Tech. innovation through new goods ① Lect 9

1) Prod. f^n of final goods by perfectly competitive firms:

$$Y_i = A L_i^{1-\alpha} \sum_{j=1}^{N} (x_{ij})^{\alpha} \qquad \alpha < 1$$

The derived demand for intermediate x_i:

$$x_i = \underset{\substack{\uparrow \\ \text{Tot. labor force}}}{L} \left(\frac{\alpha A}{P_i}\right)^{\frac{1}{1-\alpha}}$$

$$w = (1-\alpha)\frac{Y}{L} \qquad \text{labor demand}$$

2) Producers of intermediates: ⇒ optimal price

$$\boxed{P_i = \frac{1}{\alpha}} \quad (\text{marginal cost} = 1)$$

$$\boxed{x_i = \alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} L}$$

$$\boxed{\pi_i = \pi = \frac{1-\alpha}{\alpha} \alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} L}$$

R&D decision: we simplify the uncertainty around it
cost of creating a new idea: it is deterministic
on average:

η units of output to produce 1 new idea

could be $\eta(N)$ $\quad \eta' > 0$ fishing out

$\eta' < 0$ standing on shoulders

Free entry into R&D, as long as firms make positive profits (in expected value) they keep entering

Call $V(t) \to$ value of an idea at time (t)

if $V(t) > \eta$ more firms enter

$V(t) < \eta$ no more innovation done

only if

$$V(t) = \int_t^{\infty} \pi_i \, e^{-r(s-t)} \, ds = \eta$$

there is R&D and production of intermediates

$$\pi_i = \overline{\pi}$$

$$\Rightarrow V(t) = V = -\frac{1}{r}\left[e^{-\infty} - e^{0}\right]\overline{\pi} = \boxed{\frac{\overline{\pi}}{r}}$$

so the equil cond is

$$\boxed{\frac{\overline{\pi}}{r} = \eta}$$

$$\frac{\frac{1-\alpha}{\alpha}\,\alpha^{\frac{2}{1-\alpha}}\,A^{\frac{1}{1-\alpha}}\,L}{r} = \eta$$

$$\boxed{r = \frac{L}{\eta} \cdot A^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)\alpha^{\frac{2}{1-\alpha}}} \to \text{from}$$

the mkt. interest rate is pinned down by the free-entry condition in research

It is constant over time if L constant $\Rightarrow$ or increasing in L grows

Consider the consumption side of the economy:

$$\max_{C(t)} \int_0^\infty \left[\frac{C^{1-\theta}-1}{1-\theta}\right] e^{-\rho t}\, dt$$

(no pop growth)

$$\dot{a} = wL + r\,a - c$$

Usual problem, solution

$$\boxed{\frac{\dot{C}}{C} = \frac{r-\rho}{\theta}}$$

What is the aggregate budget constraint?

$$x_i = \alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} L$$

$$X = \underbrace{N x_i}_{\substack{\text{Tot. value} \\ \text{of output used} \\ \text{to produce int. inputs}}} = \alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} LN \longrightarrow \boxed{\alpha^2 \cdot Y} \longrightarrow \underline{\text{PUT ASIDE}}$$

$$Y = AL^{1-\alpha} N x_i^{\alpha} \quad \text{subst. } x_i \quad = \boxed{A^{\frac{1}{1-\alpha}}\, \alpha^{\frac{2\alpha}{1-\alpha}}\, NL} \rightarrow \text{put Aside}$$

now: Value of 1 firm $= \eta$

Assets in this world are shares in firms

So

$$a = N\eta$$

$$\dot{a} = \dot{N}\eta$$

recall 4

$$w = (1-\alpha)\frac{Y}{L}$$

$$r = \frac{1}{\eta}(1-\alpha)\alpha\frac{Y}{N}$$

from $Y = A^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} NL$

so int. Budget:

$$\eta\dot{N} = (1-\alpha)Y + \underbrace{\frac{1}{\cancel{\eta}}(1-\alpha)\alpha\frac{Y}{\cancel{N}}}_{r}\,\underbrace{\cancel{\eta}\cancel{N}}_{a} - C$$

$$\eta\dot{N} = (1-\alpha)Y + (\alpha-\alpha^2)Y - C$$

$$\eta\dot{N} = (1-\alpha^2)Y - C$$

$$\eta\dot{N} = Y - \alpha^2 Y - C$$

$$\boxed{\eta\dot{N} = Y - X - C}$$

→ total output used either to produce interm. or to consume or into R&D to increase the value of ~~new~~ fi overall firms.

from expressions

$$\frac{\dot{Y}}{Y} = \frac{\dot{X}}{X} = \frac{\dot{N}}{N}$$ (X, Y are linear in X, N).

from B.C. (divide by N) in BGP

$$\frac{\dot{C}}{C} = \frac{\dot{N}}{N}$$

$$\frac{\dot{C}}{C} = \frac{1}{\theta}\left[\frac{L}{\eta} A^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha}\right) \alpha^{\frac{2}{1-\alpha}} - \rho\right]$$

cost of R&D affects growth
$L \rightarrow$ affects growth

if cost of R&D $\quad \eta(N) = \eta \cdot \frac{Y}{N} = \eta A^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} L$

$$\Rightarrow r = \frac{\alpha(1-\alpha)}{\eta} \qquad \boxed{\frac{\dot{C}}{C} = \frac{1}{\theta}\left[\frac{\alpha(1-\alpha)}{\eta} - \rho\right]}$$

If cost of research increases with the mkt for the innovation then we get rid of scale effect

Amount of resources devoted to R&D

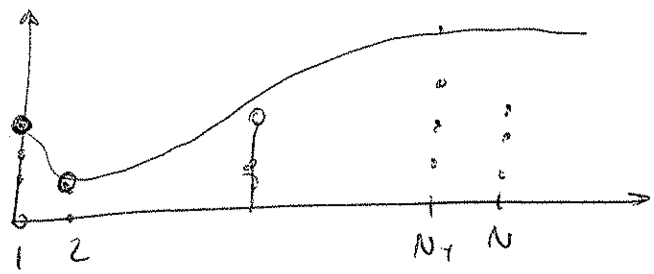
$\frac{\eta Y N}{Y}$ constant over time

SUM: innovation can break decr. ret. if

1. cost of innovation is not increasing too fast
2. New ideas have a market that does not shrink
3. growth rate: $\boxed{\gamma = f(\underset{+}{\eta}, \underset{+}{L}, \underset{+}{A}, \underset{-}{\rho})}$

Models of growth through better <u>varieties</u>, quality improvements.

Increase in N (drastic innovation). Each product (service) is subject to a continuous stream of improvements



Each good has some probability of improvement each period.

Important feature: new products displace old ones

As before:

① Producers of final output
② R&D → producers of interm.
③ Consumers.

Labor + Intermediates (N), fixed.
Only leading edge product are produced (pricing such that it drives the other goods out)

Monopoly on the most recent version of a good but only lasts for a certain period.

Researchers: invest resources to improve current quality -

Differences w/ befor

1. lower incentive to R&D (as monopoly is finite)
2. Mkt stealing effect, socially sub-optimal (marg. benefit to innovator > to society)

Final good

$$Y_i = A L_i^{1-\alpha} \sum_{j=1}^{N} (\tilde{X}_{ij})^{\alpha}$$

$\tilde{X}_{ij}$ = quality-adjusted intermediate of type j

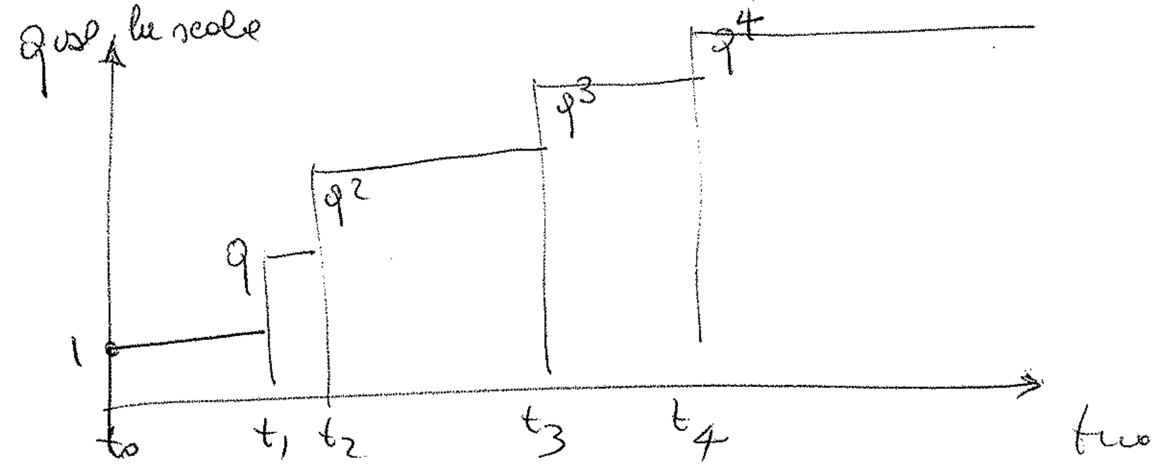
Potential quality $1, q, q^2, q^3$ so $\ln q \to q, 1, 2$ -

progression of quality is geometric (log-linear)

So K_j-th improvement implies quality q^{K_j} $(q>1)$
Improvement only take place one at a time.

Define $[t_{k-1}; t_k)$ interval over which, for good j
q^k is best quality product

Plotting quality over time:



Cost of producing intermed. is the same for all goods, $\forall$ quality

Inventor of quality q^{k_j} has a monopoly power on good q^{k_j}.

$$\tilde{X}_{ij} = q^{k_j} X_{ij}$$

↑ quality acts as a quantity-augmenting feature in the Pf.

$$Y_i = A L_i^{1-\alpha} \sum_{j=1}^{N} \left(q^{k_j} X_{ij} \right)^{\alpha}$$

profit for producer:

$$\pi_i = \left[Y_i - w L_i - \sum_{1}^{N} P_j X_{ij} \right]$$

to be maximised w/r to L_i and X_{ij}

$$\frac{\partial Y}{\partial X_{ij}} - P_i = 0$$

$$\Rightarrow \alpha A L_i^{1-\alpha} q^{k_i \alpha} X_{ij}^{\alpha-1} = P_j$$

so the derived demand for good j of quality k_i is :

$$X_{ij}^{\alpha-1} = \frac{P_j}{\alpha A L_i^{1-\alpha} q_i^{k_i \alpha}}$$

$$x_{ij} = L_i \left[\frac{\alpha A q^{k_i \alpha}}{P_j} \right]^{\frac{1}{1-\alpha}}$$

$$x_i = \sum_i x_{ij} = \underbrace{L}_{\sum L_i = L} \left[\frac{\alpha A q^{k_i \alpha}}{P_j} \right]^{\frac{1}{1-\alpha}}$$

for $q^k = 1$ we have the same as befor

The demand of good X_j increases with its quality which is a measure of its productivity in Y _

Elasticity still constant

R&D firms still have a 2-stage decision

1) Price of good
2) Decide how much R&D to do

profit of an innovation for k_i-th innovator

(cost of production $= 1$)

$$\pi(k_i) = (p_i - 1)\, x_i$$

optimal pricing still $\frac{1}{\alpha}$

The q.tity produced is:

$$x_i = L\left(\frac{A\alpha\, q^{\alpha k_i}}{1 \cancel{\;}\alpha}\right)^{\frac{1}{1-\alpha}} = L A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} \underbrace{q^{\frac{\alpha}{1-\alpha} k_i}}_{\substack{\text{this is}\\ \text{a term}\\ \text{depending on}\\ \text{quality}}}$$

profits

$$\boxed{\pi_{k_i} = \left(\frac{1-\alpha}{\alpha}\right) x_i = \underbrace{\frac{1-\alpha}{\alpha}\, \alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} L}_{\substack{= \bar{\pi}\\ \text{invariant over}\\ \text{time}}}\; \underbrace{q^{\frac{\alpha}{1-\alpha} k_i}}_{\substack{\text{increasing}\\ \text{w/ quality}}}}$$

$$\boxed{\pi_{k_i} = \bar{\pi}\, q^{\frac{\alpha}{1-\alpha} k_i}}$$

assume $q > \frac{1}{\alpha}$ otherwise at price $\frac{1}{\alpha}$ monopolist could be undercut by the next best prod $P_i = 1$ who could stay in. In that case $P = q$ (rather than $\frac{1}{\alpha}$. Monopoly right may last for ∞, but at T_{k_i+1} $\pi \downarrow 0$