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APPENDIX A.

A Simple Model of Public School Finance,

Featuring Budget Votes

When Public Schools Get Mixed Public-Private Financing

We seek to model first a household's schooling preferences and then a decisive voter's tax-and-subsidy preferences, to predict influences on tax support and school attendance. The model is kept as simple as possible, while incorporating heterogeneous preferences and a few essentials of school finance. The model features two kinds of decisionmakers. First, parents and prospective parents choose how much public schooling each child should have, given the offical length of the school year and the quality of the local public school. Second, a decisive voter chooses not only the amount of schooling for his own child but also the school budget and the length of the school year.

I. The Demand for Schooling in a Household with Children

A household utility function that is particularly handy in its tractability and relative realism is the *i*th individual's Stone-Geary function of the form

(1) $U_i = (C_i - C_{io})^{\alpha} (S_i - S_{io})^{1-\alpha}$, where

C_i = consumption of everything but primary schooling, with C_{io} being its positive necessary minimum subsistence level;

α = a preference-based coefficient for the *i*th household (we omit the *i*-subscript); and

S_i = the share of the average school-age year that each child spends in public school, with S_{io} being a base level of schooling that could be positive or negative, depending on preferences. We consider a negative sign more realistic, since it is consistent with the possibility of choosing zero public schooling, which was frequently observed in this historical period.

Note that the schooling demand being modeled here is a demand for *public* schooling. The demand for private schooling is separate and is folded into general consumption (C) here, to simplify the algebra.

For mathematical convenience, and without changing any aggregate implications, we convert the Stone-Geary utility function into its logged form:

(2) $\ln U_i = \alpha \ln(C_i - C_{io}) + (1-\alpha) \ln(S_i - S_{io})$

The consumer (couple) maximizes this utility with respect to consumption (C_i) and per-child schooling (S_i), with all children being treated equally by their parents.

The income constraint plays its usual crucial role. Full-time income (Y_i) consists of wage and property income components: $Y_i = W_i + r P_L L_i$, where W_i is the family's full-time labor earning power, r is the current-year rate of return on "land" (representing real estate, or taxable property), P_L is the purchase price (and assessed price) of land, and L_i is this *i*th household's holdings of land.

The household spends its income on consumption, on schooling above the subsidized level, and on taxes, so that the household's budget constraint is

(3) $W_i + r P_L L_i - C_i - n_i \phi (Si - s) - n_i w_c S_i - t P_L L_i \geq 0.$

There is no p_c price term in front of the real consumption term C_i because we normalize the price of consumer goods to be unity.

Here n_i = the number of children of school age,[1] and ϕ is the fee cost of a full year's normal public schooling, from both private and public funds. Institutionally, s was the share of the year for which the fees were paid by taxpayers. Typically taxes covered a number of weeks of school, say twelve weeks, and parents wanting fuller schooling during the year paid the the rest with rate bills. The annual wage rate on child labor is w_c. Having a child go to school deducts the child's labor time, as well as the school fees, from the family's potential earnings.

The local government also has a budget constraint. For simplicity, we will assume that school subsidies are the only kind of local government expenditure and a

[1] This appendix views the number of children as exogenous. The model could be extended to make fertility decisions simultaneous with schooling decisions.

proportional property (land) tax is the only source of government revenue. Ignoring any administrative costs, the budget constraint says that total revenue must cover total school expenditures:

(4) $t P_L L_{all} \geq s \phi \Sigma_i n_i S_i = s \phi N$, so that the school-tax rate $t = \frac{s\phi N}{P_L L_{all}}$

where $P_L L_{all}$ is the assessed value of all real estate (quality-adjusted price P_L times quantity L_{all}), and N is the locality's total number of children attending common schools anytime during the year. We will use Equation (4) later, to link the tax rate to the level of schooling and to model the decisive voter's self-interest.

We deal first with an interior solution in which the household demands more schooling days than the subsidized share s, and buys the extra $(S_i - s)$ per child by paying rate bills (tuition). For this interior solution, the Lagrangian is

(5) $V = \alpha \ln(C_i - C_{io}) + (1-\alpha) \ln(S_i - S_{io})$
$+ \lambda [W_i + r P_L L_i - C_i - n_i \phi (S_i - s) - n_i w_c S_i - t P_L L_i]$

and the first order conditions are

$$V_C = \frac{\alpha}{C_i - C_{i0}} - \lambda = 0$$

$$V_S = \frac{1-\alpha}{S_i - S_{i0}} - p_s \lambda = 0$$

Here the private price of schooling p_s is the sum of the opportunity cost of the child's time in school plus the private payments of any "rate bills" (tuition) beyond what is subsidized by taxpayers:

(6) $p_s = n_i w_c + n_i \phi$

We use these first-order conditions to derive the Marshallian demands for schooling and consumption in this interior-solution case where the household reaches a level of

schooling that involves some extra school days paid for by rate bills. The chosen rate of annual schooling per child is:

$$(7)\ S_i = \frac{1-\alpha}{\alpha}\left[\frac{W_i + rP_L L_i - n_i\phi(S_i - s) - n_i w_c S_i - tP_L L_i - C_{i0}}{n_i(\phi + w_c)}\right] + S_{i0}$$

Collecting all the S_i terms yields the demand equation for public school attendance:

$$(8)\ S_i = (1-\alpha)\left[\frac{W_i + rP_L L_i + n_i s\phi - tP_L L_i - C_{i0}}{n_i(\phi + w_c)}\right] + \alpha S_{i0}$$

The bracketed numerator is simply the full-time income left over after the minimum subsistence consumption has been met, and the bracketed denominator is the price of extra rate-bill schooling per child. The equation translates thus: The share of discretionary income spent on public schooling, at the observed real private price of that schooling, is shaped by the individual's relative taste for schooling (1-α and S_{i0}).

The partial derivatives of public-school attendance with respect to the exogenous parameters correspond to intuition. Raising any component of income (W_i, L_i, r, or P_L) has a positive effect, meaning that being richer leads to more demand for schooling beyond what taxes will cover. That might not be obvious at first in the case of L_i, since having more property means paying more tax as well as getting more property return. Yet the tax was less than the return (r > t), as a positive valuation of property ($P_L > 0$) requires, so more property means more parental demand for schools. By contrast, the schooling level per child is negatively affected by having more children, or by a higher local wage rate for school-age children.

Of particular interest is the effect of raising the school subsidy on different individuals' demand for schooling. The effect is not necessarily positive, since the higher subsidy must be paid for with higher taxes. The partial derivative of attendance with respect to the share of the school year covered by taxes is:

$$(9)\ \partial S_i / \partial s = (1-\alpha)\left[\frac{\phi(n_i - (NL_i / L_{all}))}{n_i(\phi + w_c)}\right].$$

The sign of this effect depends on the relationship of relative family size to relative property:

$$\text{sign}\,(\partial S_i/\partial s) = \text{sign}\,(n_i/N - L_i/L_{all}),$$

which is positive for those having relatively more children and relatively less property. That is, raising the subsidy to schooling will raise attendance by children of families that are bigger and/or less propertied. Those with smaller families and greater-than-average property will experience a net loss because their taxes will be raised by more than their use of subsidized schools, leading them to demand less public schooling for their children. Overall, a higher subsidy will make schooling more equal.

Raising the cost of public schooling of given quality (raising ϕ), e.g. by having public-school teachers become more expensive, would affect the same two groups the same way as raising the subsidy rate. That is, it would encourage public-school attendance by children from larger and poorer families, but discourage it among children of smaller and richer families, again making schooling more equal.

So far we have focused on the key group with sufficient demand for schooling to pay rate bills for extra schooling beyond what is subsidized (the interior solution with $S_i > s$). Other groups will behave differently. Moving down the scale of relative demand for schooling, the next group would be those whose demand was sufficient only to utilize the tax-paid part of the school year ($S_i = s$), because they perceive the private benefits of extra school to be somewhere between the opportunity cost of child labor and this plus the cost of private schooling (between w_c and $(w_c + \phi)$). This group's attendance will not vary in response to small movements in any parameter, except for an increase in the subsidy. A third group would be those wanting less schooling than what taxes pay for ($0 < S_i < s$). This group would respond to most parameters in the same manner as the group demanding more than the subsidized share. It too would raise attendance in response to any income variable, cut attendance either if the child wage rate rose or if schools became more costly. There is one difference, however, between the responses of this lower-demand group ($0 < S_i < s$) and the higher-demand group ($S_i > s$). If the general subsidy (s) is raised along with the tax rate, members of this lower-demand group having any property would respond

by *cutting* school attendance because the higher tax would be a pure income loss for them. Those with no property will be completely unaffected. A fourth group is the one with so little taste for schooling that they consume zero of it. For this group, as for the second group, parameter shifts will do nothing to school attendance.

The aggregate responses of school attendance to changes in market and policy parameters will be a mixture of the responses of these four groups. The functional form of the aggregate response is uncertain, but the signs of these responses are likely to resemble the high-demand first group, which actually paid some rate bills ($S_i > s$).

II. Voter Preferences Regarding Public School Subsidy and Property Tax

Our portrayal of the influence of voting and political voice on the tax support for schools, and thus on the amount and distribution of school attendance, proceeds in two main steps. First, we show how a voter's preferences regarding property tax rate and a school subsidy would align with his self-interest as a taxpayer and a parent.[2] This optimization closely follows the household model just introduced, though the individual is now free to shape public school finance as well as his own child's education. Second, we will use the clear predictions of this optimization to suggest how education finance would have been driven by the distribution of the extension of political voice in such a setting where public schools mixed public and private money.

A voter's optimization is the same problem already visited, except that the household now chooses how to vote on s and t as well as on how much schooling their children should have. We posit the same logged utility function as in Equation (2) above, sticking for now to the case in which the voter of interest has children. This time, however, the household budget constraint is different. With the freedom to set the tax rate t and the subsidized share of the school year (s), a decisive ith voter can set s equal to his demand for schooling per child (S_i), and push through a result that avoids his paying any rate bills, so that $S_i = s$. The household budget constraint is therefore a variant of Equation (3) above, specifically

[2] We set aside the case of the childless voter who does not care about schooling the children of others. Clearly, the greater the share of such voters, the lower the taxes for schools, and our econometric tests will incorporate this effect within the influence of the child/adult ratio.

(10) $W_i + r P_L L_i - C_i - n_i w_c s - s \phi N L_i/L_{all} \geq 0.$

This formulation has used the government budget balance from Equation (4) to state the tax burden on this individual as a function of the school subsidy itself, eliminating the tax rate t.[3] A derivation like that used above implies that the voter will prefer to subsidize public primary schools up to his own demand for public schooling. Setting $S_i = s$ slightly changes the Lagrangian to

(11) $V = \alpha \ln(C_i - C_{io}) + (1-\alpha) \ln(s - S_{io})$
$+ \lambda [W_i + r P_L L_i - C_i - n_i w_c s - s \phi N L_i/L_{all}]$

This makes the first-order conditions

$$V_C = \frac{\alpha}{C_i - C_{i0}} - \lambda = 0$$

$$V_s = \frac{1-\alpha}{s - S_{i0}} - \lambda[n_i w_c + \phi N L_i / L_{all}] = 0$$

Solving for the subsidized share of the public school year yields

(12) $$s = (1-\alpha)\left[\frac{W_i + rP_L L_{ii} - C_{i0}}{n_i w_c + \phi N L_i / L_{all}}\right] + \alpha S_{i0}$$

That is, the preferred subsidy share depends on the voter's tastes for schooling (1-α and αS_{i0}), his discretionary income above a bare subsistence consumption level, and a private price deflator specific to public schooling.

[3] The decisive voter might consider the possible reverse influence of schooling-cum-taxes (s) on property values, as Charles Tiebout had envisioned migrants' doing. This effect could be incorporated into the model by making P_L an inverted-U function of s, to reflect the fact that there is a most efficient level of tax-based schooling. We set aside such algebra here, and only note that it would make the decisive voter choose levels of s closer to the property-value peak of the inverted U than he would if there were no such feedback to P_L.

The parametric influences on the rate of public school subsidy resemble the influences on a household's decisions about school attendance. The greater his gross household income, the more a voter favors a longer subsidized school year and a bigger budget. On the other hand, his preferred budget per child would be smaller, the greater is his opportunity cost in terms of child labor ($n_i w_c$),or the total school cost per child (ϕ), or the average number of children in all families (N). This last effect suggests that adding disenfranchised immigrants, or having more children per family, lowers the budget per child.

The partial derivative of greatest interest here is the one that is the most complex algebraically: How is the decisive voter's choice of s affected by his owning more taxable real estate (L_i) relative to the holdings of others in the same locality? To ease the task of viewing a quotient-rule derivative, we first gather some terms on the left in a re-defined dependent variable:

$$(12)\quad s^* = \frac{s}{1-\alpha} - \alpha S_{i0} = \frac{W_i + rP_L L_i - C_{i0}}{n_i w_c + \phi N L_i / L_{all}}$$

Calling the whole right-hand denominator p_s and differentiating with respect to the voter's property holdings yields

$$(13)\quad \frac{\partial s^*}{\partial L_i} = \frac{p_s r P_L - (W_i + rP_L L_i - C_{i0})\phi N / L_{all}}{p_s^{\,2}}$$

Expanding on the definition of p_s in the numerator alone yields a cancellation of two terms in the numerator, so that

$$(14)\quad \frac{\partial s^*}{\partial L_i} = \frac{1}{1-\alpha}\frac{\partial s}{\partial L_i} = \frac{r P_L n_i w_c - (W_i - C_{i0})\phi N / L_{all}}{p_s^{\,2}}$$

The crucial result is the sign of the unfriendly-looking numerator on the right-hand side. Dividing through by parts of it yields the key sign result:

$$(15)\quad sign\left(\frac{\partial s}{\partial L_i}\right) = sign\left[\frac{n_i w_c}{W_i - C_{i0}} - \frac{\phi N}{r P_L L_{all}}\right]$$

$$= sign\left[\frac{full-time\,child\,earning\,power}{discretionary\,earning\,power\,of\,the\,whole\,family} - \frac{full-year\,school\,costs}{the\,locality's\,property\,income}\right]$$

The right-hand term is likely to have values like ten percent. In the case of antebellum America, this likelihood stems from the school tax rates and the rates of return on property. The school tax rates on assessed property value tended to by around five mills (0.05), and if the rate of return was on the order of five percent (0.05), the right-hand ratio would have been one-tenth, or ten percent.

In today's modern economy, or in the antebellum era, the left-hand term would have been well below ten percent for a decisive voter. Today children have almost no earning power during the school-age years, making the first term effectively zero. In today's setting, therefore, the more propertied the decisive voter (higher L_i), the lower would be the tax support for schooling (s). The result for antebellum America would already have approached this condition, to the extent that restrictions on political voice would have put the decisive votes in the hands of those who were rich enough that their discretionary income would not have depended at all on child earnings. Then, too, schooling would have been limited by the property ownership of the decisive voter.

The only setting in which greater property for key voters could have raised their support for school taxes would have been a setting highly dependent on child labor, so that the first term could dominate in Equation (15). Such settings would be rare. To be sure, most of human history consisted of impoverished agricultural settings in which childrens' labor would have been much needed by poor parents. But in those same settings, political voice was restricted to those owning large amounts of property. Again the reliance on child labor would have been below ten percent for those with political voice, leaving the result that restricting voice to those with above-average land holdings meant less support for schools.

III. The Implied Predictions

Given the decisive voter's setting the amount of tax support for schools, the individual households would choose their enrollments (S_i's) as modeled in Part I of

this appendix. Thus does the interplay of voting power and the distribution of property influence the enrollments and the public and private expenditures on common schools. The combined model yields the predicted coefficients for cross-sectional regressions on antebellum U.S. counties shown in Appendix Table A.1.

Note in particular that the featured voting variables are a mixture of constraints and voter preferences. The main constraint is the share of all free men who are franchised. The tighter this is, the higher is the property position of the decisive voter (L_i), and the lower the predicted support for schooling. The voter preferences are represented in the tradition of Meltzer and Richard (1981), as well as in the present model, by skewness in the distribution of property. More skewness would make the decisive voter favor more taxes and schools. Skewness is represented in mirror image by the median/mean ratio in this table.

Appendix Table A.1. Predicted Coefficients: Aligning the Model with the Regressions

Model variable	Corresponding empirical variable(s) in county samples, with predicted signs of effect on subsidies and enrollments
Dependent variables:	
s, ϕ	public expenditures per common-school pupil, in the census
S_i	share of school-age children enrolled and attending common schools
Featured independent variables:	
Decisive voter's L_i	(1) franchise share (+); (2) share owning no property (-), in the 1850 IPUMS.
Other independent variables:	
$P_L L_{all}$	real estate value per free man in that county (+)
w_c	None used here, for want of an easy proxy
W_i	None used here (could proxy with occupational mix, but at the risk of endogeneity)
N, n_i	School-age share of total population (-)
α	Religion, urban share, etc.
	(State fixed effects cannot be assigned to any one model variable.)