ECN 145 Lecture 11

Transportation Economics: Analysis of Demand II

II Discrete Choice

- Suppose that consumers n=1,…,N are choosing over a discrete number of items j=1,…,J (e.g. modes of transport)
- As a researcher, we do not exactly know consumer tastes -- tastes are “random” across consumers
- How should we model utility and demand?

Conditional utility:

- Conditional on choosing alternative j, utility is:

$$U_{jn} = V(X_j, S_n; \beta) + \varepsilon_{jn}$$

  - X_j are the characteristics of good j
  - S_n is characteristics of the decision maker
  - β is (unknown) parameters of utility
  - ε_{jn} is "random" utility from consuming item j
  - $V_{jn} = V(X_j, S_n; \beta)$ is "systematic" utility

Systematic portion of utility:

- For example, utility obtained from item j:

$$V_{jn} = \beta_1 (c_j / w_n) + \beta_2 t_j + \beta_3 t_j^2$$

- where, c_{jn} = costs of transport, β_1<0
- w_n = wage of consumer n
- t_j = time taken in travel, β_2<0
- and β_3 >< 0
-

Random portion of utility:

- Prob. that consumer n chooses item i:

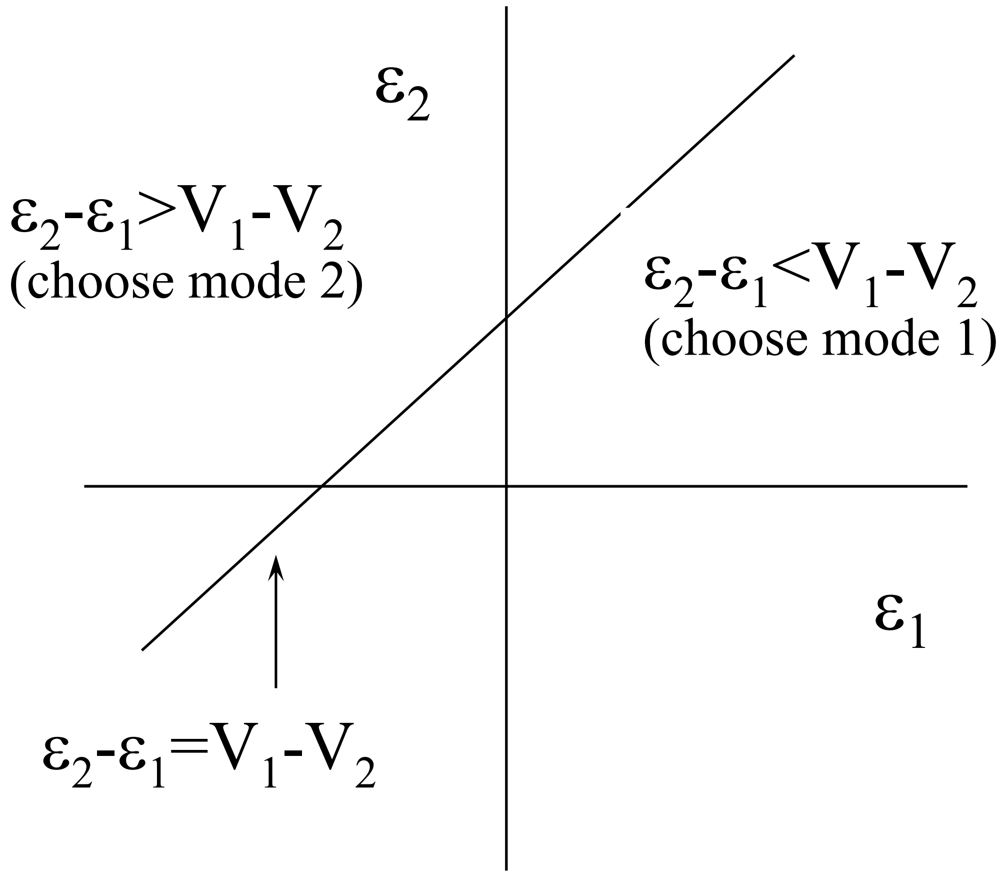
$$P_{in} = \Pr ob[U_{in} > U_{jn} \text{ for all } j \neq i]$$

$$= \Pr ob[V_{jn} + \varepsilon_{jn} < V_{in} + \varepsilon_{in} \text{ for all } j \neq i]$$

$$= \Pr ob[\varepsilon_{jn} - \varepsilon_{in} < V_{in} - V_{jn} \text{ for all } j \neq i]$$

- This will depend on the probability distribution for ε_i and ε_j

Random portion of utility - graph

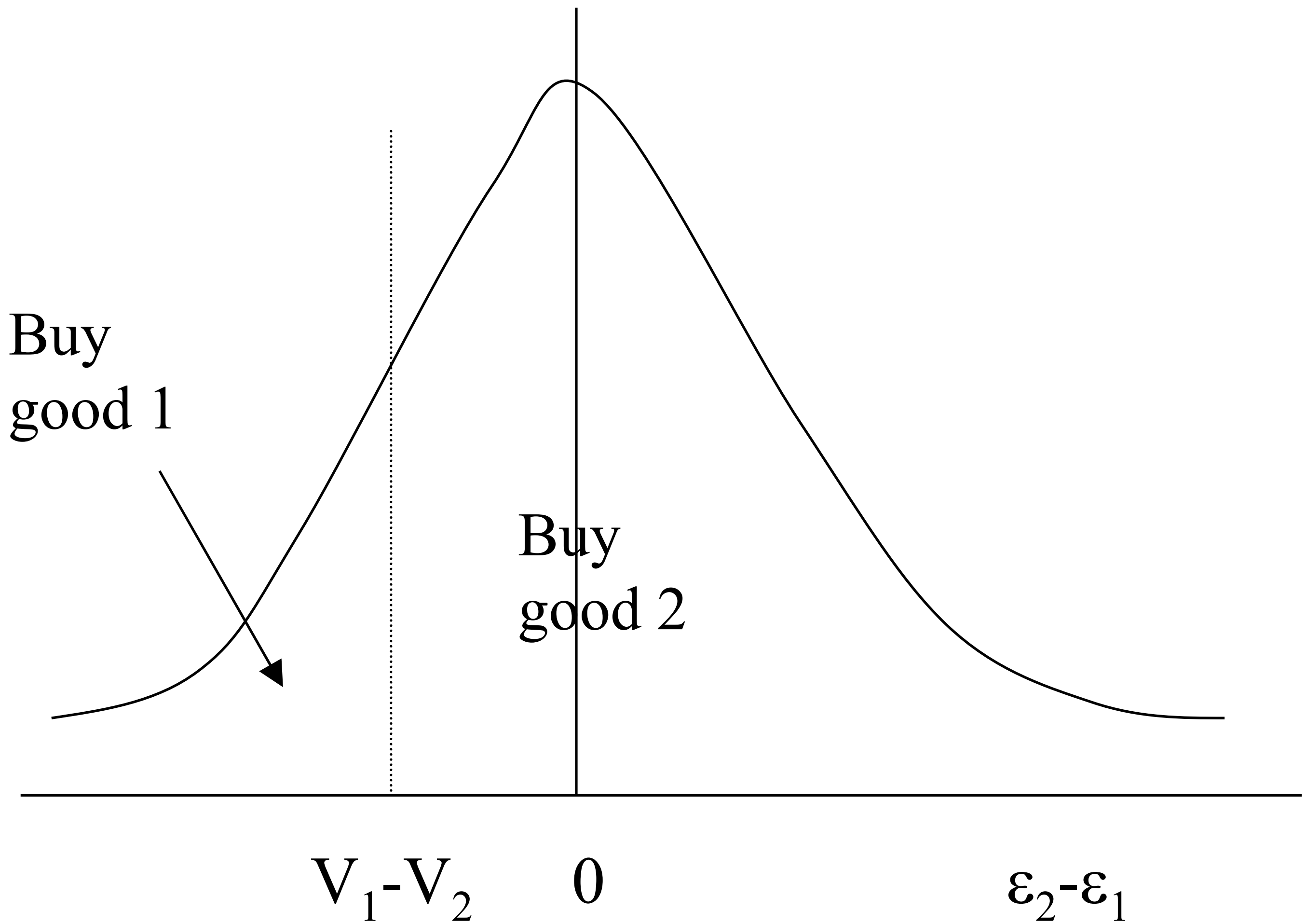


Probit distribution:

- For example, suppose there are only two items i=1,2, and that the utility difference $\varepsilon_2 - \varepsilon_1$ is normally distributed
- Denote the density function for the standard normal distribution as

$$\phi(x) = \text{Prob}[\varepsilon_2 - \varepsilon_1 = x] = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

Probit Distribution (cont'd):



Estimation of Probit (cont'd):

- Example, from a sample of 280 urban commuters choosing between two modes of transportation in Chicago:
- (1) auto and
- (2) transit
- We use data on mode characteristics (time to travel, cost, distance), and also the income of consumers.
- The estimated systematic utility function is:

Estimation of Probit (cont'd):

$$V_{jn} = -2.08D^T - 0.00759 w_n t_j - 0.0254(Inc_n Dist \cdot D^T)$$

$$-0.0186 c_j + 0.0255(Age_n D^T) - 0.057(Female_n D^T)$$

- where, D^T=1 for transit, =0 for auto
- t_j=time in travel,
- $Dist_j$=distance,
- c_j=cost of travel
- Inc_n=income, w_n=wage

Demand Curve:

- Notice that utility declines in cost:

$$\frac{\partial V_{jn}}{\partial c_j} = -0.0186$$

- So the integral under the normal curve is re-computed, and we obtain a downward sloping demand!

Value of Time:

- We can also compute the value of time as:

$$VOT_{jn} = \frac{\partial V_{jn} / \partial t_j}{\partial V_{jn} / \partial c_j} = \frac{-0.00759 w_n}{-0.0186} = 0.41 w_n$$

- so that time spent in commuting is valued at 41% of the individual's wage.
- But with more than two alternatives, Probit becomes difficult to estimate, so consider...

Multinomial Logit distribution:

- Suppose that the j=1,…,J random terms are distributed as "extreme value":

$$\text{Prob}[\varepsilon_j < x] = \exp(-e^{-\mu x})$$

- With $\mu=1$, the probability of choosing item i is:

$$P_{in} = e^{V_{in}} \Big/ \sum_{j=1}^{J} e^{V_{jn}}$$

Estimation of Logit (cont'd):

- Example, from a sample of San Francisco commuters choosing between:
- (1) auto alone,
- (2) bus + auto access,
- (3) bus + walking access,
- (4) carpool (this was before BART)

- The estimated systematic utility function is:

Estimation of Logit (cont'd):

$$V_{jn} = \underset{(0.0054)}{-0.0412}(c_j / w_n) \underset{(0.0072)}{-0.0201} t_j \underset{(0.0070)}{-0.0531} t_j^{out}$$

$$\underset{(0.26)}{-0.89} D^1 \underset{(0.24)}{-1.78} D^3 \underset{(0.25)}{-2.15} D^4$$

- where, D^j=1 for mode j, =0 otherwise
- t_j=in-vehicle time in travel,
- t_j^{out} =out of vehicle travel time
- c_j=cost of travel
- w_n=wage of traveler

Demand Curve:

- Expected demand for mode i is:

$$X_i = \sum_{n=1}^{N} P_{in}(\beta) = \sum_{n=1}^{N} e^{V_{in}} \Big/ \sum_{j=1}^{J} e^{V_{jn}}$$

- So, $\dfrac{\partial X_i}{\partial c_i} = -\sum_{n=1}^{N} \dfrac{0.0412}{w_n} P_{in}(1 - P_{in})$
- Again, we obtain downward sloping demand!
- Slope of demand will be smaller for the mode of transport with higher-wage travelers.

Value of Time:

- We can also compute the value of time as:

$$VOT_{jn} = \frac{\partial V_{jn} / \partial t_j}{\partial V_{jn} / \partial c_j} = \frac{-0.0201 w_n}{-0.0412} = 0.49 w_n$$

$$VOT_{jn}^{out} = \frac{\partial V_{jn} / \partial t_j^{out}}{\partial V_{jn} / \partial c_j} = \frac{-0.0531 w_n}{-0.0412} = 1.29 w_n$$

- so that time spent in commuting is valued at 49% of the wage, but time spent out of the vehicle is valued at (or higher) than the wage!

Problem with the Logit:

- The *relative* probability of choosing items i and k is:

$$P_{in} / P_{kn} = e^{V_{in}} / e^{V_{kn}}$$

- So P_i/P_k is *independent of other alternatives*
- This is the *"irrelevance of independent alternatives" (IIA)* property

- E.g., the "red bus, blue bus" problem

Red bus-blue bus problem:

- Suppose the *relative* probability of choosing the bus over a car is 2:1 (this is the "odds ratio"). Initially only a *red* bus is available.
- Then another bus comes available, which is blue, but otherwise *identical* to the red bus. It seems sensible that the relative probability of choosing *any* bus over a car is 2:1, so the relative probability of choosing *each* bus over the car should be 1:1

Red bus-blue bus problem (cont'd):

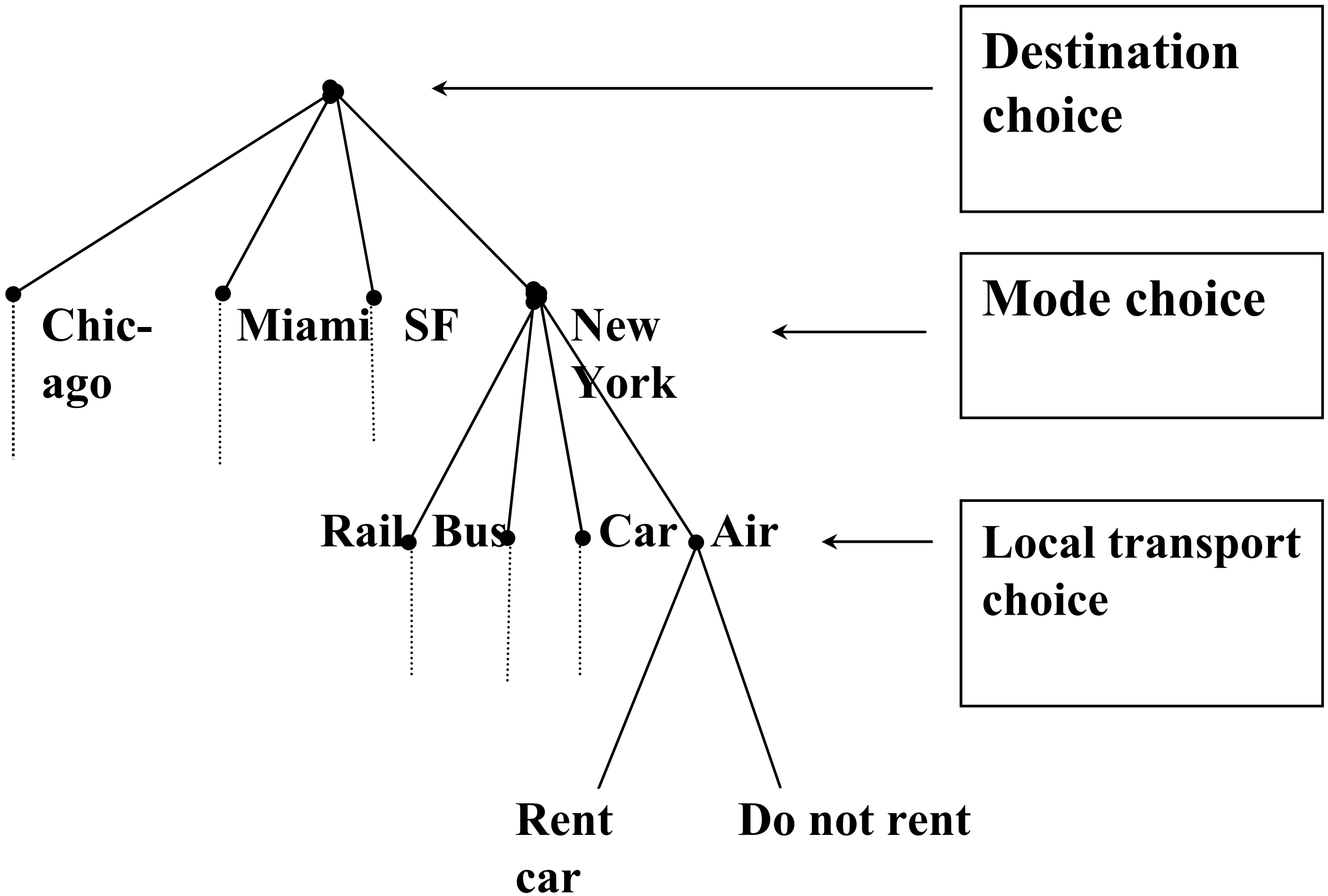
- But for the logit, the relative probability P_i/P_k is *unchanged* when a new alternative become available, so the red bus still has 2:1 odds ratio over a car.
- Therefore, the blue bus also has a 2:1 odds of being chosen (as it is identical to the red).
- So the relative probability of taking *any* bus over a car becomes 4:1 when the blue bus is added.

Red bus-blue bus problem (cont'd):

- Why does this problem with the logit arise?
- Because we have assumed that the random terms ε_j in utility are *independent!*
- Contrast with the probit, where we assumed that the *difference* $\varepsilon_1 - \varepsilon_2$ was normally distributed, so that these errors are correlated (but probit is hard to estimate for more than two modes of transport)

- Solution: use *nested* logit

Nested Logit: e.g. Vacation Choice



- *Each* type of choice is modeled as logit

Typical Elasticities (Table 2-2)

Market, model, elasticity type	*Elasticity estimates*	
Freight	***Rail***	***Truck***
Aggregate model split model[a]		
Price	-0.25 to –0.35	-0.25 to –0.35
Transit time	-0.3 to –0.7	-0.3 to –0.7
Aggregate model from translog cost function[b,c]		
Price	-0.37 to –1.16[d]	-0.58 to –1.81[e]
Disaggregate mode choice model[b,f]		
Price	-0.08 to –2.68	-0.04 to –2.97
Transit time	-0.07 to –2.33	-0.15 to –0.69

Typical Elasticities (Table 2-2, cont'd)

Market, model, elasticity type	*Elasticity estimates*			
Urban passenger[g]	***Auto***	***Bus***	***Rail***	
Price	-0.47	-0.58	-0.86	
In-vehicle time	-0.22	-0.60	-0.60	
Intercity passenger[h]	***Auto***	***Bus***	***Rail***	***Air***
Price	-0.45	-0.69	-1.20	-0.38
Travel time	-0.39	-2.11	-1.58	-0.43
Automobile utilization[i]	***One-vehicle household***	***Two-vehicle household***		
Short-run operating cost	-0.228	-0.059		
Long-run operating cost	-0.279	-0.099		

Value Time Elasticities (Table 2-3)

Transportation mode	*Estimate of value of time*			
Freight[a]		***Rail***	***Truck***	
(As percentage of daily shipment value)				
Total transit time		6-21	8-18	
Urban work trips[b]		***Auto***	***Bus***	
(As percentage of after tax wage rate)				
In-vehicle time		140	76	
Walk access time		...	273	
Transfer wait time		...	195	
Intercity passenger[c]	***Auto***	***Bus***[d]	***Rail***[e]	***Air***
(As percentage of pretax wage rate)				
Total travel time	6	79-87	54-69	149