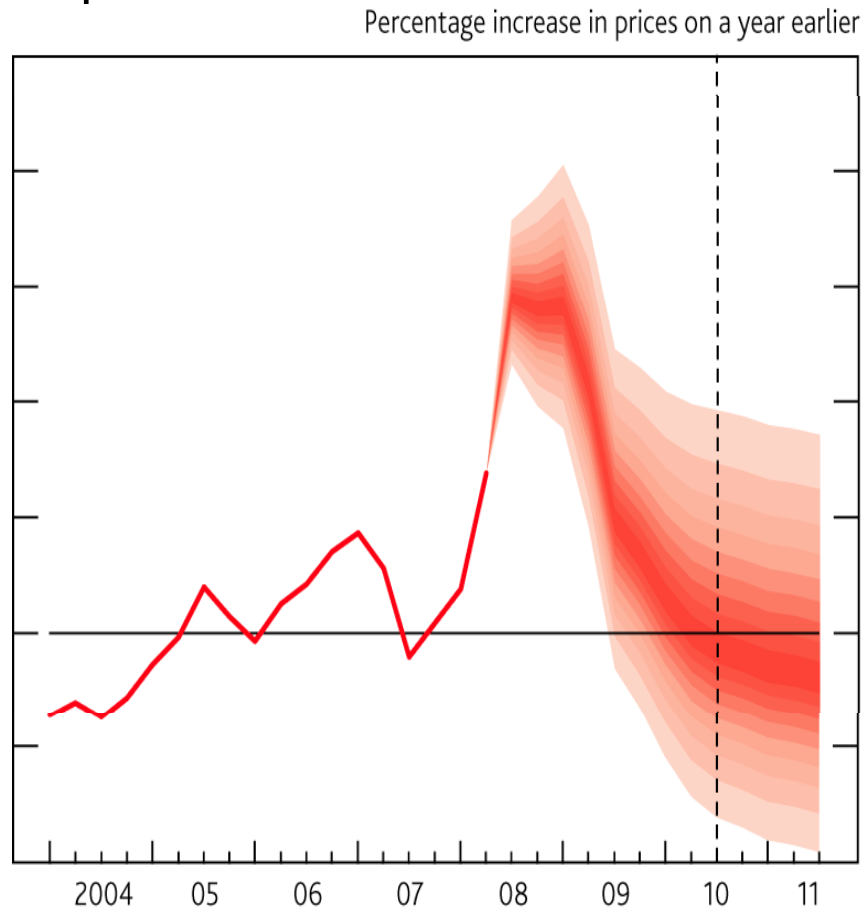
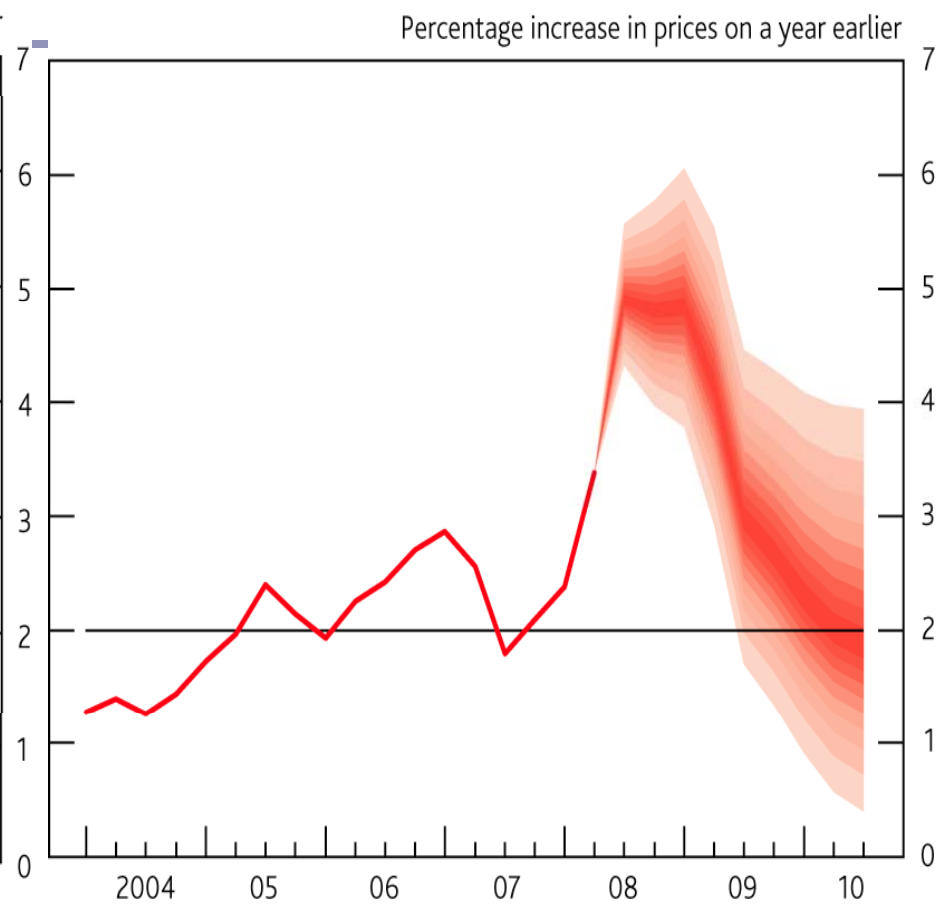


Chart 5.5 CPI inflation projection based on constant nominal interest rates at 5%



Bank of England Inflation Report: August 2008

Preview of Results

- Propose a method to construct simultaneous confidence regions based on Scheffé's (1953) S-method.
- These regions are meant to include the set of all possible *trajectories* with $(1 - \alpha)\%$ chance of happening.
- Provide Monte Carlo support.
- Provide measures of coherence and exogeneity to counterfactual forecasts.
- Empirical application to U.S. Macroeconomic data.

A Motivating Example

- Suppose

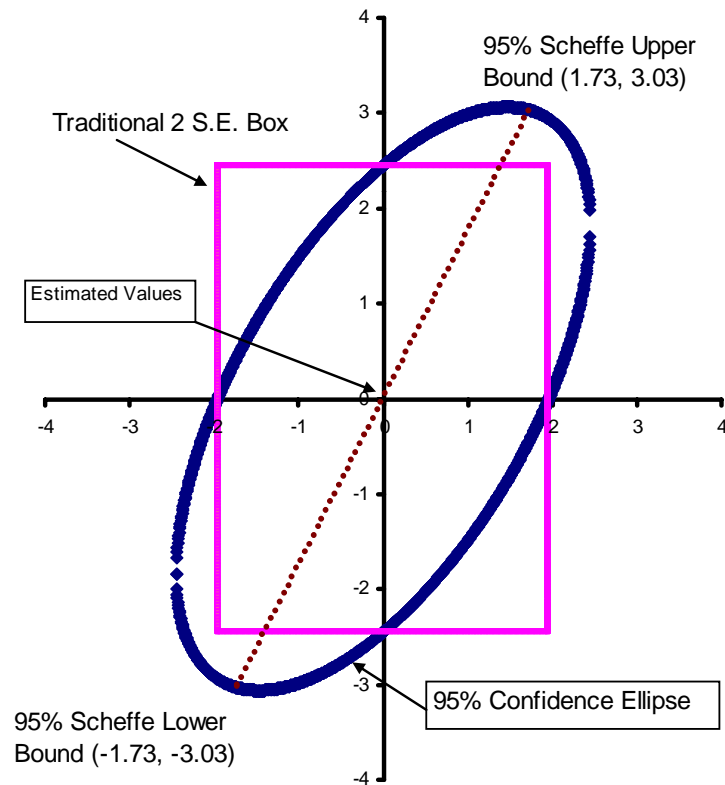
$$y_{t+1} = \rho y_t + \epsilon_{t+1}; \quad \epsilon_{t+1} \sim N(0, \sigma^2); \quad |\rho| < 1$$

- Then, two-period ahead forecast errors when ρ is known:

$$\begin{bmatrix} e_{t+1} \\ e_{t+2} \end{bmatrix} \sim N \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}; \sigma^2 \begin{bmatrix} 1 & \rho \\ \rho & 1 + \rho^2 \end{bmatrix} \right)$$

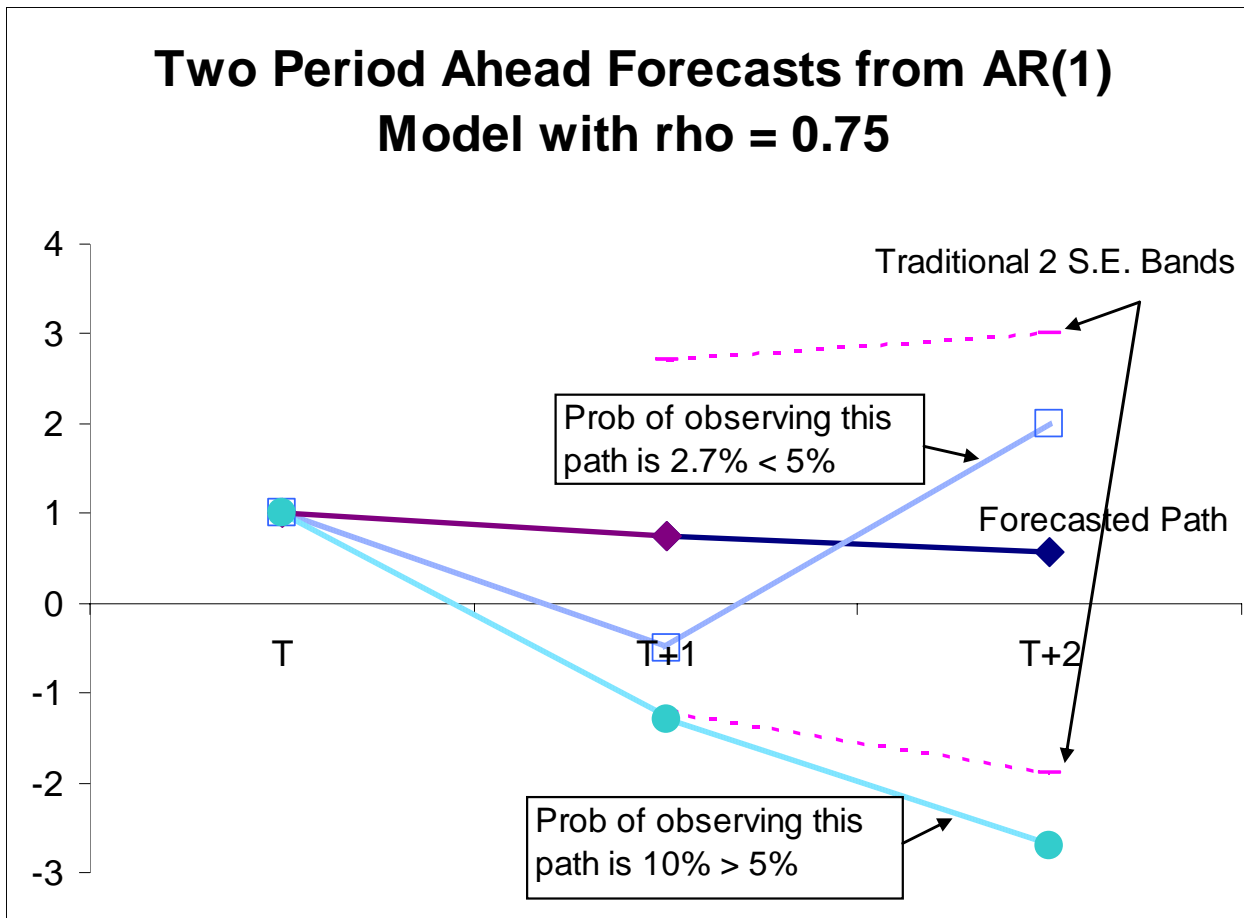
- Consider $\rho = 0.75$

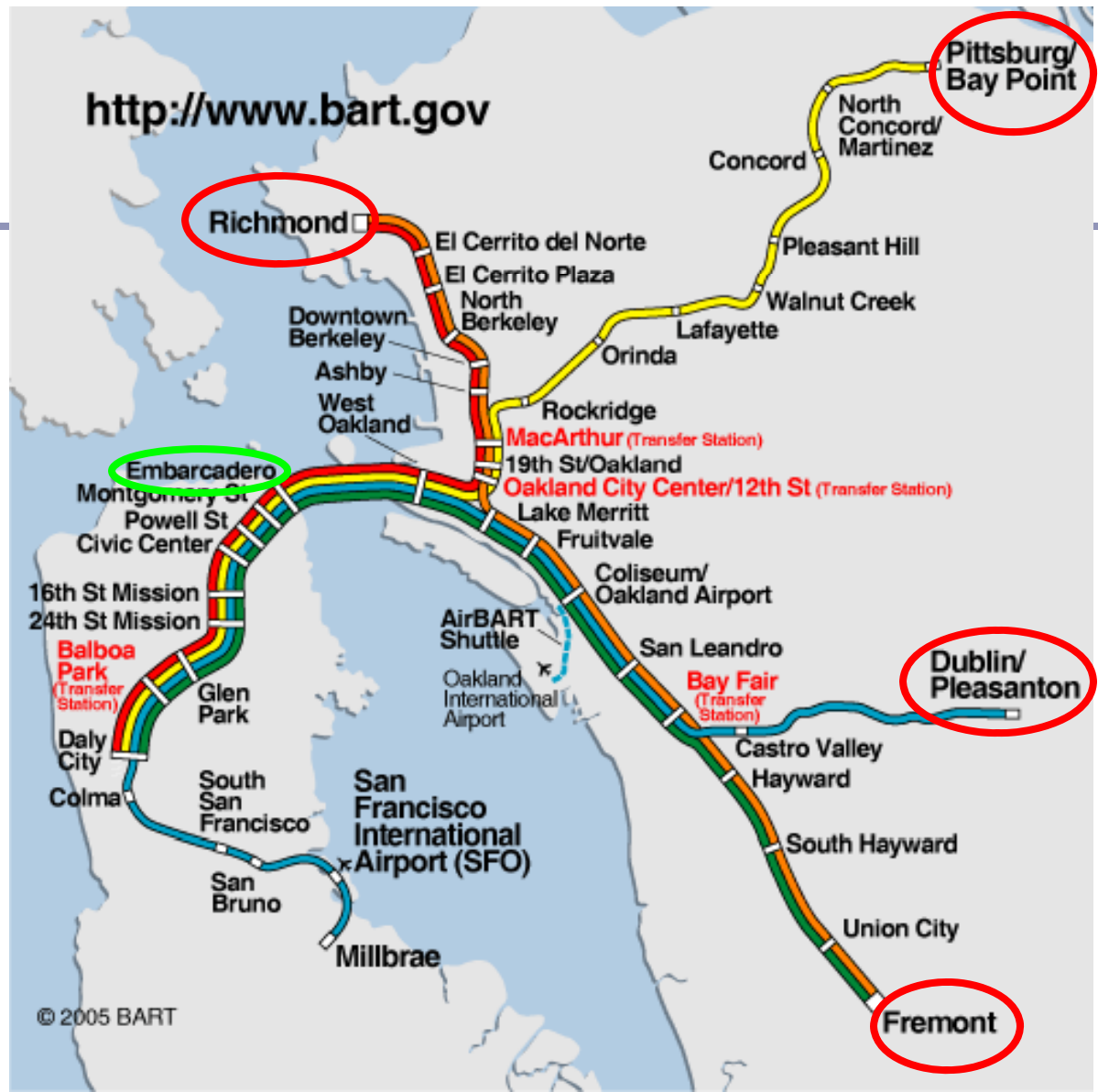
Simultaneous Confidence Ellipse



Implication

**Two Period Ahead Forecasts from AR(1)
Model with $\rho = 0.75$**





The Basics

- Let $\hat{y}_T(h)$ be the forecast for y_{T+h}
- Let $\hat{Y}_T(H)$ and $Y_{T,H}$ be the paths of the forecasts and the actual variable for $h = 1, \dots, H$
- For simplicity, suppose that

$$\sqrt{T} \left(\hat{Y}_T(H) - Y_{T,H} \right) \xrightarrow{d} N(0, \Xi_H)$$

More Basics

- Traditional error bands are the inverse of the decision problem associated with the sequence of significance t-statistics for each horizon h
- A simultaneous confidence region is instead the inverse of the decision problem associated with a *joint* significance test

The Joint Null

- A test of the null $H_0 : E(Y_{T,H}) = \mathbf{0}$ can be done with the Wald statistic:

$$W_H = T\hat{Y}_T(H)' \Xi_H^{-1} \hat{Y}_T(H) \xrightarrow{d} \chi_H^2$$

- And a confidence region can be constructed as

$$Pr [W_H \leq c_\alpha^2(H)] = 1 - \alpha$$

What are the issues?

- Traditional error bands ignore: (a) simultaneity; and (b) serial correlation in the forecasts.
- The simultaneous region from the Wald statistic is a multidimensional ellipse: cannot be displayed in two-dimensional space

Scheffé Confidence Regions

- Let $T^{-1}\Xi_H = PP'$ with P lower triangular
- Hence:

$$\left\{ \begin{array}{l} Pr \left[T\hat{Y}_T(H)' \Xi_H^{-1} \hat{Y}_T(H) \leq c_\alpha^2 \right] = 1 - \alpha \\ Pr \left[\hat{Y}_T(H)' (PP')^{-1} \hat{Y}_T(H) \leq c_\alpha^2 \right] = 1 - \alpha \\ Pr \left[\hat{V}_T(H)' \hat{V}_T(H) \leq c_\alpha^2 \right] = 1 - \alpha \\ Pr \left[\sum_{h=1}^H \hat{v}_T(h)^2 \leq c_\alpha^2 \right] = 1 - \alpha \end{array} \right.$$

Remarks

- P is a unique Cholesky decomposition of the path's covariance matrix because the forecast horizon is *ordered*
- Notice $\hat{v}_T(h) \rightarrow N(0, 1)$ and *independent* across h .
- The joint test is now a sum of squares of conditional t-tests
- We transformed the ellipse into a sphere – not quite there yet...

AR(1) Example

- The Cholesky decomposition of the forecast path's covariance matrix is

$$\sigma^2 \begin{bmatrix} 1 & \rho \\ \rho & 1 + \rho^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \rho & 1 \end{bmatrix} \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{bmatrix} \begin{bmatrix} 1 & \rho \\ 0 & 1 \end{bmatrix}$$

- and hence the orthogonal path's covariance matrix is

$$\begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{bmatrix}$$

Scheffé's S-Method

- Objective: to construct the rectangular region

$$\Pr \left[\left| \sum_{h=1}^H \frac{\hat{v}_T(h)}{h} \right| \leq \delta_\alpha \right] = 1 - \alpha$$

- So we need to find what δ_α is ...

Scheffé's S-Method (continued)

- From Bowden's (1970) lemma

$$\max \left\{ \frac{\left| \sum_{h=1}^H \frac{\hat{v}_T(h)}{h} \right|}{\sqrt{\sum_{h=1}^H \frac{1}{h^2}}} : |h| < \infty \right\} = \sqrt{\sum_{h=1}^H \hat{v}_T(h)^2}$$

which applied to the orthogonalized Wald statistic means that

$$\delta_\alpha = \sqrt{\frac{c_\alpha^2(H)}{H}}$$

Scheffé Bands

- Combining results, and using Holm's (1979) sequential procedure, we arrive at the construction of Scheffé bands:

$$\widehat{Y}_T(H) \pm P \left[\sqrt{\frac{c_\alpha^2(h)}{h}} \right]_{h=1, \dots, H}$$

...where the term in brackets is an $H \times 1$ vector with typical entry

$$\sqrt{\frac{c_\alpha^2(h)}{h}}$$

Intuition

- Suppose you displaced each estimate in the path in proportion to its variance:

$$P \left[\delta_{\alpha}^2 + \frac{H}{\dots} + \delta_{\alpha}^2 = c_{\alpha}^2 \right] = 1 - \alpha; \delta_{\alpha} = \sqrt{\frac{c_{\alpha}^2}{H}}$$

- E.g. AR(1) example:
 - Marginal Error Bands: ± 1.96 ; ± 2.45
 - Scheffé Error Bands: ± 1.73 ; ± 3.03

A Catalogue of Bands

- Marginal (traditional):

$$\hat{Y}_T(H) \pm z_{\alpha/2} \times \text{diag}(\Xi_H)^{1/2}$$

- Bonferroni:

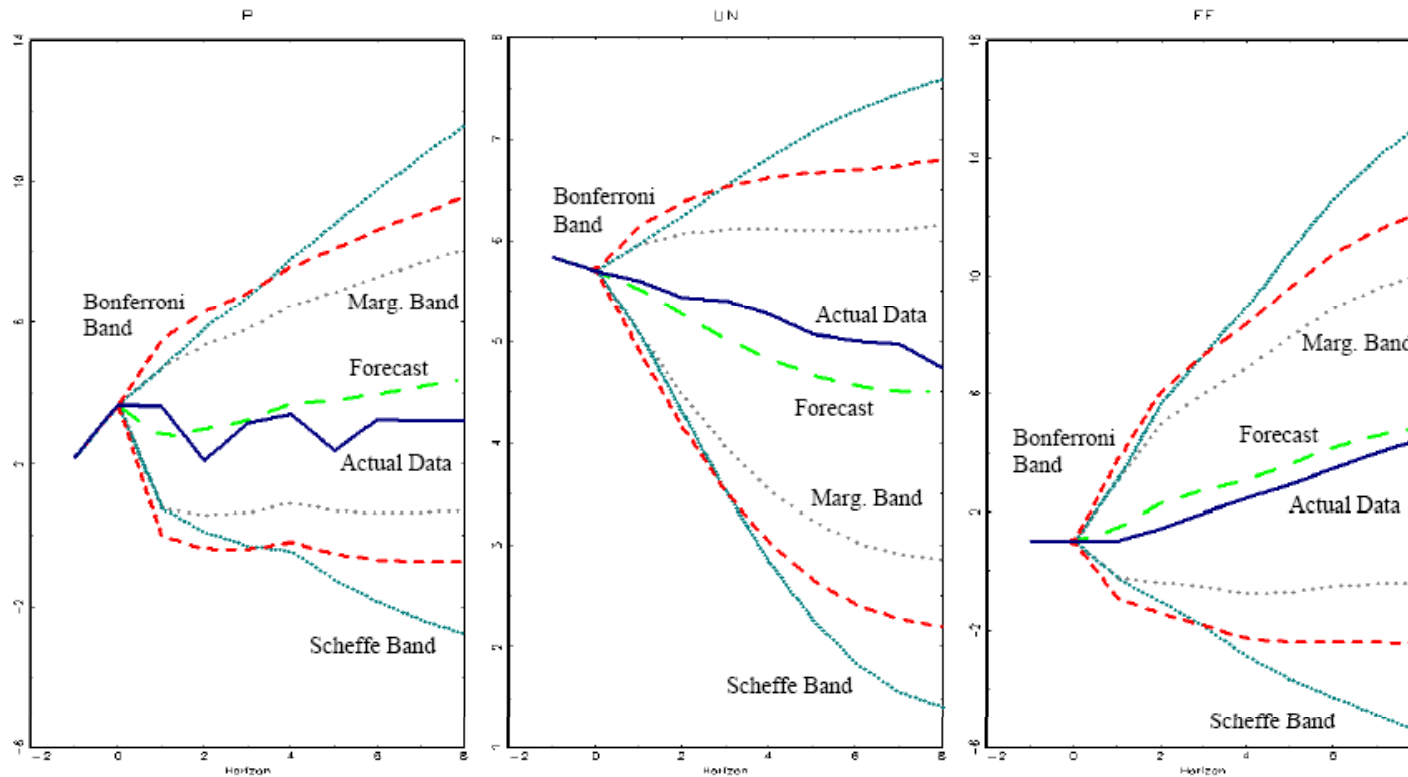
$$\hat{Y}_T(H) \pm z_{\alpha/2H} \times \text{diag}(\Xi_H)^{1/2}$$

- Scheffé:

$$\hat{Y}_T(H) \pm P \left[\sqrt{\frac{c_\alpha^2(h)}{h}} \right]_{h=1, \dots, H}$$

A Picture of the Catalogue

Figure 3. Stock and Watson (2001) Out-of-Sample Forecasts, 8-periods Ahead



Small Sample Coverage of the Bands

- Monte Carlo Set-up:
 - Stock and Watson (2001) VAR: inflation (chain weighted GDP index); unemployment; federal funds rate.
 - 1960:I-2007:I (quarterly, 188 obs)
 - 4 lags
 - Save the last 3 years for out-of-sample experiments
 - Estimate and use the parameters to generate data

Monte Carlo Set-up (continued)

- $T = 100$ and 400
- 1,000 replications
- Automatic lag-length choice with AICC
- Marginal, Bonferroni and Scheffé bands
- Two metrics: proportion of paths completely inside the bands (FWER); proportion of paths within joint Wald score (WALD) – related to FDR

The Tables

Forecast Horizon: 1

		Nominal Coverage: 68%						Nominal Coverage: 95%					
		FWER			WALD			FWER			WALD		
		Marg.	Bonf.	Schef.	Marg.	Bonf.	Schef.	Marg.	Bonf.	Schef.	Marg.	Bonf.	Schef.
T=100	P	67.5	67.5	67.5	67.5	67.5	67.5	93.8	93.8	93.8	93.8	93.8	93.8
	UN	69.5	69.5	69.5	69.5	69.5	69.5	95.8	95.8	95.8	95.8	95.8	95.8
	FF	68.4	68.4	68.4	68.4	68.4	68.4	94.6	94.6	94.6	94.6	94.6	94.6
T=400	P	66.9	66.9	66.9	66.9	66.9	66.9	93.6	93.6	93.6	93.6	93.6	93.6
	UN	69.7	69.7	69.7	69.7	69.7	69.7	96.0	96.0	96.0	96.0	96.0	96.0

Forecast Horizon: 12

		Nominal Coverage: 68%						Nominal Coverage: 95%					
		FWER			WALD			FWER			WALD		
		Marg.	Bonf.	Schef.	Marg.	Bonf.	Schef.	Marg.	Bonf.	Schef.	Marg.	Bonf.	Schef.
T=100	P	12.4	84.2	57.2	0.2	37.3	61.7	74.2	96.2	91.9	21.5	77.5	92.8
	UN	19.1	85.7	62.1	0.4	66.3	66.1	77.0	96.4	92.3	46.6	93.7	96.1
	FF	15.7	85.0	62.4	0.1	39.9	64.9	76.2	96.4	93.1	22.2	81.3	95.4
T=400	P	9.1	83.3	55.5	0.1	37.0	65.8	72.5	96.6	92.2	19.9	81.4	96.0
	UN	18.2	86.4	63.1	0.2	71.2	69.4	77.2	97.0	93.3	49.7	96.8	97.7
	FF	14.8	85.9	62.5	0.1	40.0	68.4	77.2	97.5	93.6	21.3	84.7	97.7

An AR(1) Monte Carlo

- Simulate data from a simple AR(1) for different values of the AR coefficient.
- The higher the AR parameter, the more correlated the forecasts are.
- The exercise provides a good rule-of-thumb

More Tables

Table 3. Coverage Rates of Marginal, Bonferroni, and Scheffé Bands in Simple AR(1) Model

Nominal Coverage Level: 68%										
Horizon = 1						Horizon = 4				
FWER	$\rho = 0.5$	$\rho = 0.6$	$\rho = 0.7$	$\rho = 0.8$	$\rho = 0.9$	$\rho = 0.5$	$\rho = 0.6$	$\rho = 0.7$	$\rho = 0.8$	$\rho = 0.9$
Marg.	68	67.5	67.8	68.5	68.2	27.3	28.3	28.1	34.2	33.2
Bonf.	68	67.5	67.8	68.5	68.2	73.8	77.3	75	79	77.8
Schef.	68	67.5	67.8	68.5	68.2	53	54.3	55.9	62.3	60.6
WALD	$\rho = 0.5$	$\rho = 0.6$	$\rho = 0.7$	$\rho = 0.8$	$\rho = 0.9$	$\rho = 0.5$	$\rho = 0.6$	$\rho = 0.7$	$\rho = 0.8$	$\rho = 0.9$
Marg.	68	67.5	67.8	68.5	68.2	26.3	24.7	20.5	20.6	15.5
Bonf.	68	67.5	67.8	68.5	68.2	83	79	72.6	69.1	61.8
Schef.	68	67.5	67.8	68.5	68.2	67.4	69.2	65.7	68.2	66.6
Nominal Coverage Level: 68%										
Horizon = 8						Horizon = 12				
FWER	$\rho = 0.5$	$\rho = 0.6$	$\rho = 0.7$	$\rho = 0.8$	$\rho = 0.9$	$\rho = 0.5$	$\rho = 0.6$	$\rho = 0.7$	$\rho = 0.8$	$\rho = 0.9$
Marg.	6.8	8.9	9.9	15.4	22.4	1.8	3.6	3.8	6.6	11.6
Bonf.	76.8	76	76.1	79.5	82.8	74.6	75.2	79.4	80.5	85.3
Schef.	42.3	49.6	52.3	59.8	59.3	33.3	42.2	53.4	59.5	59.5
WALD	$\rho = 0.5$	$\rho = 0.6$	$\rho = 0.7$	$\rho = 0.8$	$\rho = 0.9$	$\rho = 0.5$	$\rho = 0.6$	$\rho = 0.7$	$\rho = 0.8$	$\rho = 0.9$
Marg.	8.2	6.2	3.1	2.2	1	3.3	1.7	0.4	0.3	0.1
Bonf.	93	85.4	71.9	62.7	50.7	98	90.6	78	56.1	37.1
Schef.	69.4	68.4	65.9	68.3	69.2	69.2	67.2	69.6	68.5	69.9

... and then some more

Nominal Coverage Level: 95%										
Horizon = 1						Horizon = 4				
FWER	$\rho = 0.5$	$\rho = 0.6$	$\rho = 0.7$	$\rho = 0.8$	$\rho = 0.9$	$\rho = 0.5$	$\rho = 0.6$	$\rho = 0.7$	$\rho = 0.8$	$\rho = 0.9$
Marg.	94.7	94.8	95.3	96.2	94.4	82.6	85.9	84.1	86.7	84.2
Bonf.	94.7	94.8	95.3	96.2	94.4	95.2	95.5	95.9	96.6	95.5
Schef.	94.7	94.8	95.3	96.2	94.4	90.4	93.3	93.8	95.2	92.9
WALD	$\rho = 0.5$	$\rho = 0.6$	$\rho = 0.7$	$\rho = 0.8$	$\rho = 0.9$	$\rho = 0.5$	$\rho = 0.6$	$\rho = 0.7$	$\rho = 0.8$	$\rho = 0.9$
Marg.	94.7	94.8	95.3	96.2	94.4	91	87.7	83.3	80.2	74.9
Bonf.	94.7	94.8	95.3	96.2	94.4	99.3	97.9	96.8	96.5	93.1
Schef.	94.7	94.8	95.3	96.2	94.4	98.1	97.2	97	97.5	95.3

Nominal Coverage Level: 95%										
Horizon = 8						Horizon = 12				
FWER	$\rho = 0.5$	$\rho = 0.6$	$\rho = 0.7$	$\rho = 0.8$	$\rho = 0.9$	$\rho = 0.5$	$\rho = 0.6$	$\rho = 0.7$	$\rho = 0.8$	$\rho = 0.9$
Marg.	72.6	70.5	71.2	75	79.4	57.8	59.1	64	69.9	75.7
Bonf.	95.7	95.3	96	92.4	97.6	95.4	95.6	95.3	96.8	97.2
Schef.	87.7	91.7	92.2	95.9	95.2	80.2	87.5	92.2	92.3	93.9
WALD	$\rho = 0.5$	$\rho = 0.6$	$\rho = 0.7$	$\rho = 0.8$	$\rho = 0.9$	$\rho = 0.5$	$\rho = 0.6$	$\rho = 0.7$	$\rho = 0.8$	$\rho = 0.9$
Marg.	89.1	80.3	65.4	56.3	43.7	89.1	72.9	55.8	34.5	18.7
Bonf.	99.9	99.6	97.8	92.4	90	100	99.8	98.8	93.2	81
Schef.	98.6	98.4	97.8	95.9	97.2	99.6	98.4	98.3	97.3	97.3

Internal Consistency of the Forecasting Exercise

- Scheffé bands are useful in communicating forecast uncertainty about the path of a particular variable
- Other times, it is of interest to stress the forecasting exercise by examining how variables in a system respond to the alternative paths of other variables (think of inflation forecasts based on different assumptions on future oil prices, etc.)
- Examples: the Bank of England reports inflation forecasts based on “interest rate market expectations” and forecasts based on “constant interest rate” paths

Chart 5.3 CPI inflation projection based on market interest rate expectations

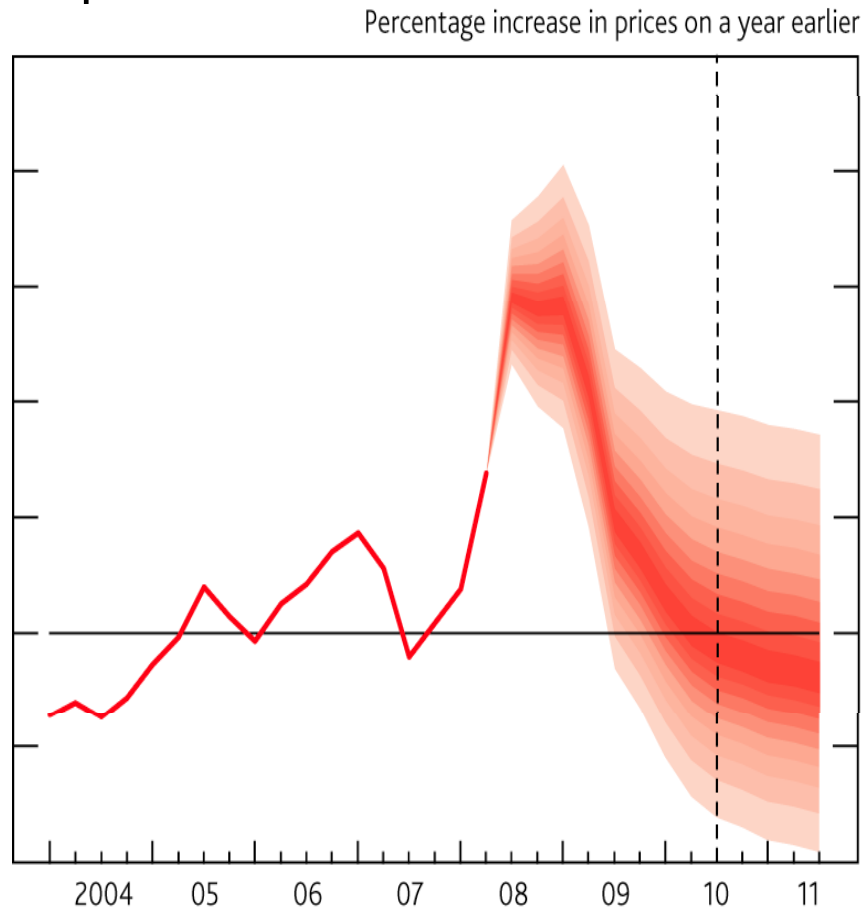
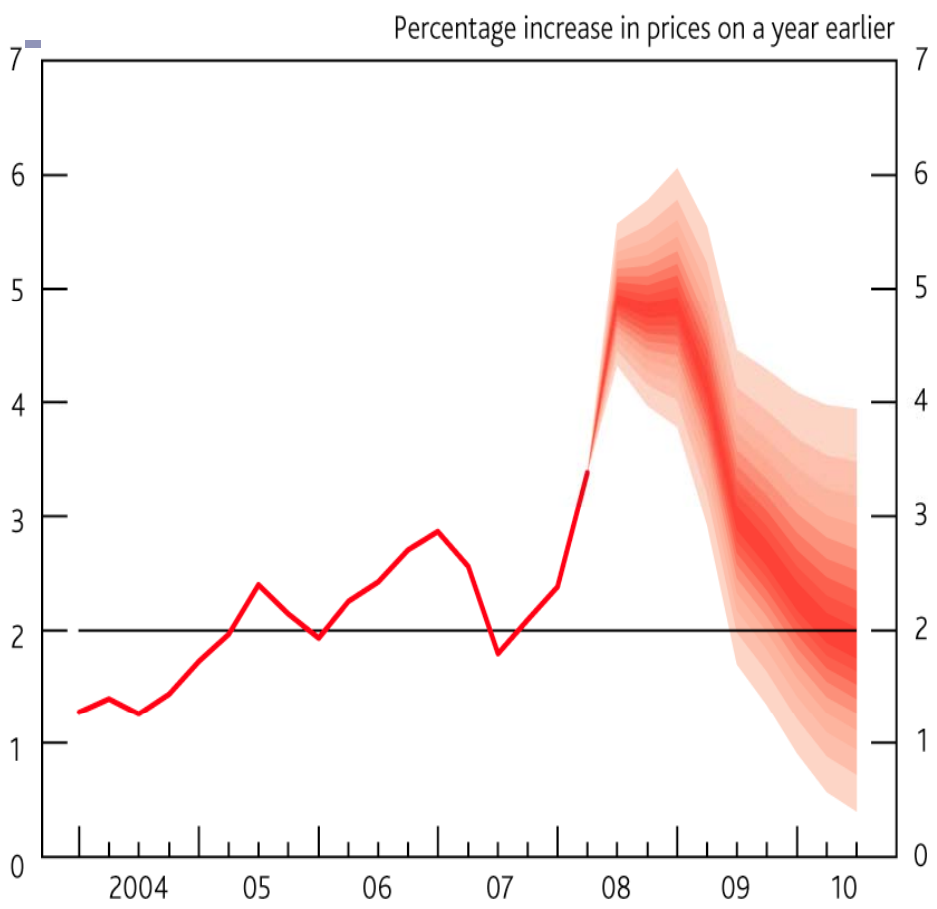


Chart 5.5 CPI inflation projection based on constant nominal interest rates at 5%



Bank of England Inflation Report: August 2008

Constructing Counterfactual Forecasts

- Easy - use projection formulas. It helps when forecasts paths are at least approximately multivariate Gaussian.
- Let $\hat{Y}_T(H)$ be an $kH \times 1$ vector of forecast paths of dimension H for k variables of interest.
- Assume that it is multivariate Gaussian (at least, approximately)

Computing Conditional Forecasts

- Recall: properties of the multivariate normal and linear projections

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{bmatrix} \sim N \left(\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}; \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix} \right)$$

$$\mathbf{y}_1 | \mathbf{y}_2 = \mathbf{y}_2^c \sim N \left(\mu_{1|2}; \Sigma_{11|2} \right)$$


$$\mu_{1|2} = \mu_1 + \Sigma_{12} \Sigma_{22}^{-1} (\mathbf{y}_2^c - \mu_2)$$

$$\Sigma_{11|2} = \Sigma_{11} - \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{21}$$

The Formulas

- Let S_0 and S_1 be selector matrices where the index 1 refers to the variables whose forecast paths we experiment with.
- Hence

$$\tilde{Y}_T^0(H) = S_0 \hat{Y}_T(H) + S_0 \Xi_H S_1' (S_1 \Xi_H S_1')^{-1} \left(\tilde{Y}_T^1(H) - S_1 \hat{Y}_T(H) \right)$$

Counterfactual


- With covariance matrix

$$\Xi_H^0 = S_0 \Xi_H S_0' - S_0 \Xi_H S_1' (S_1 \Xi_H S_1')^{-1} S_1 \Xi_H S_0$$

Two Useful Wald Scores

- *Coherence* of the counterfactual with the historical experience:

$$W_1 = T \left(S_1 \hat{Y}_T(H) - \tilde{Y}_T^1(H) \right)' (S_1 \Xi_H S_1')^{-1} (S_1 \hat{Y}_T(H) - \tilde{Y}_T^1(H))$$

- *Sensitivity* (exogeneity) to the counterfactual paths

$$W_0 = T \left(S_0 \hat{Y}_T(H) - \tilde{Y}_T^0(H) \right)' (S_0 \Xi_H S_0')^{-1} (S_0 \hat{Y}_T(H) - \tilde{Y}_T^0(H))$$

Large Sample Approximations

- Basic Assumptions:
 - Infinite order stationary VAR
 - Estimate a finite order VAR whose truncation lag grows at an appropriate rate along with the sample
 - Use either recursive formulas or direct methods to construct the path forecasts

Basic Result

$$\sqrt{\frac{T - p - H}{p}} \text{vec} \left(\hat{Y}_T(H) - Y_{T,H} \right) \xrightarrow{d} N(\mathbf{0}; \Xi_H)$$

$$\Xi_H = \left\{ \frac{p}{T - p - H} \Omega_H + \Psi_H \right\}$$

$$\Omega_H = \Phi(\mathbf{I}_H \otimes \Sigma_u) \Phi'$$

$$\Psi_H \equiv \frac{\partial \text{vec} \left(\hat{Y}_T(H) \right)}{\partial \text{vec} \left(\hat{\mathbf{A}} \right)} \Sigma_A \frac{\partial \text{vec} \left(\hat{Y}_T(H) \right)}{\partial \text{vec} \left(\hat{\mathbf{A}} \right)'}$$

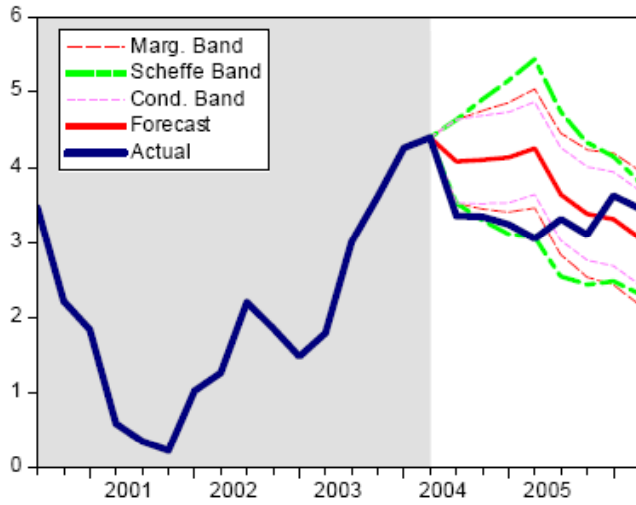
Application

- A macroeconomic, out-of-sample forecasting exercise on U.S. real GDP growth; (PCE) inflation; fed funds rate; 10-Year T-Bond (quarterly)
- Estimation sample: 1953:I-2004:I
- Forecast: 2004-II -2006:II
- Method: Direct Forecasts

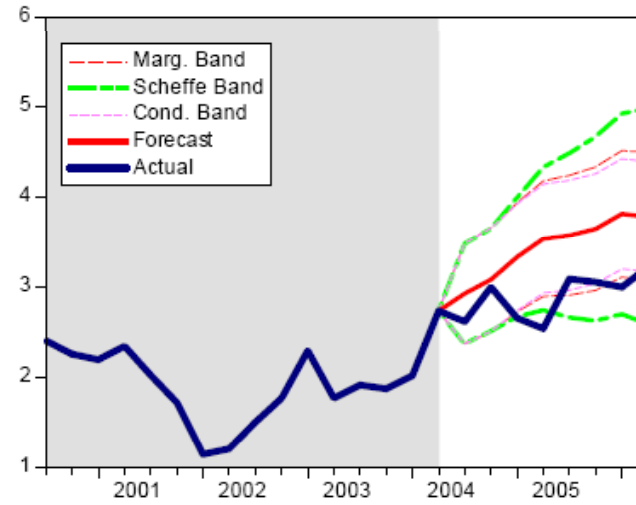
Background

- The fed funds rate had remained at 1% since November 6, 2002. The increase to 1.25% came in June 2004.
- Present forecasts on the eve of this increase about the macroeconomic outlook
- Experiment with some alternative scenarios

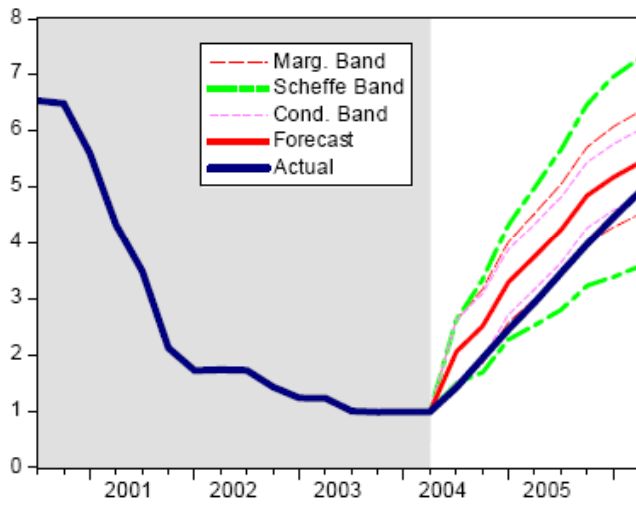
GDP Growth



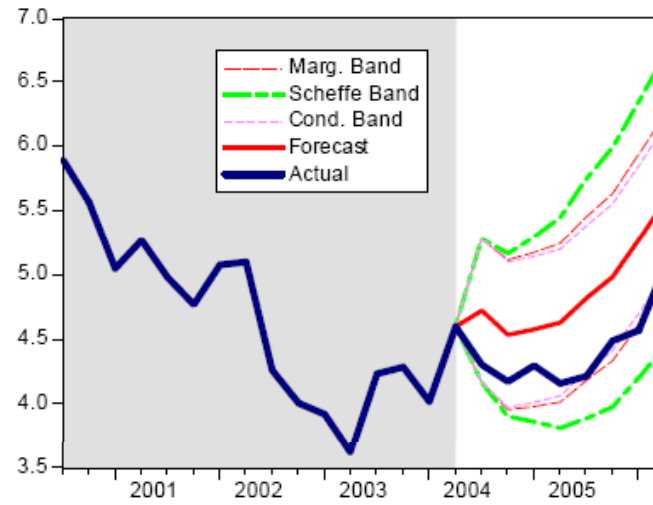
PCE Inflation



Federal Funds Rate



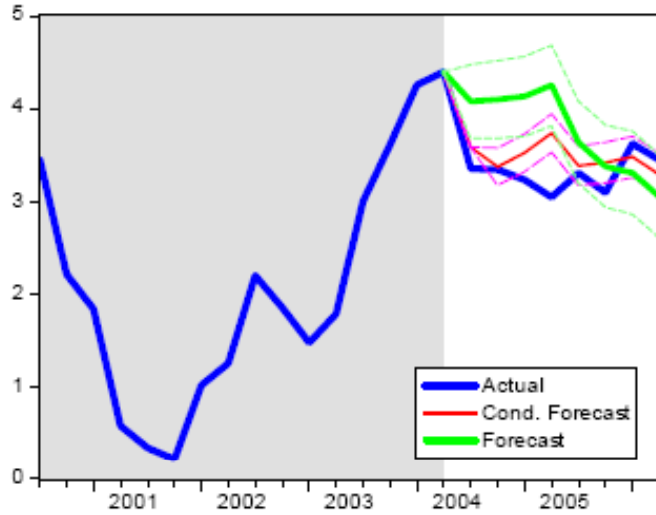
10 Year T-Bond



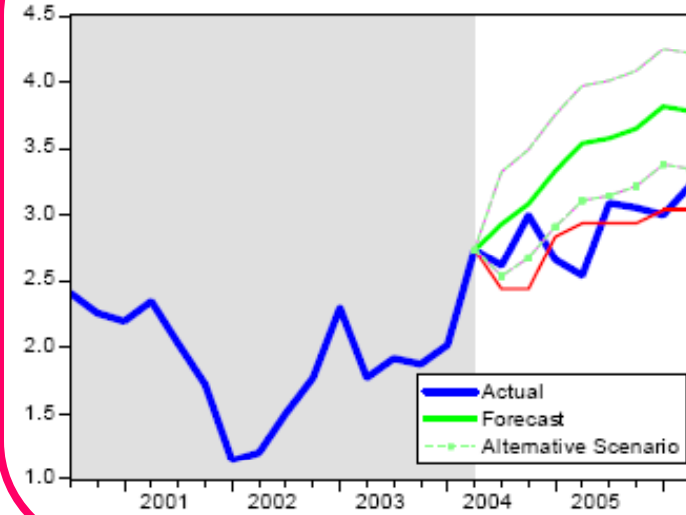
A Stress Test

- Consider a more benign path for inflation (the end of major operations in Iraq may have suggested more stability of oil prices would be forthcoming).
- Use the lower conditional band as the counterfactual: distance to the historical average according to W_1 (*coherence score*) is 29% in probability units – pretty reasonable.
- *Sensitivity score* of the remaining variables overwhelmingly rejected

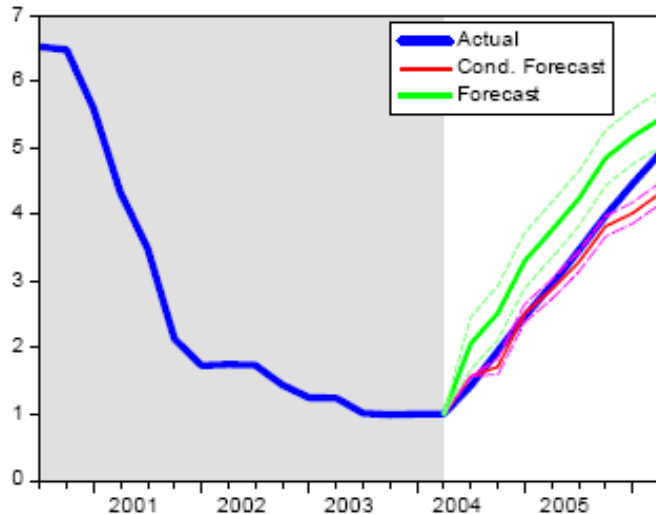
GDP Growth



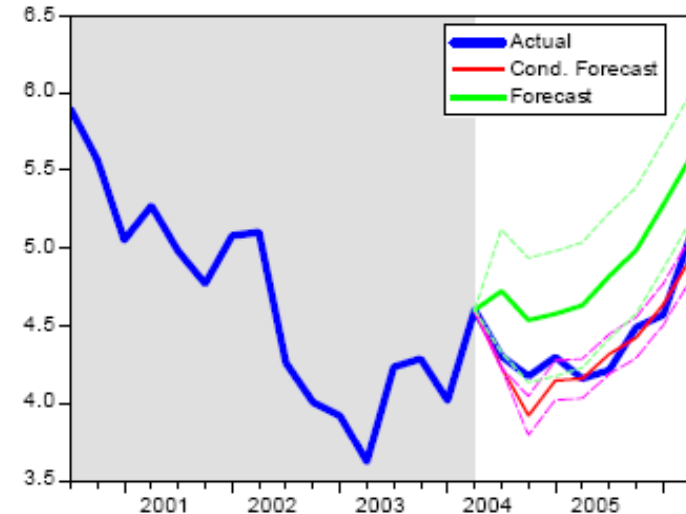
PCE Inflation



Federal Funds Rate



10 Year T-Bond



Not in the paper (but baking in the oven...)

Model Comparison: Mean Square Forecast Path

- Usual MSFE:

$$MSFE_h = \frac{1}{N} \sum_{i=1}^N (\hat{\mathbf{y}}_{T+i}(h) - \mathbf{y}_{T+i+h})' (\hat{\mathbf{y}}_{T+i}(h) - \mathbf{y}_{T+i+h})$$

forecast evaluation sample

- Notice that:

$$W_H = T \left(\hat{\mathbf{Y}}_T(H) - Y_{T,H} \right)' \Xi_H^{-1} \left(\hat{\mathbf{Y}}_T(H) - Y_{T,H} \right) \xrightarrow{d} \chi_{kH}^2$$

- Define:

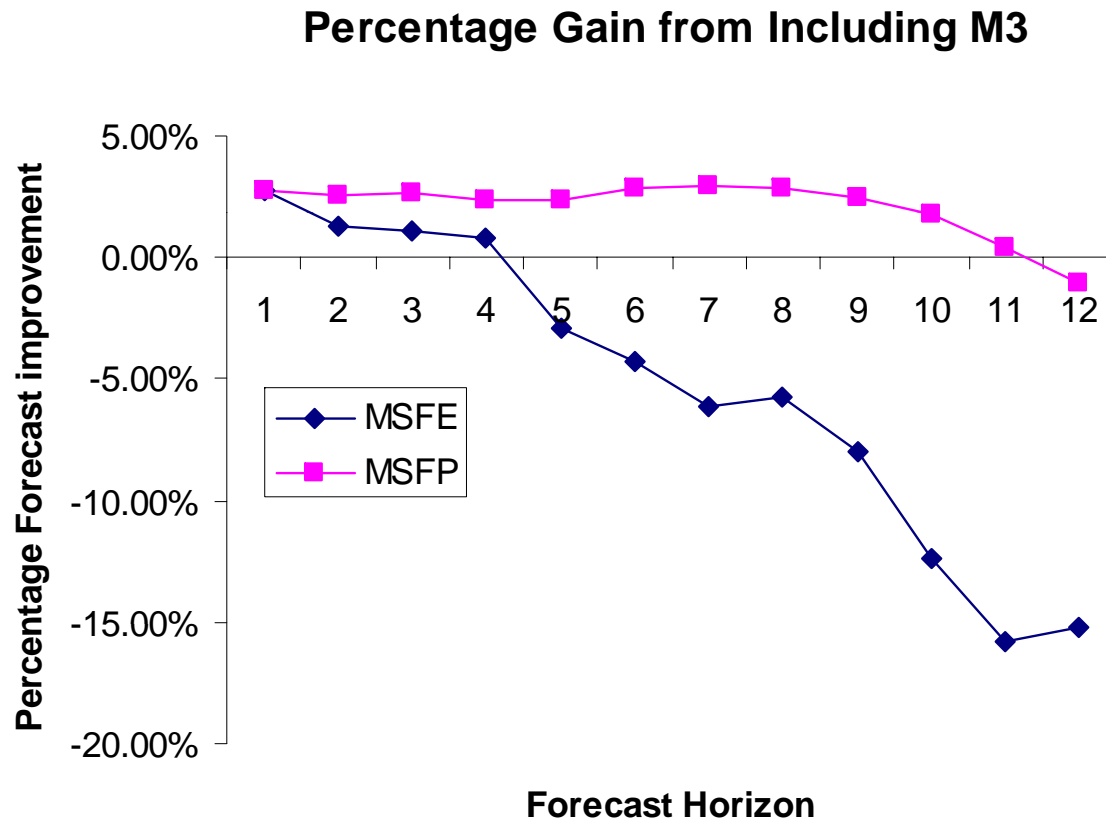
Correlation Matrix

$$MSFP_{1,H} = \left(\frac{1}{HN} \sum_{i=1}^N \left(\hat{\mathbf{Y}}_{T+i}(H) - Y_{T+i,H} \right)' \hat{\Lambda}_H^{-1} \left(\hat{\mathbf{Y}}_{T+i}(H) - Y_{T+i,H} \right) \right)$$

MSFP in AR(1) Example

$$\begin{aligned} MSFP_{1,2}^{AR} &= \frac{1}{2N} \sum_{i=1}^N \left(\hat{Y}_{T+i}(2) - Y_{T+i,2} \right)' \begin{bmatrix} 1 & \rho \\ \rho & 1 + \rho^2 \end{bmatrix}^{-1} \left(\hat{Y}_{T+i}(2) - Y_{T+i,2} \right) \\ &= \frac{1}{2N} \sum_{i=1}^N \left(\begin{array}{cc} \varepsilon_{T+i+1} & \varepsilon_{T+i+2} + \rho\varepsilon_{T+i+1} \end{array} \right) \begin{bmatrix} 1 & \rho \\ \rho & 1 + \rho^2 \end{bmatrix}^{-1} \left(\begin{array}{c} \varepsilon_{T+i+1} \\ \varepsilon_{T+i+2} + \rho\varepsilon_{T+i+1} \end{array} \right) \\ &= \frac{1}{2N} \sum_{i=1}^N (\varepsilon_{T+i+1}^2 + \varepsilon_{T+i+2}^2) \xrightarrow{p} \sigma^2 \text{ as } N \rightarrow \infty \end{aligned}$$

Can Monetary Aggregates Predict Inflation?



Path Predictive Ability: Diebold-Mariano-West

- Null: $H_0 : p \lim \left[\overline{\widehat{Y}}_T^1(H) - \overline{\widehat{Y}}_T^2(H) \right] = \mathbf{0}, N \rightarrow \infty$

- where: $\overline{\widehat{Y}}_T^j(H) = \frac{1}{N} \sum_{i=1}^N \widehat{Y}_{T+i}^j(H); j = 1, 2$

- What we need:

$$\sqrt{N} \text{vec} \left[\overline{\widehat{Y}}_T^1(H) - \overline{\widehat{Y}}_T^2(H) \right] \xrightarrow{d} N(0; \Upsilon_H)$$

Path Diebold-Mariano-West Test

$$PDMW_H = N \left[\overline{\widehat{Y}}_T(H)^1 - \overline{\widehat{Y}}_T(H)^2 \right]' \Upsilon_H^{-1} \left[\overline{\widehat{Y}}_T(H)^1 - \overline{\widehat{Y}}_T(H)^2 \right] \rightarrow \chi_{kH}^2$$

- As estimate of Υ_H can be obtained from the forecast evaluation sample with a HAC covariance estimator
- PDMW is a Hausman Test

Conclusions

- We have provided measures of forecast uncertainty that emphasize uncertainty about a forecast path rather than individual elements in that path
- All the relevant information is contained in the multivariate density:
 - Scheffé bands
 - Coherence and sensitivity metrics for counterfactuals
- Depending on the relevant question, forecasting assessments based on marginal predictive densities can be very misleading
- We provide asymptotic results for two common estimation/forecasting methods for quite general DGPs.

Further Research

- Generalizations of mixing conditions; heterogeneity; and stationarity assumptions of all the theorems
- *Confidence tunnels* for evaluation of path dependent options
- Stress-testing of credit default models