
ECN 135 Lecture 5 Money, Banks & Financial Institutions

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Understanding Interest Rates

- Present Value
- Credit Instruments
- Interest Rates: Real and Nominal

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Present Value

- **Concept of Present Value or Present discounted value**

- Simple loan of \$1 at 10% interest
- Year 1 2 3 n
- \$1.10 \$1.21 \$1.33 $\$1 \times (1 + i)^n$
- \$1 PV of future \$1 = $(1 + i)^{-n}$
- **Why Present discounted Value is important?**
 - \$ in a year from now is less than \$ today
 - We need to compare dollars across time [dollar today with dollar at other dates]

Types of Credit Instruments

- **Four Types of Credit Instruments**
- 1. Simple loan (borrower repays the principal + interest)
- 2. Fixed-payment loan (auto, mortgage) [called also fully amortized loan]
- 3. Coupon bond pays
 - C - yearly coupon (or half a year coupon), n =years to maturity
 - F - face value
 - P - price of coupon bond $P = C/(1+i) + \dots + C/(1+i)^n + F/(1+i)^n$ where i is yield to maturity
 - If Coupon rate is 10%, $C=100$ for $F=1000$
- 4. Discount (zero coupon) bond (same as for a simple loan)
From $F = P(1+i)^n$ we have $i = (F/P)^{1/n} - 1$
- 5. Console or Perpetuity $P = C/i$
- Yield to maturity = interest rate that equates today's value with present value of all future payments
- Yield to maturity reflects TODAY value of interest rate most accurately

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Yield to maturity = interest rate that equates today's value with present value of all future payments

[M] p.66 – 6, see Table 1

When coupon rate and yield to maturity are equal, $P = F$

If yield to maturity rises, price of bond falls. Intuition: when interest rates increase, bonds with the same face value should cost less → fall in price ↔ If yield rises, price falls

↔ $P < F$ ↔ yield rate $i >$ coupon rate

Yield to Maturity: Loans

Yield to maturity = interest rate that equates today's value with present value of all future payments

1. Simple Loan ($i = 10\%$)

$$\$100 = \$110 / (1 + i) \Rightarrow$$

$$i = \frac{\$110 - \$100}{\$100} = \frac{\$10}{\$100} = 10\%$$

2. Fixed Payment Loan ($i = 10\%$)

$$\$1000 = \frac{\$126}{(1+i)} + \frac{\$126}{(1+i)^2} + \frac{\$126}{(1+i)^3} + \dots + \frac{\$126}{(1+i)^{25}}$$

$$LV = \text{Loan Value} = \frac{FP}{(1+i)} + \frac{FP}{(1+i)^2} + \frac{FP}{(1+i)^3} + \dots + \frac{FP}{(1+i)^n}$$

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Yield to Maturity: Bonds

3. Coupon Bond (Coupon rate = 10% = C/F)

$$P = \frac{\$100}{(1+i)} + \frac{\$100}{(1+i)^2} + \frac{\$100}{(1+i)^3} + \dots + \frac{\$100}{(1+i)^{10}} + \frac{\$1000}{(1+i)^{10}}$$

$$P = \frac{C}{(1+i)} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \dots + \frac{C}{(1+i)^n} + \frac{F}{(1+i)^n}$$

Consol: Fixed coupon payments of \$C forever

$$P = \frac{C}{i} \quad i = \frac{C}{P}$$

4. Discount Bond ($P = \$900$, $F = \$1000$), one year

$$\$900 = \frac{\$1000}{(1+i)} \quad \Rightarrow$$

$$i = \frac{\$1000 - \$900}{\$900} = 0.111 = 11.1\%$$

$$i = \frac{F - P}{P}$$

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Relationship Between Price and Yield to Maturity

Table 1 Yields to Maturity on a 10%-Coupon-Rate Bond Maturing in Ten Years (Face Value = \$1,000)

Price of Bond (\$)	Yield to Maturity (%)
1,200	7.13
1,100	8.48
1,000	10.00
900	11.75
800	13.81

Three Interesting Facts from Table 1

1. When bond is at par, yield equals coupon rate
2. Price and yield are negatively related
3. Yield greater than coupon rate when bond price is below par value

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Current Yield

$$i_c = \frac{C}{P}$$

Two Characteristics

1. Is better approximation to yield to maturity, nearer price is to par and longer is maturity of bond
2. Change in current yield *always* signals change in same direction as yield to maturity

Yield on a Discount Basis

$$i_{db} = \frac{(F - P)}{F} \times \frac{360}{(\text{number of days to maturity})}$$

One year bill, $P = \$900$, $F = \$1000$

$$i_{db} = \frac{\$1000 - \$900}{\$1000} \times \frac{360}{365} = 0.099 = 9.9\%$$

Two Characteristics

1. Understates yield to maturity; longer the maturity, greater is understatement
2. Change in discount yield *always* signals change in same direction as yield to maturity

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Rate of Return & Interest Rate

- Rate of Return NOT equal to the interest rate of the bond
- Rate of Return for a security

$$RET = (C + P_{t+1} - P_t) / P_t = i_c + g,$$

- where:
 - $i_c = C / P_t$ = current yield
 - $g = (C + P_{t+1} - P_t) / P_t$ = rate of capital gain
 - P_t is price of the bond at time t
 - P_{t+1} is price of the bond at time $t+1$
- Long term bonds are considered not safe because their prices fluctuate substantially with interest rate changes
- Do you Remember that bond prices fall when interest rates increase and vice versa?
- Short term bonds prices and returns are less volatile than long term [M], p. 78
- Short term bonds are less risky relative to long term bonds when holding period and the maturity are not identical.

Key Facts about Relationship Between Interest Rates and Returns

Table 2 One-Year Returns on Different-Maturity 10%-Coupon-Rate Bonds When Interest Rates Rise from 10% to 20%

(1) Years to Maturity When Bond Is Purchased	(2) Initial Current Yield (%)	(3) Initial Price (\$)	(4) Price Next Year* (\$)	(5) Rate of Capital Gain (%)	(6) Rate of Return (2 + 5) (%)
30	10	1,000	503	-49.7	-39.7
20	10	1,000	516	-48.4	-38.4
10	10	1,000	597	-40.3	-30.3
5	10	1,000	741	-25.9	-15.9
2	10	1,000	917	-8.3	+1.7
1	10	1,000	1,000	0.0	+10.0

*Calculated using Equation 3.

Maturity & Volatility of Bond Returns

- **Key Findings from Table 2**

- 1. Only bond whose return = yield is one with maturity = holding period
- 2. For bonds with maturity > holding period, $i \uparrow$ $P \downarrow$ implying capital loss
- 3. Longer is maturity, greater is % price change associated with interest rate change
- 4. Longer is maturity, more return changes with change in interest rate
- 5. Bond with high initial interest rate can still have negative return if $i \uparrow$

- **Conclusion from Table 2 Analysis**

- 1. Prices and returns more volatile for long-term bonds because have higher interest-rate risk
- 2. No interest-rate risk for any bond whose maturity equals holding period

Real and Nominal Interest Rates

- Fisher's equation $(1+i)=(1+i_r)(1+\pi^e)$ & for small π^e and $i_r \rightarrow$

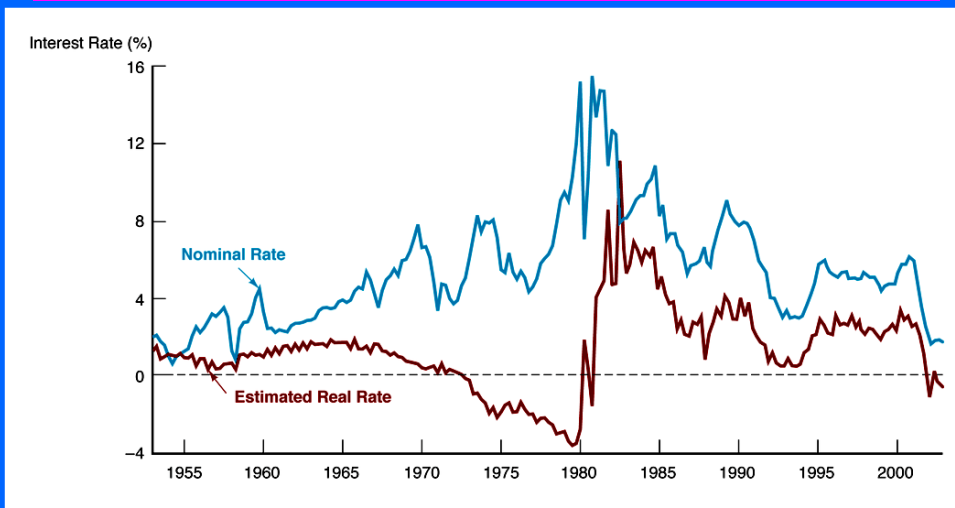
$$i = i_r + \pi^e$$

- **Real Interest Rate** is Interest rate that is adjusted for expected changes in the price level

$$i_r = i - \pi^e$$

1. Real interest rate more accurately reflects true cost of borrowing
2. When real rate is low, greater incentives to borrow and less to lend
 - if $i = 5\%$ and $\pi^e = 3\%$ then:
 - $i_r = 5\% - 3\% = 2\%$
 - if $i = 8\%$ and $\pi^e = 10\%$ then
 - $i_r = 8\% - 10\% = -2\%$

U.S. Real and Nominal Interest Rates



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1. If real interest rate is low → incentives to borrow increase, and to lend decrease (because costs of borrowing are low in this case/ borrowing is cheap)
2. In real terms, costs of borrowing in 1980 was low.

Interest Rates and Taxes

- Need to adjust Fisher equation to taxes because after tax interest rate matters, i.e. after taxes are subtracted

$$i_r = i(1-\tau) - \pi^e,$$

- Where τ is a tax rate
- If tax rate is $\tau=30\%$, and $\pi^e=5\%$, after tax rate for $i=10\%$ bond is 7%, and

$$i_r = 2\%$$

- After-tax real rate is the effective cost of borrowing. After-tax rate is below real rate

TIPS= Treasury Inflation Protected Securities

- New instrument in US in 1997 (introduced by US Treasury (Summers))
- TIPS are Indexed Bonds or inflation protected bonds
- What TIPS do?
 - Provide a measure of real interest rate, i.e. TIPS make i_r observable
 - Serve to estimate expected inflation (or market expectations about future inflation)
- Why TIPS are useful? TIPS generate information
 - → can be used to hedge against changes in inflation
 - → estimate credit tightness (show real interest rate)
- US was following example of other countries (UK, Canada, Australia & Sweden)
- If for 10 year TIPS interest rate is 2.19%, and for 10 year T-bill 3.84 → $\pi^e = 1.65\%$
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Next Lecture

- Your preparation: read [M] Ch. 4, 5, 6
- More on interest rates & bonds

Summary of Today

- Present Value
- Types of Debt Instruments
- Have a Nice Night