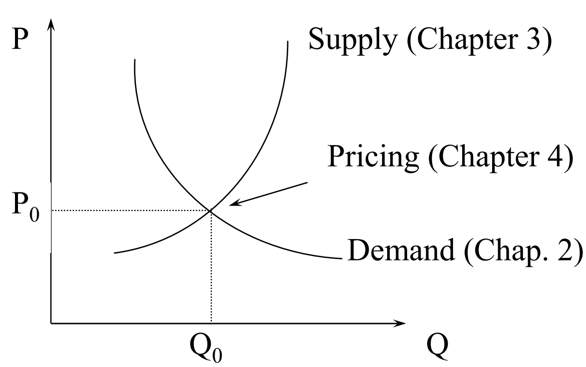
Economics 145 Lecture 10

Transplementation Economics:

Analysis of Demand I

Demand and Supply

- Chapters from *Essays* textbook:



Types of Demand

- 1)When to travel? – trip generation
- 2)To what destination? – trip distribution
- 3)What mode of transport to use? – mode choice
- 4)What route to take? – trip assignment
- **We will focus on <u>mode choice</u>.**

Continuous versus Discrete Choice

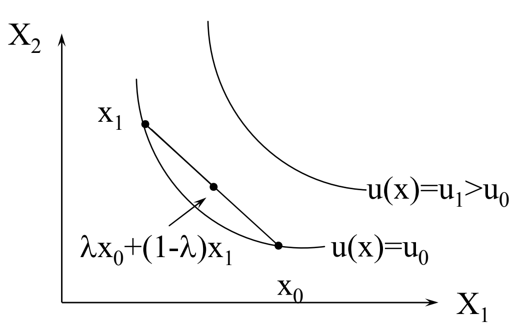
- **Q:** why does demand slope down?
- **A:** As price decreases, you will purchase more of this good
- Problem: What if good is only available in <u>one size</u>, e.g. a car. Then you are choosing over <u>discrete</u> alternatives.
- **Thus, look at: (I) Continuous choice; (II) Discrete choice**.

(I) Continuous choice:

- Utility function: $u(x_1,\ldots x_n) = u(x)$
- Properties:
  - **Increasing:** $\partial u / \partial x > 0$
  - **Quasi-concave**:
  - if $u(x_0) = u(x_1)$ and $0 \le \lambda \le 1$
  - then $u(\lambda x_0 + (1-\lambda)x_1) \ge u(x_0)$

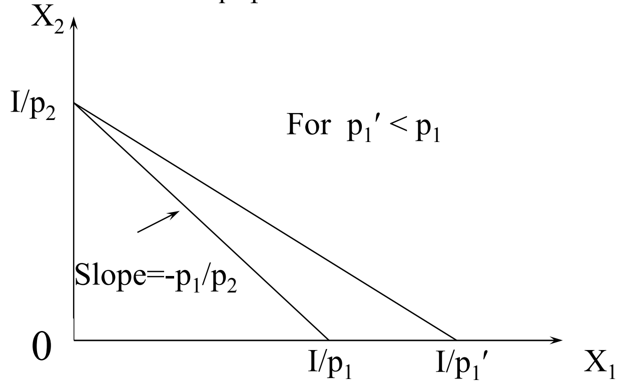
Utility Function

- Illustrate with "indifference curves":



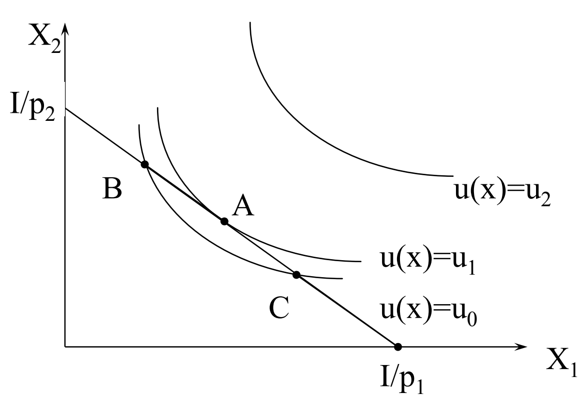
Budget constraint

$$\sum_{i=1}^{n} p_i x_i \le I$$



The consumer problem:

Max u(x) subject to $\sum_{i=1}^{n} p_i x_i \le I$



The consumer problem (cont'd):

- Solution is at A
  - B or C are feasible but not optimal
- Write solution as **demand functions**:

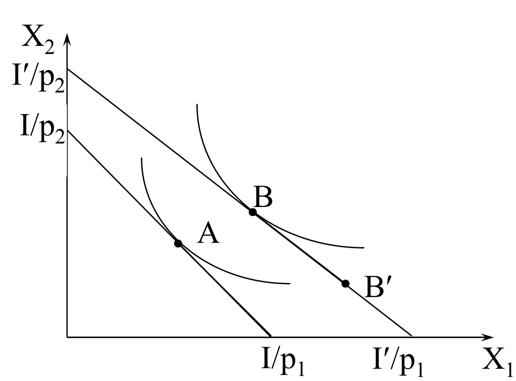
$$x_i\,(p_1,\ldots,p_n,I) = x_i(p,I), \quad i = 1,\ldots,n$$

- What can we say about the derivatives:

$$\text{(a)} \quad \partial x_i / \partial I \quad \text{and} \quad \text{(b)} \quad \partial x_i / \partial p_j$$

Change in Income:

- Suppose income rises from I to I′:

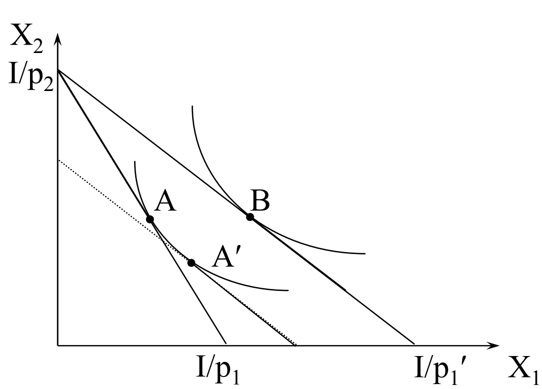


Change in Income (cont'd):

- If consumption moves from A to B
  – Both goods increase in demand, so that both goods are "normal $\partial x_i / \partial I > 0$
- If consumption moves from A to B′
  – Demand for good 2 has fallen;
  – Good 2 is "inferior", $\partial x_i / \partial I < 0$.
- For example: bus transportation?

Change in Price:

- Suppose price falls from p_1 to p_1':

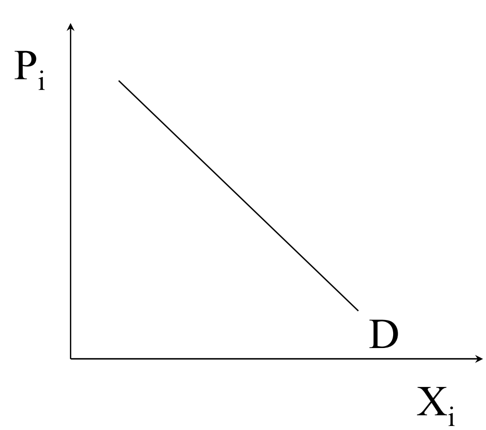


Change in Price (cont'd):

- Consumption moves from A to B.
- Decompose as:
  - "substitution" effect, A to A′
    - This increases x_1 and reduces x_2.
  - "income" effect, A′ to B
    - This increases x_1 if it is normal;
    - This decreases x_1 if it is inferior.
- Thus, x_1 will increase provided that substitution effect exceeds income effect

Demand Curve

- This gives us downward sloping demand:

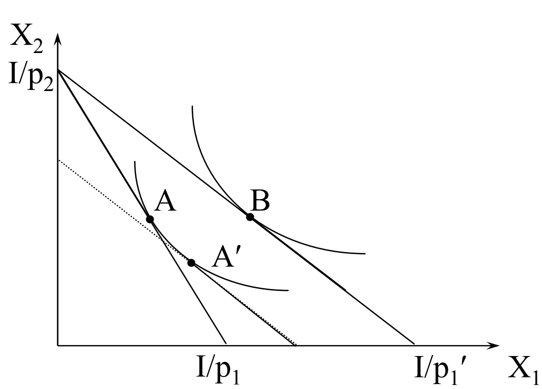


Example:

- Effect of *rise in the price of gasoline*:
- *Elasticities of demand*:
- 1% rise in gas price will *reduce* gas consumption by 0.27% in the short run (within several months), and by 0.71 % in the long run (say, several years)
- Long-run elasticity is 2-3x larger than short- run elasticity

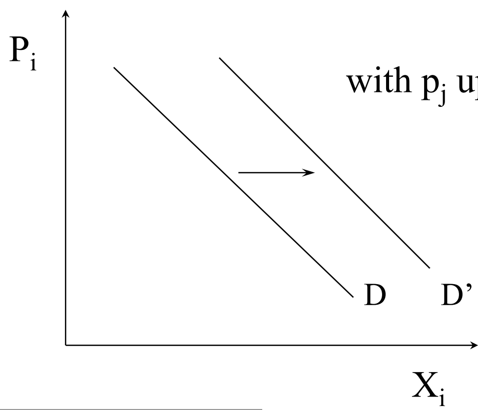
Change in another good's price:

- Effect on the demand for x_2 (+ or -):



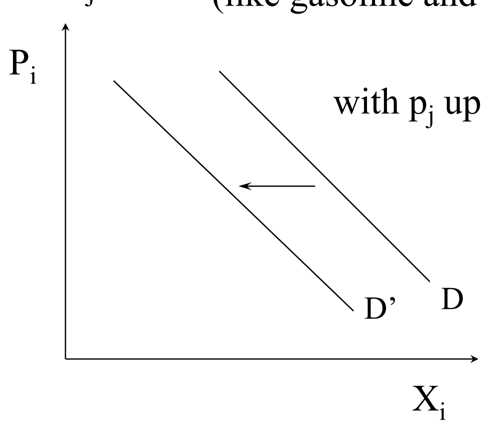
Substitutes:

– if $\frac{\partial x_i}{\partial p_j} > 0$ goods i and j are substitutes
– (like bus and subway)
– with p_j up:



Complements:

– if $\frac{\partial x_i}{\partial p_j} < 0$ goods i and j are complements
– (like gasoline and large cars)
– with p_j up:



Example: Gasoline and Traffic Levels

- A 1% increase in the price of gas will *reduce* traffic levels by 0.16 % in the short-run, and by 0.33 % in the long-run
- These are “cross-price” elasticities: driving and gasoline are *complements.*
- **Implication** (compared to gas elasticities):
- people must be switching to more fuel-efficient vehicles (and trips) in SR and especially in the LR

Example: Public Transport and Gasoline

- A 1% increase in bus fares will *reduce* demand by 0.41 %
- A 1% increase in rail fares will *reduce* demand by 0.79 %
- Rail demand is *more elastic* than bus
- A 1% increase in the price of gas will *increase* demand for public transportation by 0.34 %
- Public Transportation and gasoline are *substitutes.*

Example: Cobb-Douglas Utility Function

$$\max \ u(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha} \ \text{ s.t. } \ p_1 x_1 + p_2 x_2 \le I$$

- Use a Lagrangian, with λ=Lagrange multiplier

$$L = x_1^{\alpha} x_2^{1-\alpha} + \lambda(I - p_1 x_1 - p_2 x_2)$$

- Find first-order condition w.r.t x_1, x_2:

$$\frac{\partial L}{\partial x_1} = \alpha x_1^{\alpha-1} x_2^{1-\alpha} - \lambda p_1 = 0 \Rightarrow \alpha u = \lambda p_1 x_1$$

$$\frac{\partial L}{\partial x_2} = (1-\alpha) x_1^{\alpha} x_2^{-\alpha} - \lambda p_2 = 0 \Rightarrow (1-\alpha) u = \lambda p_2 x_2$$

Cobb-Douglas Utility Function (cont'd)

- From these two conditions, we solve for,

$$u = \lambda(p_1 x_1 + p_2 x_2)$$

- Also find first-order condition w.r.t λ:

$$\frac{\partial L}{\partial \lambda} = I - (p_1 x_1 + p_2 x_2) = 0 \Rightarrow u = \lambda I$$

- so that we have solved for $\lambda = u / I$
- This is the "marginal utility of income", i.e. effect on utility of loosening budget constraint

Cobb-Douglas Utility Function (cont'd)

- substitute this into the earlier conditions:

$$x_1 = \frac{\alpha u}{\lambda p_1} = \frac{\alpha I}{p_1}, \quad \frac{\partial x_1}{\partial p_1} = -\frac{\alpha I}{p_1^2} < 0, \quad \frac{\partial x_1}{\partial p_2} = 0$$

- so demand slopes down, and zero effect wrt. other price

$$x_2 = \frac{(1-\alpha)u}{\lambda p_2} = \frac{(1-\alpha)I}{p_2}, \quad \frac{\partial x_2}{\partial p_2} = -\frac{(1-\alpha)I}{p_2^2} < 0$$

Elasticity of Demand

$$E_i = -\frac{\partial x_i}{\partial p_i}\frac{p_i}{x_i} = -\frac{\partial \ln x_i}{\partial \ln p_i}$$

- For the Cobb-Douglas function:

$$E_i = -\frac{\partial x_i}{\partial p_i}\frac{p_i}{x_i} = 1$$

- In general, $E_i > 1$ means that demand is *elastic*
- $E_i < 1$ means that demand is *inelastic*

Elasticity of Demand (cont'd)

- For the Cobb-Douglas function:
- E_i =1 and the budget shares $p_i x_i/I$ are constant

- In general, $E_i > 1$ means that demand is *elastic,* and the budget share $p_i x_i/I$ will *increase* as we move down the demand curve

- While $E_i < 1$ means that demand is *inelastic,* and the budget share $p_i x_i/I$ will *decrease* as we move down the demand curve