A Contribution to the Empirics of Economic Growth

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QJE 1992

Taking Solow Seriously: Questions

- 1) Do the prediction of the Solow model hold qualitatively and quantitatively in explaining cross-countries differences in long-run income per person?

- 2) What if we add human capital to the explanatory variables?

- 3) Does the prediction of conditional convergence hold? At what speed?

Taking Solow Seriously: Answers

- 1) The simple Solow model has the right qualitative implication but cannot explain the quantitative differences well

- 2) Augmenting Solow with human capital explains most of the differences in cross-country income per person. Still there is an important role for differences in productivity.

- 3) Conditional convergence exists but is slow.

Basic Solow

$$Y = K^{\alpha}(AL)^{1-\alpha}$$

$$\frac{\dot{A}}{A} = g$$

$$\frac{\dot{L}}{L} = n$$

Usual capital dynamic equation:

$$\dot{\tilde{k}} = s\tilde{k}^{\alpha} - (n + g + \delta)\tilde{k}$$

In Balanced Growth Path

$$\tilde{y}^{*} = \left(\frac{s}{n + g + \delta} \right)^{\frac{\alpha}{1-\alpha}}$$

Explaining long-run income per person

$$\ln(Y/L) = \ln A(0) + gt + \frac{\alpha}{1-\alpha}\ln(s) - \frac{\alpha}{1-\alpha}\ln(n+g+\delta)$$

The above relation expresses the relation between income per person, investment rate, population growth and depreciation, assuming common values of A(0).

Issues:

a) Correlation between A(0) and savings-population growth
b) The model predicts the magnitudes of the coefficients, hence one can test the bias
c) The above equation can be used as “basis for cross-country accounting”.

Data

- Summers and Heston Data set, now Penn World Table, are used to measure variables
- 3 samples: all countries, Excluding those with a very poor data quality, OECD
- Averaged 1960-85
  - s=investment rate,
  - n=growth rate of the population.
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- The analysis has been updated several times with similar results

Textbook Solow

TABLE I
ESTIMATION OF THE TEXTBOOK SOLOW MODEL

Dependent variable: log GDP per working-age person in 1985			
Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	5.48	5.36	7.97
	(1.59)	(1.55)	(2.48)
ln(I/GDP)	1.42	1.31	0.50
	(0.14)	(0.17)	(0.43)
$\ln(n + g + \delta)$	−1.97	−2.01	−0.76
	(0.56)	(0.53)	(0.84)
$\bar{R}^2$	0.59	0.59	0.01
s.e.e.	0.69	0.61	0.38
Restricted regression:			
CONSTANT	6.87	7.10	8.62
	(0.12)	(0.15)	(0.53)
$\ln(I/GDP) - \ln(n + g + \delta)$	1.48	1.43	0.56
	(0.12)	(0.14)	(0.36)
$\bar{R}^2$	0.59	0.59	0.06
s.e.e.	0.69	0.61	0.37
Test of restriction:			
p-value	0.38	0.26	0.79
Implied α	0.60	0.59	0.36
	(0.02)	(0.02)	(0.15)

Note. Standard errors are in parentheses. The investment and population growth rates are averages for the period 1960–1985. $(g + \delta)$ is assumed to be 0.05.

How did the model perform?

- Success:
  - Signs of coefficient
  - Restriction of same magnitude and opposite sign
  - Explain 60-70% of variation across all countries

- Failures
  - Magnitude of coefficient, α is too large to be the share of physical capital
  - Explanatory power in OECD

Adding Human Capital

Production Function

$$Y(t) = K(t)^{\alpha} H(t)^{\beta} (A(t)L(t))^{1-\alpha-\beta},$$

Accumulation of capital and human capital

$$\dot{k}(t) = s_k y(t) - (n + g + \delta)k(t),$$
$$\dot{h}(t) = s_h y(t) - (n + g + \delta)h(t),$$

Balanced Growth Paths

$$k^* = \left(\frac{s_k^{1-\beta} s_h^{\beta}}{n + g + \delta} \right)^{1/(1-\alpha-\beta)}$$

$$h^* = \left(\frac{s_k^{\alpha} s_h^{1-\alpha}}{n + g + \delta} \right)^{1/(1-\alpha-\beta)}.$$

Estimable equation

$$\ln\left[\frac{Y(t)}{L(t)}\right] = \ln A(0) + gt - \frac{\alpha+\beta}{1-\alpha-\beta}\ln(n+g+\delta) + \frac{\alpha}{1-\alpha-\beta}\ln(s_k) + \frac{\beta}{1-\alpha-\beta}\ln(s_h).$$

S_h should measure investments in human capital
share of people in age 15-19 that is enrolled in school

Advantages of this equation
1) Human capital accumulation may explain some differences
2) Coefficient on population is higher
3) Phisical and human capital accumulation are related, explaining the upward bias of previous share estimates

Solow with Human Capital

TABLE II
ESTIMATION OF THE AUGMENTED SOLOW MODEL

Dependent variable: log GDP per working-age person in 1985			
Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	6.89	7.81	8.63
	(1.17)	(1.19)	(2.19)
ln(I/GDP)	0.69	0.70	0.28
	(0.13)	(0.15)	(0.39)
$\ln(n + g + \delta)$	−1.73	−1.50	−1.07
	(0.41)	(0.40)	(0.75)
ln(SCHOOL)	0.66	0.73	0.76
	(0.07)	(0.10)	(0.29)
$\bar{R}^2$	0.78	0.77	0.24
s.e.e.	0.51	0.45	0.33
Restricted regression:			
CONSTANT	7.86	7.97	8.71
	(0.14)	(0.15)	(0.47)
$\ln(I/GDP) - \ln(n + g + \delta)$	0.73	0.71	0.29
	(0.12)	(0.14)	(0.33)
$\ln(SCHOOL) - \ln(n + g + \delta)$	0.67	0.74	0.76
	(0.07)	(0.09)	(0.28)
$\bar{R}^2$	0.78	0.77	0.28
s.e.e.	0.51	0.45	0.32
Test of restriction:			
p-value	0.41	0.89	0.97
Implied α	0.31	0.29	0.14
	(0.04)	(0.05)	(0.15)
Implied β	0.28	0.30	0.37
	(0.03)	(0.04)	(0.12)

Note. Standard errors are in parentheses. The investment and population growth rates are averages for the period 1960–1985. $(g + \delta)$ is assumed to be 0.05. SCHOOL is the average percentage of the working-age population in secondary school for the period 1960–1985.

Conditional Convergence

Growth

Initial condition

$$\ln(y(t)) - \ln(y(0)) = (1 - e^{-\lambda t}) \ln(y^*) - (1 - e^{-\lambda t}) \ln(y(0)).$$

As we can calculate the BGP values of income per person we can also

1) Assess if there is conditional convergence
2) Calculate its speed

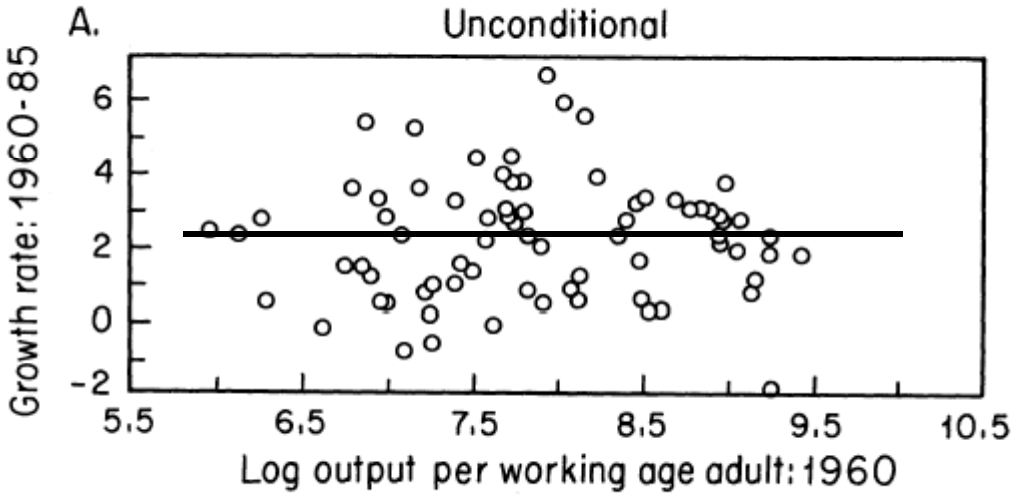
We can Test absolute convergence and conditional convergence

Absolute convergence

TABLE III
TESTS FOR UNCONDITIONAL CONVERGENCE

Dependent variable: log difference GDP per working-age person 1960–1985			
Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	−0.266	0.587	3.69
	(0.380)	(0.433)	(0.68)
ln(Y60)	0.0943	−0.00423	−0.341
	(0.0496)	(0.05484)	(0.079)
$\bar{R}^2$	0.03	−0.01	0.46
s.e.e.	0.44	0.41	0.18
Implied λ	−0.00360	0.00017	0.0167
	(0.00219)	(0.00218)	(0.0023)

Note. Standard errors are in parentheses. Y60 is GDP per working-age person in 1960.



Conditional convergence

TABLE V
TESTS FOR CONDITIONAL CONVERGENCE

Dependent variable: log difference GDP per working-age person 1960–1985			
Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	3.04	3.69	2.81
	(0.83)	(0.91)	(1.19)
ln(Y60)	−0.289	−0.366	−0.398
	(0.062)	(0.067)	(0.070)
ln(I/GDP)	0.524	0.538	0.335
	(0.087)	(0.102)	(0.174)
$\ln(n + g + \delta)$	−0.505	−0.551	−0.844
	(0.288)	(0.288)	(0.334)
ln(SCHOOL)	0.233	0.271	0.223
	(0.060)	(0.081)	(0.144)
$\bar{R}^2$	0.46	0.43	0.65
s.e.e.	0.33	0.30	0.15
Implied λ	0.0137	0.0182	0.0203
	(0.0019)	(0.0020)	(0.0020)

Note. Standard errors are in parentheses. Y60 is GDP per working-age person in 1960. The investment and population growth rates are averages for the period 1960–1985. $(g + \delta)$ is assumed to be 0.05. SCHOOL is the average percentage of the working-age population in secondary school for the period 1960–1985.

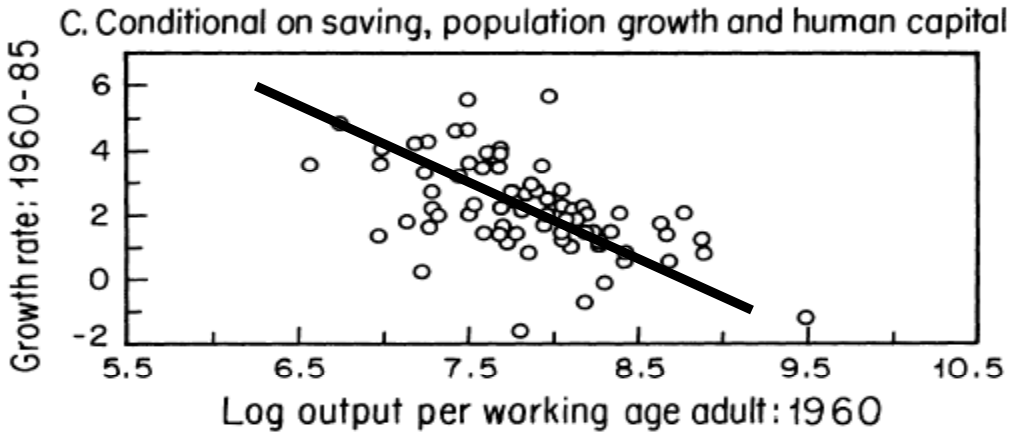


FIGURE I
Unconditional versus Conditional Convergence

Conclusions

- Solow model with a production function as

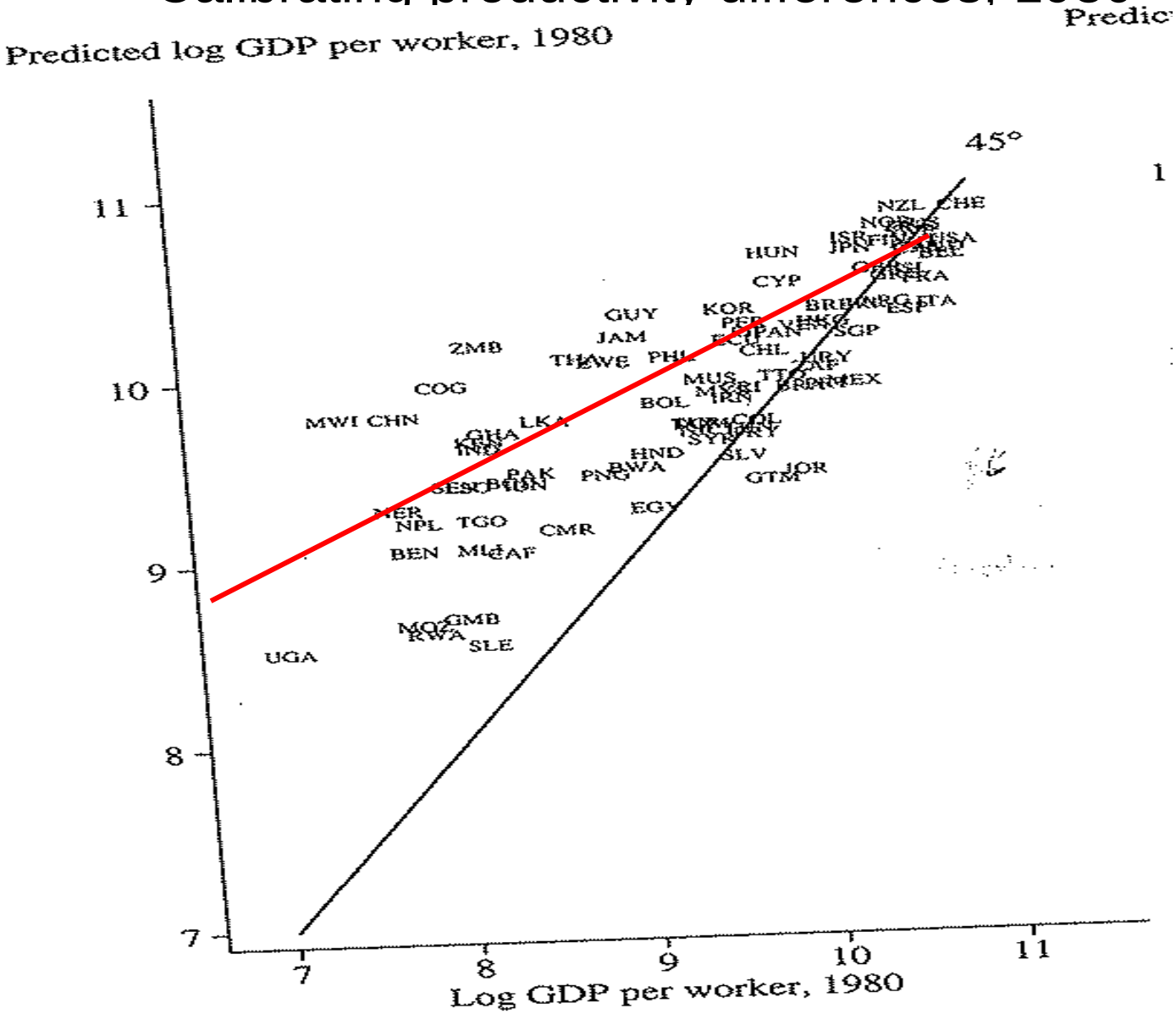
$$Y = K^{1/3} H^{1/3} L^{1/3}.$$

Explains most of the cross-country income variation

It also predicts conditional convergence, although at a rather slow rate. Speed of convergence is about 2% which I consistent with the Solow formula using human and physical capital.

What determines savings, education and population growth?

Calibrating productivity differences, 1980



Calibrating productivity differences, 2000

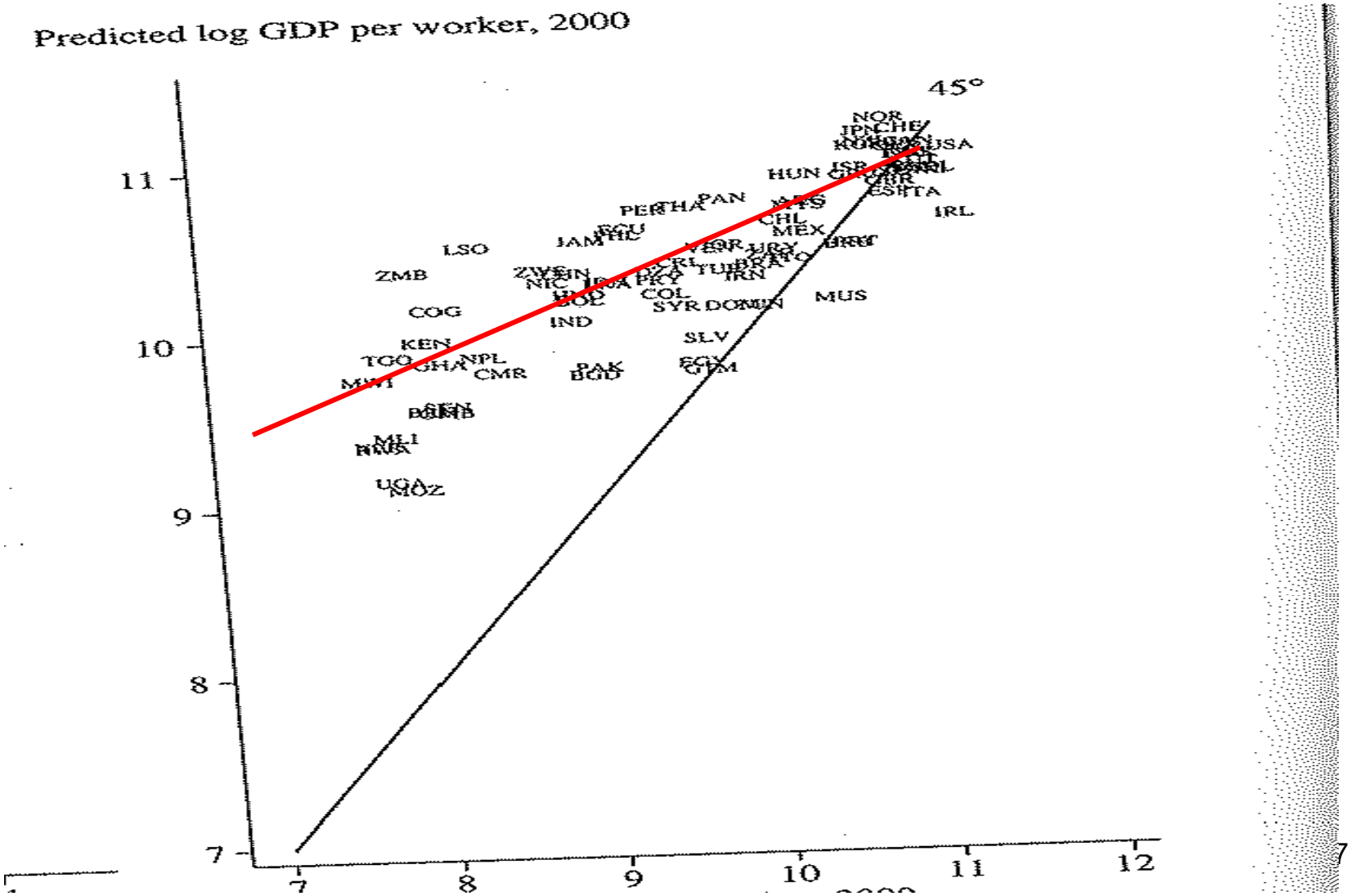
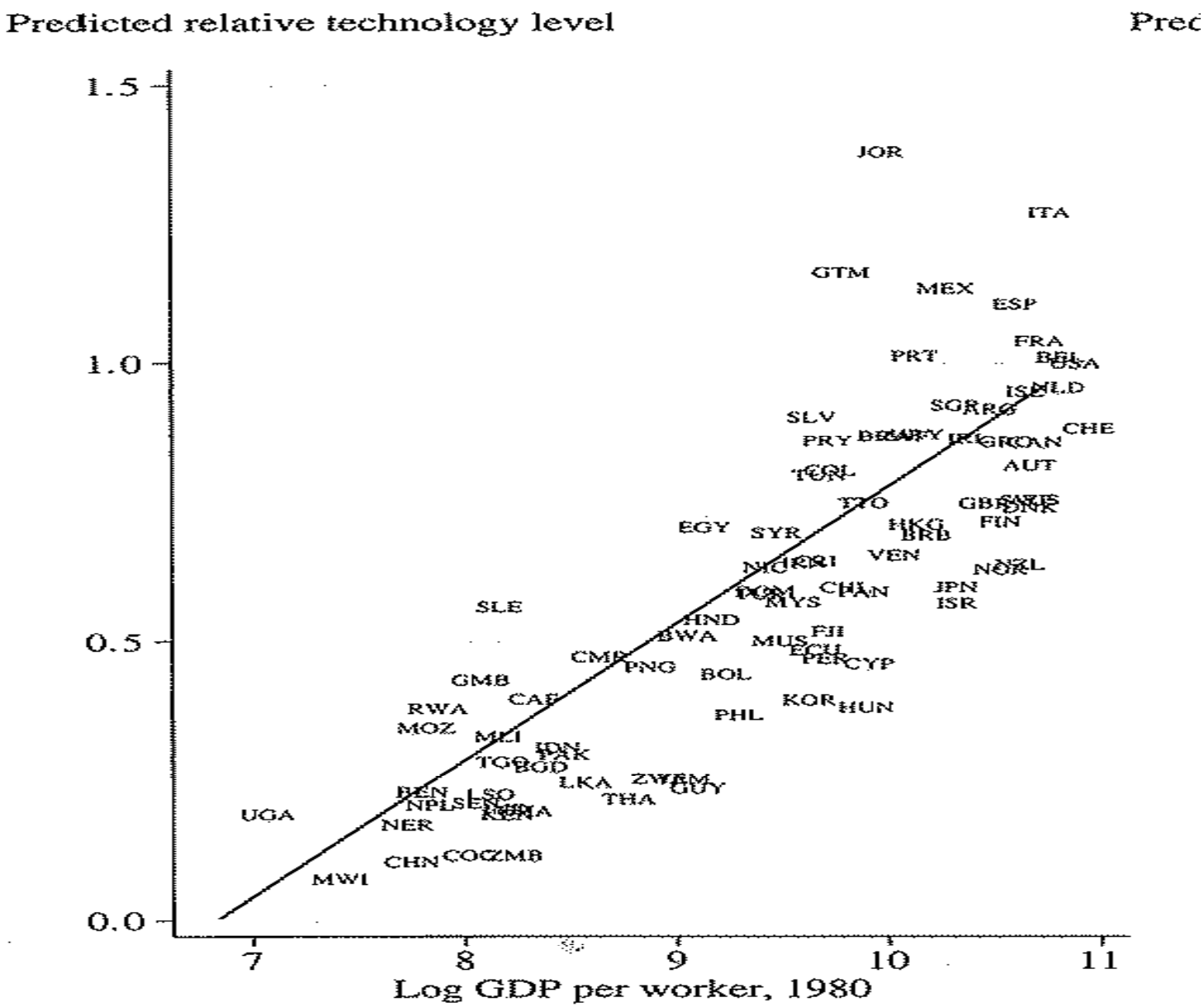


Table 1: Productivity Calculations: Ratios to U.S. Values

Country	Y/L	Contribution from $(K/Y)^{\alpha/(1-\alpha)}$	Contribution from H/L	Contribution from A
United States	1.000	1.000	1.000	1.000
Canada	0.941	1.002	0.908	1.034
Italy	0.834	1.063	0.650	1.207
West Germany	0.818	1.118	0.802	0.912
France	0.818	1.091	0.666	1.126
United Kingdom	0.727	0.891	0.808	1.011
Hong Kong	0.608	0.741	0.735	1.115
Singapore	0.606	1.021	0.545	1.078
Japan	0.587	1.119	0.797	0.658
Mexico	0.433	0.868	0.538	0.926
Argentina	0.418	0.953	0.676	0.648
U.S.S.R.	0.417	1.231	0.724	0.468
India	0.086	0.709	0.454	0.267
China	0.060	0.891	0.632	0.106
Kenya	0.056	0.747	0.457	0.165
Zaire	0.033	0.499	0.408	0.160
Average, 127 Countries:	0.296	0.853	0.565	0.516
Standard Deviation:	0.268	0.234	0.168	0.325
Correlation w/ Y/L (logs)	1.000	0.624	0.798	0.889
Correlation w/ A (logs)	0.889	0.248	0.522	1.000

Note: The elements of this table are the empirical counterparts to the components of equation (3), all measured as ratios to the U.S. values. That is, the first column of data is the product of the other three columns.


Predicted relative technology level
Prec
1.5
1.0
0.5
0.0
7
8
9
10
11
Log GDP per worker, 1980
JOR
ITA
GTM
MEX
ESP
FRA
PRT
SLV
PRY
CHE
AUT
EGY
SYR
FIN
VEN
JPN
ISR
SLE
HND
FJI
BWA
MUS
CMR
PNG
BOL
CYP
GMB
CAF
KOR
HUN
RWA
PHL
MOZ
MLI
IDN
PAK
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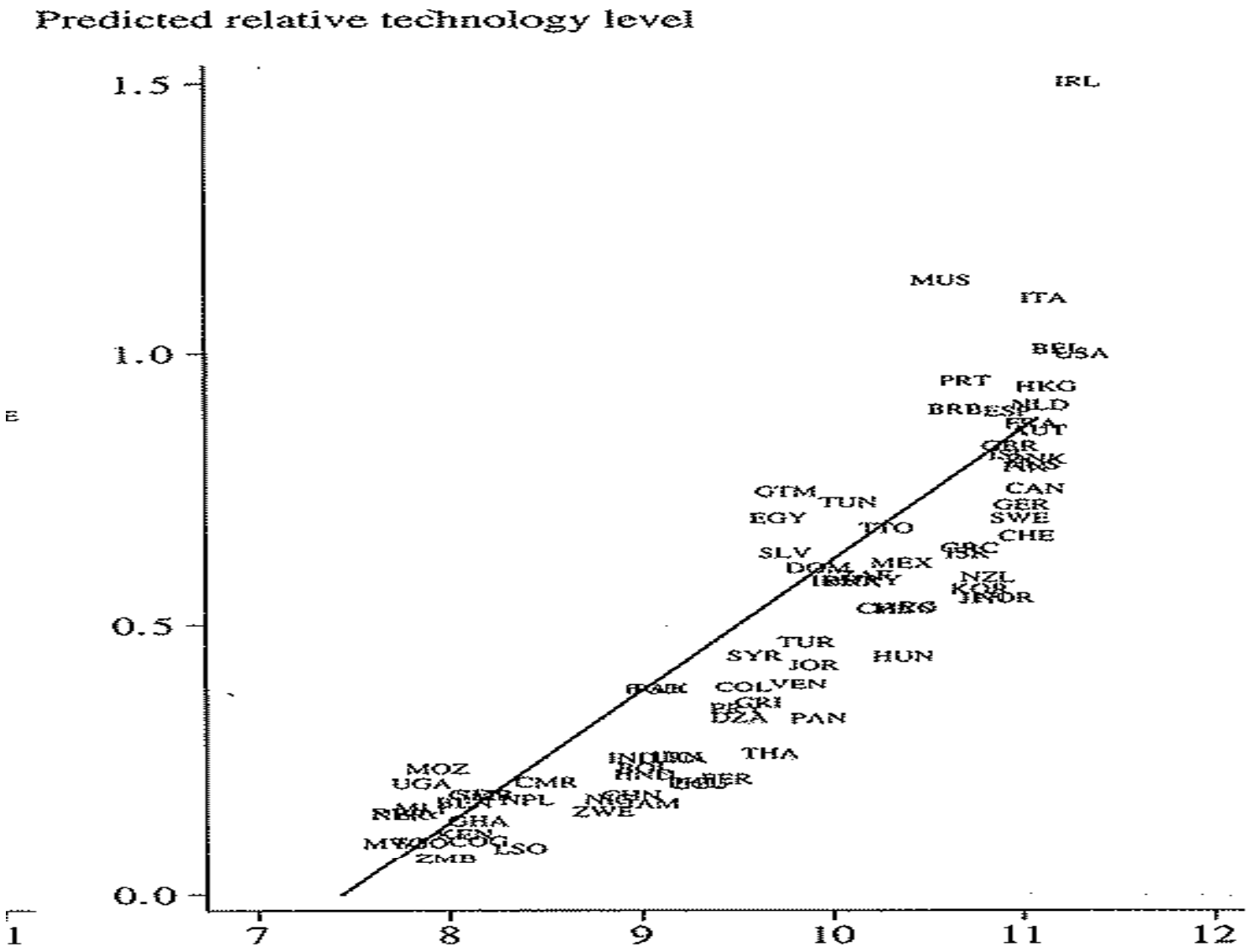
MWI

Year 2000


Predicted relative technology level
1.5
1.0
0.5
0.0
1
7
8
9
10
11
12
IRL
MUS
ITA
USA
PRT
HKG
CAN
GER
SWE
CHE
GTM
TUN
EGY
SLV
MEX
NZL
TUR
SYR
JOR
HUN
COL
VEN
PAN
DZA
THA
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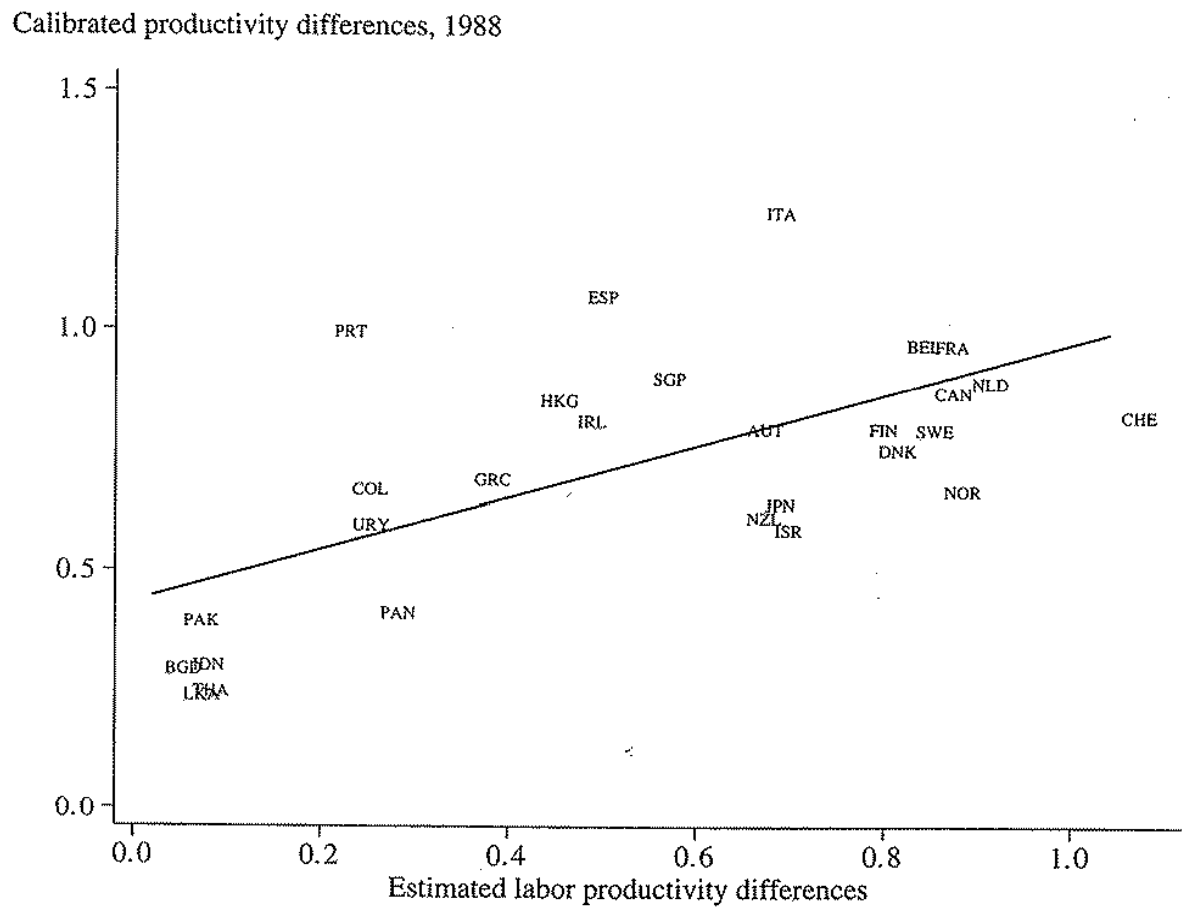
Comparing the Trefler (Trade-based) approach and the calibrated (Hall and Jones) approach in estimating labor productivity



Comparing the Trefler (Trade-based) approach and the calibrated (Hall and Jones) approach in estimating capital productivity