How Big is China?

And Other Puzzles in The Measurement of Real GDP

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1. Introduction

The latest International Comparison Project (ICP) numbers from the World Bank show that real GDP per capita China was 40% smaller in 2005 than real GDP for the same year in earlier rounds of the ICP.[1] Maddison (2007) argues that such a downward revision for China is implausible because extrapolating backwards it would imply per capita income below subsistence levels in early years. This observation raises the questions of why the downward revision occurred and whether alternative calculations would give noticeably different results. The first of these questions has been addressed by Heston (2007) and Deaton and Heston (2010).[2] We will provide some theoretical structure to help understand their critique, and then address the second question: whether alternative, theoretically consistent calculations of real GDP make a difference, for China or other countries.

Among the reasons provided by Deaton and Heston for why the relative position of Asia was lowered in the most recent ICP round are the following:

1) The price data provided by China for the most recent ICP was for urban areas, and may have overstated the actual prices faced by rural consumers.

This first issue is a well-recognized feature of the price data provided for the 2005 ICP, which was the first time that China participated fully in the round. While Deaton and Heston note that there is a theoretical issue of aggregating over large countries with diverse consumer prices between regions, we do not address that issue here. Rather, we treat the urban bias in the prices report from China to the 2005 ICP as empirical issue to be investigated.

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[1] Deaton and Heston (2008) compare the most recent ICP round for 2005, published in 2008, with earlier figures: "the 2007 version of the World Development Indicators (WDI), World Bank (2007), lists 2005 per capita GDP for China as $6,757 and for India as $3,452, both in current international dollars. The 2008 version, World Bank (2008a), which includes the new ICP data, gives, for the same year, and the same concept $4,088 for China and $2,222 for India. For comparison, GDP per capita at market exchange rates is $1,721 for China and $797 for India."

[2] See also the comments by Diewert (2010).

2) Different index number methods provide different relative sizes of countries.

There are very substantial differences in the index numbers methods used by the ICP, for example, and the methods used by Penn World Table (PWT). We focus here on just one aspect of those differences, namely, the use of fixed-weight indexes used in PWT with flexible-weight index used by the ICP. Neary (2008) has recently proposed an alternative that requires estimating the expenditure function across countries to obtain indexes of real consumption. In section 2 we use all these methods, along with a recommendation by Barnett, Diewert and Zellner (2009), to calculate the size of real consumption in China. In addition to showing the differences between the various index number approaches we use two differences sets of prices for China: the prices they reported to the 2005 ICP, and our own estimates of alternative prices that adjust for urban versus rural differentials. We find that the impact of adjusting China's prices is quite large: real consumption in China is 10 to 20% higher using our adjusted prices than using the ICP prices, which is much greater than the differences arising from the index number methods.

Closely related to the index number issue but conceptually distinct is:

3) There are several different concepts of real GDP that can be measured, and each of which can provide different relative sizes of countries.

Shifting attention from just real consumption to real GDP, in sections 3 and 4 we incorporate investment, government spending, exports and imports. This raises new conceptual issues. For example, Deaton and Heston discuss the assumption of "equal productivity" across countries that is used to measure the output of service sectors, including the government. We do not deal with that issue, but instead focus on incorporating exports and imports into the measurement of real GDP.

Feenstra et al (2009) have recently contrast real GDP measured on the expenditure-side of the economy, as in the ICP and in PWT, with real GDP measured on the output-side. These two concepts differ by countries terms of trade, i.e. by the prices of their exports and imports. In section 4, we provide calculations of China's real GDP measured on the production side, taking into account its terms of trade. Feenstra et al (2009) dealt only with fixed-weight indexes, and here we generalize their results to obtain flexible-weight indexes of real GDP on the output-side.

The final issue we shall investigate is:

4) The relative size of China today should depend on its growth rate from prior years, but that growth rate changes with each new ICP benchmark.

The issue of changing growth rates in PWT as each new benchmark is incorporated was highlighted by Johnson et al (2009) and is discussed by Deaton and Heston. In section 4, we make a new recommendation that would eliminate this dependence on the benchmark, not just for China but for all other countries, too.

In section 2 we focus on the measurement of real consumption across countries, using both fixed and flexible-weight indexes and Neary's (2008) method that involves estimating the expenditure function. We compare results obtained with the price data reported by China to the 2005 ICP and our own alternative prices. Then we shift to real GDP in total, discussing real GDP on the expenditure-side in section 3 and real GDP on the output-side in section 4. Those concepts are applied again to the 2005 ICP data to re-compute the relative size of China. In section 5 we generalize the discussion to include the measurement of real GDP across countries and over time, making a specific suggestion for how the growth of real GDP in PWT should be computed. Conclusions are given in section 6. Additional empirical results are in Appendix A and the proofs of propositions are in Appendix B.

2. Real Consumption

In Table 1 and the Appendix Table A1 we show various calculations of real consumption using data for 124 countries from the 2005 International Comparisons Project (ICP).[3] There are 12 categories of consumption goods that we aggregate to compute real consumption. We report results for selected countries in Table 1, with results for all 124 countries shown in Appendix Table A1.

2.1 Fixed and Flexible Weight Indexes

The first calculation in Table 1 is the GK (Geary, 1958, Khamis, 1970, 1972) system, as used by the Penn World Table (PWT), but applied here to only i =1,…,M final consumption goods, with prices p_{ij} and quantities q_{ij} across countries j = 1,…,C. The reference prices π_i^{GK} and the purchasing power parities or real exchange rate RE_j are defined as the solution to the simultaneous system:[4]

$$\pi_i^{GK} = \sum_{j=1}^{C} RE_j p_{ij} q_{ij} \Big/ \sum_{j=1}^{C} q_{ij} \ , \qquad i = 1,\ldots,M, \tag{1}$$

$$RE_j = \sum_{i=1}^{M} \pi_i^{GK} q_{ij} \,/\, \sum_{i=1}^{M} p_{ij} q_{ij} \,, \qquad j = 1,\ldots,C, \tag{2}$$

subject to a normalization. Notice that the real exchange rate in (2) is a Laspeyres price index calculated using the fixed quantities q_{ij}. It is used to adjust expenditure in national currency to obtain that in reference prices, or real consumption:

$$\sum_{i=1}^{M} \pi_i^{GK} q_{ij} = RE_j \sum_{i=1}^{M} p_{ij} q_{ij} \,, \qquad j = 1,\ldots,C. \tag{3}$$

[3] The total number of countries in the 2005 ICP is 146, but we omitted 22 countries with missing prices for some consumption goods.
[4] Neary refers to (2) as a real exchange rate, and distinguishes it from purchasing power parity as used in PWT, which is instead defined as the reciprocal of (2), i.e. expenditure at domestic prices relative to expenditure at reference prices. We shall use Neary's convention in (2) throughout this paper.

Because real consumption for different countries is computed as in (3) using fixed quantities q_{ij}, we refer to it as a "fixed-weight" index. It is reported in column (2) of Table 1, relative to the United States. We see that real consumption in China is only 5.9% of that in the U.S. in 2005. This estimate is more than twice as large as what we get from comparing nominal consumption in U.S. dollars using official exchange rates in column (1), but is *much smaller* than the figure for *total* real GDP per capita (including C, I G, and X–M) relative to the U.S. of 9.8% from the 2005 ICP, let alone the estimate of 18.5% for real GDP per capita that Maddison (2007, Table 5) claims is needed to avoid having Chinese living standards below subsistence in past decades.[5]

In column (3)-(5) we report various "flexible-weight" indexes, so named because they use index formulas that are known to be exact for underlying expenditure functions.[6] The first method, which is used by the ICP, is the so-called EKS system (or GEKS after Gini, 1931, Eltetö and Köves, 1964, and Szulc, 1964), which relies on Fisher quantity indexes in country *j* relative to a base country k:

$$Q_{jk}^{F} \equiv \left(\frac{\sum_{i=1}^{M} p_{ij} q_{ij}}{\sum_{i=1}^{M} p_{ij} q_{ik}} \right)^{0.5} \left(\frac{\sum_{i=1}^{M} p_{ik} q_{ij}}{\sum_{i=1}^{M} p_{ik} q_{ik}} \right)^{0.5}, \qquad j = 1,\ldots,C.$$

Because this comparison between countries is not transitive in general, we apply the following transformation to obtain the EKS system:

$$Q_{jk}^{EKS} \equiv \prod_{\ell=1}^{C} \left(Q_{\ell k}^{F} Q_{j\ell}^{F} \right)^{1/J}, \qquad j = 1,\ldots,C. \tag{4}$$

The EKS estimates of real consumption are shown in column (3), from which we see that

[5] As explained in note 1, World Bank (2008a), which includes the 2005 ICP data, gives real GDP per capita in China of $4,088 in 2005, as compared to $41,674 in the United States. The ratio of these gives the 9.8% figure.

[6] See Balk (2008), Diewert (1976, 1999) and Neary (2004). Neary questions whether the exact results for bilateral comparisons really extend to a multilateral context.

China is 5.7% of the U.S., which is even lower than in the GK system. More generally, the GK estimates of real consumption understate the EKS estimate for rich countries and overstate the EKS estimates for poor countries. Specifically, when measured relative to the United States, the GK estimates are less than the EKS estimates for most countries with higher nominal GDP per capita than South Korea (ranked 31st out of 124 countries), and greater than the EKS estimates for most countries with lower nominal GDP per capita than Macedonia (ranked 61st out of 124), with a mixed pattern in-between. Those two countries are included in Table 1, and we will argue below that the GK reference prices can be thought of as lying in-between the prices of South Korea and the United States.

An alternative flexible-weight system is the CCD (Caves, Christensen, Diewert , 1982a,b) index, which makes use of the Törnqvist index instead of the Fisher index. The index of real consumption in country *j* relative to consumption in country *k* measured using the Törnqvist index is given by:

$$Q_{jk}^{T} \equiv \prod_{i=1}^{M} \left(\frac{q_{ij}}{q_{ik}} \right)^{\frac{s_{ij}+s_{ik}}{2}} \quad \text{where } \ s_{ij} = \frac{p_{ij} q_{ij}}{\sum_{i=1}^{M} p_{ij} q_{ij}} \qquad j,k = 1,2,\ldots,C.$$

It is easy to see that these quantity indexes are not transitive unless the expenditure shares are the same in all the countries. The CCD index is a transitive index generated from the matrix of all binary Törnqvist indexes and is defined as:

$$Q_{jk}^{CCD} \equiv \prod_{\ell=1}^{C} \left(Q_{\ell k}^{T} Q_{j\ell}^{T} \right)^{1/C} \qquad \text{for } j,k = 1,2,\ldots,C. \tag{5}$$

The CCD indexes are reported in column (4) of Table 1, and are quite close to the EKS indexes.

We now turn to *indirect* CCD indexes. These are similar to the indexes defined in (5) except that the quantity indexes used on the right-hand side are derived indirectly. We first define an indirect real consumption or quantity index for country *j* relative to country *k* as the ratio of nominal expenditures deflated by a Törnqvist price index:

$$Q_{jk}^{IT} \equiv \left(\frac{\sum_{i=1}^{M} p_{ij} q_{ij}}{\sum_{i=1}^{M} p_{ik} q_{ik}} \right) \Bigg/ \prod_{i=1}^{M} \left(\frac{p_{ij}}{p_{ik}} \right)^{\frac{s_{ij}+s_{ik}}{2}} \qquad \text{for } j,k=1,2,\ldots,C.$$

The denominator on the right hand side is the Törnqvist price index. The indirect quantity indexes are again not transitive, so we can construct transitive indexes using the EKS approach leading to the indirect CCD indexes given by:

$$Q_{jk}^{ICCD} \equiv \prod_{\ell=1}^{C} \left(Q_{\ell k}^{IT} Q_{j\ell}^{IT} \right)^{1/C} \qquad \text{for } j,k=1,2,\ldots,C. \qquad (6)$$

The indirect CCD measures of real consumption are shown in column (5), and when measured relative to the U.S., are somewhat lower than the direct CCD measure for nearly every country in the sample.[7]

2.2 Indexes Based on the Expenditure Function

Neary (2004) has recently proposed that real consumption should be measured with an expenditure function. We shall use the expenditure function corresponding to the Almost Ideal Demand System (AIDS) (Banks et al, 1997):

$$\ln e(p,u) = \alpha_0 + \alpha' \ln p + \tfrac{1}{2} \sum_{i=1}^{M} \sum_{j=1}^{M} \gamma_{ij} \ln p_i \ln p_j + b(p) \ln u\,, \qquad (7)$$

where $b(p) = \eta \prod_i p_i^{\beta_i}$. We impose "money metric scaling", which leads to the following restrictions on the parameters: $\alpha_0 = 0, \;\; \eta = 1$. To ensure that the expenditure function is homogeneous of degree one in prices we require that $\sum_i \alpha_i = 1$ and $\sum_i \beta_i = \sum_i \gamma_{ij} = 0$ for all *j*,

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[7] This pattern likely reflects the fact that the United States itself is an outlier, and for most other choices of the base country we would find that the direct and indirect CCD indexes fluctuate above and below each other.

and to ensure that expenditure is increasing in utility we require that $b(p) > 0$. Neary provides software for estimating the AIDS expenditure function using the semi-parametric approach of Diewert and Wales (1988). We report the estimated parameter values using data on 124 countries and 12 commodity groups in Appendix Table A2.

The measure of real consumption at reference prices π is then:

$$e(\pi, u_j) = \sum_{i=1}^{M} \pi_i \, q_{ij}^{*}, \qquad j = 1,\ldots,\mathrm{C}, \tag{8}$$

where we use as asterisk to denote optimally chosen quantities, as contrasted with the fixed quantities in (1). From (7), the ratio of real consumption in country *j* relative to *k* is then:

$$\frac{e(\pi, u_j)}{e(\pi, u_k)} = \left(\frac{u_j}{u_k}\right)^{b(\pi)}. \qquad j = 1,\ldots,\mathrm{C}. \tag{9}$$

For any reference prices, we refer to (8) or (9) as a measure of real consumption based on the expenditure function.

It is helpful to relate these measures of real consumption from the expenditure function to the earlier flexible-weight indexes. Suppose that we let the reference price vector differ across countries, and set it equal to the geometric mean $\overline{p}_{jk}$, with components $\ln \overline{p}_{ijk} = \frac{1}{2}(\ln p_{ij} + \ln p_{ik})$. The following result draws on Feenstra, Hong and Prasada Rao (2009):

<u>Lemma 1</u>

If the expenditure function corresponds to the AIDS and the observed consumption is expenditure-minimizing, so that $q_{ij} = \partial e(p_j, u_j) / \partial p_{ij}$, $i = 1,\ldots,\mathrm{M}$, $j = 1,\ldots,\mathrm{C}$, then:

$$Q_{jk}^{IT} \approx \frac{e(\overline{p}_{jk}, u_j)}{e(\overline{p}_{jk}, u_k)}. \tag{10}$$

Proof: See Appendix B.

The approximation in Lemma 1 comes from the fact that the Törnqvist price index is not exact for the AIDS expenditure function. Rather, the exact price index for the AIDS system requires using an average value for the shares that differs slightly from the arithmetic mean (see Feenstra and Reinsdorf, 2000). In practice this adjustment to the shares will have minimal impact on the value of the price index, and any difference between the indirect CCD index and the expenditure-function-based index using the prices $\overline{p}_{jk}$ will come from violations of the hypothesis in Proposition 1, that the data are consistent with expenditure minimization.

We have computed (10) using the United States as the base country *k*, and find that the two sides of (10) are very close with the exception of a few countries – China in particular. In Table 1 we report the two sides of (10) *after further applying the EKS transformation* in (6), obtaining the indirect CCD comparison in column (5) and the transitive reference price comparison using the geometric-mean $\overline{p}_{j,us}$ in column (6). In practice, the indirect CCD and the transitive $\overline{p}_{j,us}$ comparison in columns (5) and (6) are not that close, which indicates that the EKS transformation makes a difference. But China remains one of the countries with the greatest difference between the indirect CCD and the transitive $\overline{p}_{j,us}$ comparison, reported as 0.055 and 0.052 compared to the U.S. in Table 1, or a difference of 6%, which is about the same as the difference between the two sides of equation (10) for China.

The finding that the approximation in (10) does not hold for China strongly suggests that its consumption shares are not optimal for the prices it reported to the 2005 ICP. It is clear from Asian Development Bank (2007) that the price surveys in China were restricted to 11 capital cities and the rural areas surrounding these 11 cities. The ADB, following the recommendation of an Expert Group, constructed a national average price using a method of extrapolation also described in ADB (2007). However, the extrapolation method did not make any explicit allowance for spatial price differences across different regions and across rural and urban regions of China. As a result the general consensus is that national average prices have a tendency to overstate the actual prices. Accordingly, in Appendix A we make an adjustment to the Chinese prices by predicting them from a simple model based on data for the other Asian countries. Of the 12 categories of consumption goods, we adjust five prices downwards (for food and non-alcoholic beverages; clothing and footwear; education; restaurants; and other goods and services), four prices upwards (for gross rent, fuel, power; medical and health services; transport; and recreation), and leave three prices unchanged due to lack of data to adjust them.

Real income comparisons based on the adjusted prices for China are presented at the bottom of Table 1. In the GK calculation, real consumption in China relative to the U.S. rises from 5.9% to 6.4%, which is a rise of 9% in Chinese real consumption. Similar increase are seen for the flexible-weight indexes using the EKS, CCD direct and CCD indirect methods. But interestingly, the comparison based on the expenditure function and the geometric-mean of country prices $\overline{p}_{j,us}$ in column (6) gives a *larger* increase in real consumption in China due to the adjustment in its prices: from 5.2% to 6.1%, for a increase of 16.3%. We will find even larger increases due to the adjustment in Chinese prices as we consider alternative reference prices.

2.3 Alternative Reference Prices

The calculation we have made so far using the expenditure function and the geometric-mean of country prices $\overline{p}_{j,us}$ is not a true reference-price comparison across countries, because the reference prices $\overline{p}_{j,us}$ themselves change. To obtain a fixed-reference-price vector, Neary (2004) proposes that it should be computed as the solution to:

$$\pi_i^{GAIA} = \sum_{j=1}^{J} (RE_j^* p_{ij}) q_{ij} \bigg/ \sum_{j=1}^{J} q_{ij}^* \ , \qquad i = 1,\ldots,\text{M}, \tag{11}$$

$$RE_j^* = \sum_{i=1}^{M} \pi_i^{GAIA} q_{ij}^* / \sum_{i=1}^{M} p_{ij} q_{ij} = \frac{e(\pi^{GAIA}, u_j)}{e(p_j, u_j)}, \ j = 1,\ldots,\text{C}. \tag{12}$$

which extends the GK system in (1)-(2) by using optimal quantities q_{ij}^* in the denominator of (11) and the numerator of (12). Notice that RE_j^* is the ratio of the expenditure function at two different prices, but constant utility, so it can be viewed as an exact cost-of-living index in the spirit of the Allen index. For this reason, Neary (2004) refers to (11)-(12) as the Geary-Allen International Accounts (GAIA).

We have computed the GAIA reference prices π^{GAIA} for the 124 countries and 12 consumption goods using Neary's software. We follow his procedure of first normalizing the prices of each goods by the arithmetic mean of the country prices, so that $\pi = 1$ is the sample mean of prices. In Table 2 we report this sample mean along with the actual U.S. prices, the Geary-Khamis reference prices π^{GK}, the GAIA reference prices π^{GAIA}, and the actual prices for South Korea. It can be seen that the GK and GAIA reference prices fall in-between those of the United States and South Korea for most commodities.

From (9), it is evident that real income at any reference prices π^A can be computed from real income at the geometric mean of country prices by:

$$\frac{e(\pi^A, u_j)}{e(\pi^A, u_k)} = \left[\frac{e(\overline{p}_{jk}, u_j)}{e(\overline{p}_{jk}, u_k)} \right]^{b(\pi^A)/b(\overline{p}_{jk})}, \tag{13}$$

so it is very easy to make the transformation between one reference price vector to another. At the bottom of Table 2 we report the values of $b(\pi^{GK})$=0.996 and $b(\pi^{GAIA})$=1.026, which can be compared to $b(1)=1$ at the sample mean.[8] With these values as well as $b(\overline{p}_{jk})$, we can easily compute the real consumption values shown in columns (7) and (8) of Table 1.[9]

Using both the GK and GAIA reference prices, we see in columns (7) and (8) of Table 1 that real consumption per capita in China is increased by 21% due to the adjustment in prices. Notice that columns (7) and (8) are themselves just simple power transformations of each other: from (13), we could compute the comparison at reference prices π^B from that at prices π^A by applying the power $b(\pi^B)/b(\pi^A)$. For this reason, it is perhaps not surprising that the correction to Chinese real consumption we find from adjusting its prices does not differ much across the various reference prices in columns (7), (8), or column (9), considered next.

A final reference price calculation we shall make is due to a suggestion by Barnett, Diewert and Zellner (2009). To compare the real consumption of two countries *j* and *k*, they recommend that *every country's price vector* p_ℓ, ℓ =1,…,C, be used, and then take the geometric mean of the resulting comparisons. From (9), this procedure results in:

$$\left[\prod_{\ell=1}^{C}\frac{e(p_\ell,u_j)}{e(p_\ell,u_k)}\right]^{1/C}=\left[\prod_{\ell=1}^{C}\left(\frac{u_j}{u_k}\right)^{b(p_\ell)}\right]^{1/C}=\left(\frac{u_j}{u_k}\right)^{\sum_{\ell=1}^{C}b(p_\ell)/C}. \qquad (14)$$

Thus, it is apparent that in the AIDS case this recommendation corresponds to the use of

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8 It should be stressed that high versus low values for *b* do not indicate high versus low country prices. Recall that $b(p)=\prod_i p_i^{\beta_i}$ and $\sum_{i=1}^{M}\beta_i=0$, so that $b(\lambda p)=b(p)$ is homogeneous of degree zero, meaning that this function does not distinguish between price vectors that are scalar multiples of each other.

9 Note that there is a highly *non-monotonic* relationship between country income per capita and $b(p_j)$, as illustrated for some of the countries in Table 1: the highest value $b(p_{us})$=1.054 is obtained for the U.S., which falls to $b(p_{kor})$=0.929 for South Korea, rising to $b(p_{mac})$=0.988 for Macedonia, falling again to $b(p_{ch})$=0.956 for the original Chinese prices and rising to $b(p_{in})$=0.992 for India and $b(p_{ch})$=1.033 for the adjusted Chinese prices.

reference prices π^D where $b(\pi^D) = \frac{1}{C}\sum_{\ell=1}^{C} b(p_\ell)$; we refer to these as the Diewert reference prices for brevity. For our sample of 124 countries we obtain $b(\pi^D) = 0.988$, and it turns out that $b(\pi^D) \approx [b(\pi_{us})b(\pi_{kor})]^{0.5}$, which means that the Diewert reference prices are equivalent to using the geometric mean of the U.S. and South Korean prices. With this values for $b(\pi^D)$ then column (9) of Table 1 is readily computed. Like the other reference price methods, real consumption in China is revised upwards by about 21% due to the adjustment in its prices. Using these prices, we find that real consumption per capita in China is 6.6% of that in the United States, which is the highest of any estimate shown in Table 1.

3. Real GDP on the Expenditure Side

PWT defines real GDP by using the fixed-weight index in (3) to convert nominal exports and imports, or nominal GDP, to common units across countries:

$$RGDE_j^{pwt}(\pi) \equiv (\text{Nominal } GDP_j)\, RE_j(\pi)$$

$$= \sum_{i=1}^{M} \pi_{ij} q_{ij} + (X_j - M_j)\, RE_j(\pi) \qquad (15)$$

where the equality follows from nominal $GDP_j = \sum_{i=1}^{M} p_{ij} q_{ij} + (X_j - M_j)$, where X_j and M_j are the values of export and imports. This is reported as the Laspeyres index of real GDP in PWT.

In future work we will make the calculation in (15) using the 2005 ICP data.

4. Real GDP on the Output-side

We will now establish results for real output. Suppose that the M final goods now include those used for consumption, investment and government purchases, all of which are non-traded. In addition, suppose that there are i =M+1,…,M+N intermediate inputs that can be both imported and exported (imports and exports are different varieties). This convention that all traded goods are by definition intermediate inputs follows the "middle products" approach of Sanyal and Jones (1983) and the "production approach" to modeling imports and exports of Diewert and Morrison (1986) and Kohli (2004), and we shall also follow it here.

Specifically, let us denote three groups of commodities:

- those for final domestic demand (quantities $q_{ij} \geq 0$ and prices $p_{ij} > 0$, for $i = 1,\ldots,M$);
- those for export (quantities $x_{ij} \geq 0$ and prices $p_{ij}^{x} > 0$, for $i = M+1,\ldots,M+N$);
- imported intermediate inputs (quantities $m_{ij} \geq 0$ and prices $p_{ij}^{m} > 0$, $i = M+1,\ldots,M+N$).

The free trade price vectors for exports and imports are p_j^x and p_j^m in country j, and domestic prices are $p_j^x + s_j$ and $p_j^m + t_j$, where s_j is the vector of export subsidies and t_j is the vector of import tariffs. The column vector of prices is then $P_j = (p_j, p_j^x + s_j, p_j^m + t_j)$, and we let $y_j \equiv (q_j, x_j, -m_j)$ denote the corresponding column vector of outputs and inputs. Then the revenue function for the economy is defined as:

$$r_j(P_j, v_j) \equiv \max_{q_{ij}, x_{ij}, m_{ij} \geq 0,} \left\{ P_j' y_j \;\middle|\; F_j(y_j, v_j) = 1 \right\}, \tag{16}$$

where $F_j(y_j, v_j)$ is a transformation function for each country, which depends on the vector v_j representing primary factor endowments in country j.

3.1 Real Output with Reference Prices

We will distinguish the reference prices π_i for final goods, i =1,…,M, and two sets of reference prices π_i^x, π_i^m for exports and imported intermediate inputs, i =M+1,…,M+N. Denote the M+2N dimensional vector of reference prices by $\Pi = (\pi, \pi^x, \pi^m)$. We suppose that the country is engaged in *free trade* at these reference prices, and evaluate GDP on the output-side using the revenue function:

$$RGDO_j^*(\Pi) \equiv r_j(\Pi, v_j) \equiv \max_{q_{ij}, x_{ij}, m_{ij} \geq 0,} \left\{ \Pi' y_j \;\middle|\; F_j(y_j, v_j) = 1 \right\}. \tag{17}$$

Provided that $F_j(y_j, v_j)$ is sufficiently concave we can expect (17) to have a well-defined solution, which we denote by $q_{ij}^*, x_{ij}^*, m_{ij}^*$. Let us make this assumption on the revenue function:

Assumption 1

For all $\Pi > 0$, $r_j(\Pi, v_j)$ is positive, bounded above and continuously differentiable.

In economic terms, this assumption states that even if the price of an imported intermediate input is very high, the country can economize on its imports to still produce positive revenue. Conversely, the upper bound means that the economy cannot make arbitrarily high revenue by importing some inputs and exporting other goods.

To obtain real GDP on the output-side, we shall use the reference prices Π to obtain final demands, net outputs, exports and imports as $q_{ij}^* \equiv \partial r_j(\Pi, v_j) / \partial \pi_i$, i =1,…,M, and, $x_{ij}^* \equiv \partial r_j(\Pi, v_j) / \partial \pi_i^x$, $-m_{ij}^* \equiv \partial r_j(\Pi^*, v_j) / \partial \pi_i^m$, i =M+1,…,M+N. We assume that the sum across countries of each of these quantities is strictly positive:

Assumption 2:

For all $\Pi > 0$, $\sum_{j=1}^{J} q_{ij}^* > 0$, i = 1,…,M, and $\sum_{j=1}^{J} m_{ij}^* > 0$, $\sum_{j=1}^{J} x_{ij}^* > 0$, for i = M+1,…,N.

Notice that this assumption applies to the observed prices $P_j > 0$ and quantities, too.

Consider computing the reference prices as a weighted average of free-trade prices:

$$\pi_i = \sum_{j=1}^{J} (RO_j^* p_{ij}) q_{ij} \Big/ \sum_{j=1}^{J} q_{ij}^*, \qquad i = 1,\ldots,M, \tag{18a}$$

$$\pi_i^x = \sum_{j=1}^{J}(RO_j^* p_{ij}^x)x_{ij} \Big/ \sum_{j=1}^{J} x_{ij}^* \ , \qquad i = \text{M+1},\ldots,\text{M+N}, \tag{18b}$$

$$\pi_i^m = \sum_{j=1}^{J}(RO_j^* p_{ij}^m)m_{ij} \Big/ \sum_{j=1}^{J} m_{ij}^* \ , \qquad i = \text{M+1},\ldots,\text{M+N}, \tag{18c}$$

and,

$$RO_j^* = \frac{r_j(\Pi, v_j)}{r_j(P_j, v_j)}, \qquad j = 1,\ldots,\text{C}. \tag{19}$$

Thus, in (18) –(19) we use the *optimal* quantities in the denominators, but the *observed* quantities in the numerators. In (19), the real exchange rate on the output-side, RO_j^*, is computed by comparing real GDP at free-trade reference prices to nominal GDP. The system defined in (18a,18b,18c) and (19) is an extension of the GAIA system in Neary (2004) by introducing exports and imports explicitly into the system. In order to be able to use the system (18) – (19), we need to demonstrate the existence of a positive solution for π_i, π_i^x, π_i^m and RO_j^*. We follow Neary (2004) in proving the following result.

Proposition 1

Under Assumptions 1 and 2, there exists a positive solution for π_i, π_i^x, π_i^m and RO_j^* satisfying the system (18) – (19).

Proof: See Appendix B.

Using the reference prices coming from Proposition 1, or any other, we can make comparisons across countries of real GDP on the output-side – or *real output* for short – using the ratio of revenue functions:

$$\frac{r_j(\Pi, v_j)}{r_k(\Pi, v_k)}. \tag{20}$$

In practice, however, we should not expect to estimate the revenue functions as we did quite easily for the expenditure function, because the revenue functions are indexed by the country *j,* indicating technological differences across countries. Even with a parsimonious specification of such technological differences it would be difficult to estimate differing revenue functions while pooling across countries. For this reason, we now turn to the fixed-weight and flexible-weight indexes that can be used to establish bounds on real output.

3.2 Fixed and Flexible Weight Indexes

The simplest fixed-weight index that could be used to measure real output across countries is the ratio of quantities in the two evaluated at the prices of one country or the other. According to the Gerschenkron effect, we expect that these ratios are related by:

$$\left(\frac{P_j' y_j}{P_j' y_k}\right) < \left(\frac{P_k' y_j}{P_k' y_k}\right). \tag{21}$$

This inequality states that real GDP is higher when measured with the prices of another country, or "the grass is greener on the other side." Another way to interpret (21) is that the Laspeyres quantity index (on the right) exceeds the Paasche quantity index (on the left). That inequality is familiar from consumer theory, but what is surprising is that the same inequality tends to hold empirically even for production data. That inequality was originally found by Gerschenkron (1951) and has been confirmed in many subsequent studies.

Diewert (1983) refers to the measure of real output in (20) as a Samuelson-Swamy-Sato index. An alternative measure of real output, referred to as the Malmquist index, can be obtained by using the distance between the transformation functions. Specifically, Caves, Christensen and Diewert (1982a,b) define two measures of the growth in real output δ_j and δ_k as follows:

$$F_j(y_k\delta_k, v_j) = 1 \text{ and } F_k(y_j / \delta_j, v_k) = 1. \qquad (22)$$

To interpret the first of these conditions, it states that if we start with the observed output vector for country *k*, and inflate it by δ_k, then we obtain an output vector that is feasible to produce with the technology and endowments of country *j*. Likewise, the second condition states that by deflating the observed output vector in country *j* by δ_j, we obtain an output vector that is feasible to produce in country k.

The question then arises as to how to measure Malmquist distance factors defined by (22) without full knowledge of the transformation function. Caves, Christensen and Diewert (1982a,b) provide a powerful result by showing that if the transformation function is translog, then the geometric mean of δ_j and δ_k is measured by a Törnqvist quantity index of the outputs. We instead provide a result that does not depend on the form of the transformation function, but just relies on having observed output vectors be revenue-maximizing at the observed prices:

Proposition 2

Suppose that the outputs are revenue-maximizing and the Gerschenkron Effect in (21) holds. Then: there exists a reference price vector Π in-between P_j and P_k such that:

$$\delta_k \leq \frac{r_j(\Pi, v_j)}{r_k(\Pi, v_k)} = \left[\left(\frac{P_j' y_j}{P_j' y_k}\right)\left(\frac{P_k' y_j}{P_k' y_k}\right)\right]^{0.5} \leq \delta_j. \qquad (23)$$

Proof: See Appendix B.

This result says that computing a Fisher quantity index between the countries will be a valid comparison of real GDP between them. Remarkably, it does not depend on the functional form of the revenue function but only on optimizing behavior. It gives a strong rationale for extending the

EKS calculation to incorporate exports (with positive sign) and imports (with negative sign) within the calculation of Fisher quantity index.

In future empirical work, we intend to compute the GK system as well as the EKS indexes, extended to include export and imports, for all 124 countries in the 2005 ICP.

4. Comparisons over Time

We conclude with a discussion of the comparison of real GDP over time. The practical implications of this issue have recently been raised by Johnson et al (2009), who point out that each new ICP benchmark leads to not only a new set of real GDP levels in one year, but also a nontrivial revision of past PWT *growth rates*. Therefore, results from studies making use of one version of PWT may change when a new version is released. This issue also impacts China, in so far as the current level of real GDP is judged to be "reasonable" or not based on extrapolating backwards to see what real GDP has been in years past. As noted earlier, Maddison (2007) criticizes the 2005 World Bank estimate for China as being "too low" because they imply real GDP well below subsistence in 1950. So the question of the appropriate level and the appropriate growth-rates for real GDP appear to be linked. We will make a new recommendation here that eliminates such a link between new ICP benchmark data and past growth rates, while being theoretically consistent with the analysis earlier in the previous section.

Our recommendation follows quite directly from Proposition 2, but applied in a time-series rather than a cross-country context. Specifically, suppose that we start in situation where we have two reference price vectors at two points in time, Π_{t-1} and Π_t, used to make comparisons across countries. In order to also compare real output over time, it would be desirable to use a single vector Π and compute the ratios:

$$\frac{r_{jt}(\Pi, v_{jt})}{r_{jt-1}(\Pi, v_{jt-1})}, \tag{24}$$

for each country. Notice that the endowments in (24) can change over time, as well as the revenue function themselves due to technological change.

We can apply Proposition 2 by treating the bilateral comparison as between country j using reference prices Π_{t-1} and Π_t in the two periods. The optimal outputs at these prices are $y^*_{j\tau} \equiv \partial r_{j\tau}(\Pi_\tau, v_{j\tau}) / \partial \Pi_\tau, \tau = t-1, t.$ We use these optimal outputs to define the distance between the transformation functions in each period, as follows:

$$F_{jt}(y^*_{jt-1}\delta_{jt-1}, v_{jt}) = 1 \text{ and } F_{jt-1}(y^*_{jt} / \delta_{jt}, v_{jt-1}) = 1 .$$

These are analogous to the definitions in (22) in the cross-country context, but now use the optimal inputs at the reference prices each period.

Then an immediate application of Proposition 2 is that there will exist reference prices Π lying in-between Π_{t-1} and Π_t such that:

$$\delta_{t-1} \leq \frac{r_{jt}(\Pi, v_{jt})}{r_{jt-1}(\Pi, v_{jt-1})} = \left[\left(\frac{\Pi'_{jt-1} y^*_{jt}}{\Pi'_{jt-1} y^*_{jt-1}} \right) \left(\frac{\Pi'_{jt} y^*_{jt}}{\Pi'_{jt} y^*_{jt-1}} \right) \right]^{0.5} \leq \delta_t . \tag{25}$$

In words, this result states that we can take the geometric mean of the growth rates obtained at each vector of reference prices to obtain a constant-reference-price growth rate. To see the usefulness of this result, suppose that instead of using the *optimal* quantities in (25) we apply this formula using the *observed* quantities. Then the geometric mean appearing in (25) is taken over the growth in the fixed-weight Geary-Khamis measures of real GDP. Since PWT uses the GK method, this Corollary suggests that we should take geometric mean of growth rates in PWT as the reference prices changes. *In other words, we should take the geometric mean of the growth rates in PWT, or arithmetic mean of log growth rates, across different versions.*

This recommendation differs radically, however, from current practice in PWT. To see this, we introduce the notation that is used in PWT to compute the Laspeyres growth rate of real GDP on the expenditure-side.[10] We start with the fundamental additive property of the GK system shown by (15) for a benchmark year denoted by *t=0*. Breaking up the final goods into three groups C, I and G, and defining RC_{j0}^{pwt}, RI_{j0}^{pwt} and RG_{j0}^{pwt} as real expenditure $\sum_i \pi_{ij0} q_{ij0}$ for the goods in each group, along with $RX_{j0}^{pwt} \equiv X_{j0} PPE_{j0}$ and $RM_{j0}^{pwt} \equiv M_{j0} PPE_{j0}$ as real exports and imports as shown in (15), we have:

$$RGDPE_{j0}^{pwt} \equiv RC_{j0}^{pwt} + RI_{j0}^{pwt} + RG_{j0}^{pwt} + RX_{j0}^{pwt} - RM_{j0}^{pwt}. \tag{26}$$

To obtain real GDP in period *t,* PWT then relies on national-accounts growth rates for the components, C, I, G, X and M. Specifically, each component RZ_{jt} is updated as:

$$RZ_{jt}^{pwt} = RZ_{j0}^{pwt} \left(\frac{Z_{jt}^{rna}}{Z_{j0}^{rna}} \right), \qquad Z = C,I,G,X,M, \tag{27}$$

where Z_{jt}^{rna} is the real value of that component from the national accounts. Then the Laspeyres measures of real GDP in PWT is computed as:

$$RGDPE_{jt}^{pwt} \equiv RC_{jt}^{pwt} + RI_{jt}^{pwt} + RG_{j1}^{pwt} + RX_{j1}^{pwt} - RM_{j1}^{pwt}. \tag{28}$$

Using (26)-(28), simple algebra shows that:

$$\frac{RGDPL_{jt}^{pwt}}{RGDPL_{jt-1}^{pwt}} - 1 = SC_{jt-1}^{0}\left(\frac{C_{jt}^{rna}}{C_{jt-1}^{rna}} - 1 \right) + SG_{jt-1}^{0}\left(\frac{G_{jt}^{rna}}{G_{jt-1}^{rna}} - 1 \right) + SI_{jt-1}^{0}\left(\frac{I_{jt}^{rna}}{I_{jt-1}^{rna}} - 1 \right)$$

$$+ SX_{jt-1}^{0}\left(\frac{X_{jt}^{rna}}{X_{jt-1}^{rna}} - 1 \right) - SM_{jt-1}^{0}\left(\frac{M_{jt}^{rna}}{M_{jt-1}^{rna}} - 1 \right), \tag{29}$$

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[10] The extension to the growth of real GDP on the output-side will be evident.

where $SZ_{jt}^{0} \equiv RZ_{jt}^{pwt} / RGDPE_{jt}^{pwt}$ is the share of component Z in real GDP, which is based on the benchmark year data as in (27). The magnitude in (29) is reported as the *growth rate of Laspeyres real GDP* in PWT. It is a weighted average of the national-account growth rates of the components, using the real shares based on benchmark period. As benchmarks change in each version of PWT, then the weights change and so, too, will past growth rates of real GDP. This is the criticism that is raised by Johnson et al (2009).

To address this, suppose we take the simple average of growth rates from two successive versions of PWT, using benchmarks years *0* and *1.* Then for years *t-1* and *t* in-between these benchmarks, the simple average of the reported growth rates from these versions of PWT is:

$$\sum_{Z=C,I,G,X} \frac{1}{2}(SZ_{jt-1}^{0} + SZ_{jt}^{1})\left(\frac{Z_{jt}^{rna}}{Z_{jt-1}^{rna}} - 1\right) - \frac{1}{2}(SM_{jt-1}^{0} + SM_{jt}^{1})\left(\frac{M_{jt}^{rna}}{M_{jt-1}^{rna}} - 1\right). \qquad (30)$$

We view (30) as the natural fixed-weight analogue to equation (25), expressed as an arithmetic mean over log growth rates of the components of real GDP. Proposition 2 therefore provides a theoretical rationale for using the simple average of the shares in (30), i.e. for using the simple average of growth rates from successive versions of PWT. Furthermore, by computing real growth in this fashion, *future benchmarks will not affect growth rates before the previous benchmark,* since those past growth rates are fixed by (30).

6. Conclusions

In this paper we have attempted to address a number of unresolved questions in the current practice of international comparisons as reflected in the 2005 round of the International Comparison Program. Drawing on our recent work (Feenstra, Ma and Rao, 2009) on constant-price based real income comparisons, our first objective was to examine the sensitivity of real consumption comparisons to the choice of the index number methods used. We include the fixed weight GK index, several variations of flexible weight indexes including the EKS, CCD and the indirect CCD indices, expenditure function based indexes due to Neary, and, finally the method suggested by Barnett, Diewert and Zellner (2009). We have shown that their suggestion comes under the general class of constant-price based comparisons in Feenstra, Ma and Rao (2009), and that implementation of their suggestion is roughly equivalent to the geometric mean of the U.S. and South Korean prices.

All the empirical results reported here are based on data drawn from the 2005 ICP benchmark comparisons reported in World Bank (2008b). Based on the framework discussed here, the paper also attempts to provide revised estimates for real consumption in China after making adjustments to prices reported and used in the 2005 ICP comparisons in the Asia-Pacific. Making use of a regression model to explain commodity-specific price levels as a function of real per capita income (in index form), and applying the resulting adjustments to price data for China, we measure the revisions to be anywhere between 9% to 21% depending upon the index number methodology and the reference price vector used. Our results confirm that an upward revision of real consumption in China somewhere between 10 to 20 percent may be quite realistic. However such a revision does not necessarily imply a corresponding revision to real GDP as consumption is only a part of the GDP.

On the theoretical side, the major contribution of the paper lies in the framework it provides to make real GDP comparisons from the output side using revenue functions. The approach is similar to the expenditure side approach in Neary (2004) and the paper shows that under mild conditions there exist a set of positive output, export and import price vectors that can be used in making real output comparisons using revenue functions that allow for substitution. We have been able to show that in the context of binary comparisons involving two countries, there exists a reference price vector such that real income comparison based on the reference price vector equals the Fisher price index. The surprising aspect of this result is that it is derived without making any assumptions concerning the functional form of the revenue function, instead it just depends upon the optimality of the observed quantities for the two countries.

The final part of the paper focused on real income comparisons over time. The paper recommends a new method of updating real GDP between two benchmarks within the general framework of the Penn World Tables, which addresses the current problem where the past growth rates of real GDP are affected by the introduction of new benchmarks. The paper provides a theoretical rationale for using the arithmetic average of the shares observed at two given benchmarks for all the years in between the two benchmarks, thus ensuring that these growth rates are not affected by future benchmarks.

Appendix A: Data and Estimates

In Table A1 we report the calculations of real consumption for all 124 countries. In Table A2 we report the estimates of the AIDS expenditure function, listing first the R^2 values for the share equations for each product and then the parameters α, β and Γ.[11]

In Table A3 we report the correction made to Chinese prices for 2005. As noted in the text, we used an regression approach to determine predicted prices for China. A simple regression model is used with the price level (ratio of PPP to exchange rate) for each commodity group is expressed as a function of the real income per capita index. The real income per capita index is computed using the real per capital income estimates from the ICP Asia-Pacific 2005 which are based on the EKS index number formula. The real incomes are expressed as index relative to the Asian Region =100. All the relevant data are drawn from the Final Report of the ICP Asia-Pacific (ADB, 2007) and the sample is restricted to all Asian countries except China. The total number of countries in the Asia-Pacific comparison was 23 including China..[12]

A log-linear model with a dummy variable for Fiji is our chosen model, and a separate regression is run for each of the commodities except alcoholic beverages. [13] The predicted prices from these regressions are shown in Table A2. Using these predicted or "adjusted" prices for China we also re-estimated the AIDS expenditure function, but it is essentially unchanged from Table A1 because data for only one country out of 124 has been adjusted. Using the AIDS estimates we can compute the estimated shares for China, using actual or predicted prices. These are shown in Table A2 together with the actual shares. It can be seen that the adjustment to the

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[11] The R^2 values are computed for each commodity group as the square of the correlation coefficient of the observed and predicted shares from the estimated share equations.
[12] As fixity of the regional comparisons was maintained in the global comparisons and that there could be regional differences in the regression relationships, it was decided that only data on countries from the Asian region would be utilized in the regressions.
[13] A linear model with a dummy variable for Fiji also performed well. Results from the log-linear model are quite similar to those obtained using a linear model.

shares is not that large, but nevertheless, the estimated shares using predicted prices are closer to the actual shares for food and non-alcoholic beverages, recreation, and restaurants.

Table A1: Comparisons of Real Consumption

Countries	Nominal consump.	G-K	EKS	CCD direct	CCD indirect	Using expenditure function and π: $\overline{p}_{j,us}$	Geary	Neary	Diewert
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Iceland	**1.248**	**0.767**	**0.794**	**0.795**	**0.784**	**0.768**	**0.790**	**0.784**	**0.791**
Norway	1.100	0.737	0.758	0.768	0.751	0.736	0.755	0.749	0.757
Luxembourg	1.071	0.925	0.919	0.924	0.923	0.896	0.912	0.909	0.913
Switzerland	1.043	0.726	0.739	0.749	0.735	0.727	0.749	0.743	0.751
United States	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
Denmark	0.982	0.663	0.685	0.690	0.681	0.677	0.686	0.678	0.688
United Kingdom	**0.895**	**0.772**	**0.793**	**0.807**	**0.796**	**0.767**	**0.778**	**0.772**	**0.779**
Ireland	0.843	0.638	0.662	0.648	0.658	0.652	0.663	0.655	0.665
Sweden	0.838	0.670	0.693	0.696	0.686	0.684	0.693	0.686	0.695
Austria	0.814	0.758	0.779	0.792	0.777	0.751	0.768	0.761	0.769
France	0.775	0.693	0.716	0.726	0.712	0.698	0.704	0.696	0.706
Finland	0.773	0.601	0.623	0.630	0.618	0.612	0.620	0.611	0.623
Netherlands	**0.756**	**0.696**	**0.720**	**0.736**	**0.722**	**0.707**	**0.697**	**0.689**	**0.699**
Japan	0.743	0.626	0.618	0.625	0.609	0.592	0.658	0.650	0.660
Belgium	0.735	0.653	0.671	0.679	0.667	0.659	0.671	0.663	0.673
Canada	**0.731**	**0.722**	**0.735**	**0.733**	**0.729**	**0.715**	**0.739**	**0.733**	**0.741**
Germany	0.727	0.662	0.670	0.684	0.674	0.658	0.669	0.660	0.671
Australia	0.722	0.678	0.701	0.702	0.694	0.686	0.703	0.695	0.704
Italy	0.679	0.622	0.636	0.648	0.633	0.613	0.631	0.623	0.634
New Zealand	0.606	0.574	0.590	0.599	0.584	0.571	0.594	0.585	0.597
Cyprus	0.597	0.647	0.670	0.691	0.666	0.642	0.667	0.659	0.669
Spain	0.582	0.624	0.648	0.677	0.647	0.622	0.632	0.623	0.634
Greece	0.541	0.610	0.637	0.667	0.633	0.608	0.629	0.620	0.631
Hong Kong, China	0.504	0.643	0.634	0.635	0.624	0.596	0.634	0.625	0.636
Portugal	0.441	0.489	0.501	0.515	0.500	0.491	0.501	0.491	0.504
Israel	0.422	0.501	0.516	0.520	0.509	0.505	0.516	0.506	0.519
Malta	0.397	0.548	0.573	0.593	0.570	0.555	0.575	0.565	0.577
Slovenia	0.378	0.484	0.499	0.505	0.494	0.484	0.503	0.493	0.506
Singapore	0.377	0.506	0.498	0.503	0.495	0.476	0.506	0.495	0.508
Taiwan, China	0.324	0.566	0.549	0.554	0.537	0.517	0.572	0.563	0.575
South Korea	**0.297**	**0.375**	**0.375**	**0.378**	**0.366**	**0.350**	**0.381**	**0.370**	**0.383**
Bahrain	0.277	0.414	0.387	0.393	0.388	0.374	0.385	0.374	0.388
Czech Republic	0.238	0.425	0.429	0.436	0.424	0.413	0.432	0.421	0.434
Hungary	0.237	0.391	0.399	0.404	0.394	0.387	0.403	0.392	0.406
Macao, China	0.236	0.340	0.348	0.360	0.343	0.325	0.345	0.334	0.348
Croatia	0.226	0.341	0.347	0.353	0.342	0.332	0.349	0.338	0.352
Estonia	0.220	0.366	0.378	0.384	0.372	0.363	0.380	0.369	0.383

Poland	0.183	0.332	0.329	0.335	0.326	0.318	0.330	0.319	0.333
Lithuania	0.179	0.357	0.368	0.373	0.362	0.356	0.372	0.362	0.375
Slovak Republic	0.179	0.354	0.354	0.360	0.349	0.339	0.355	0.344	0.358
Mexico	0.175	0.289	0.292	0.298	0.289	0.280	0.290	0.279	0.292
Lebanon	0.169	0.284	0.277	0.283	0.274	0.274	0.285	0.274	0.287
Oman	0.153	0.249	0.237	0.240	0.236	0.227	0.237	0.227	0.239
Latvia	0.153	0.302	0.305	0.310	0.300	0.293	0.307	0.297	0.310
Chile	0.144	0.225	0.231	0.234	0.229	0.227	0.237	0.227	0.240
Mauritius	0.123	0.244	0.247	0.249	0.242	0.236	0.247	0.237	0.250
Uruguay	0.122	0.214	0.220	0.223	0.218	0.216	0.224	0.215	0.227
South Africa	0.118	0.180	0.185	0.188	0.183	0.182	0.190	0.181	0.193
Turkey	0.116	0.183	0.183	0.185	0.180	0.175	0.183	0.174	0.186
Romania	0.113	0.227	0.231	0.233	0.227	0.221	0.233	0.223	0.236
Brazil	0.101	0.174	0.177	0.180	0.175	0.177	0.185	0.175	0.187
Argentina	0.099	0.232	0.237	0.243	0.234	0.229	0.239	0.229	0.242
Bosnia & Herzegovina	0.099	0.208	0.210	0.215	0.207	0.201	0.212	0.202	0.214
Russian Federation	0.093	0.243	0.248	0.253	0.244	0.243	0.255	0.245	0.258
Fiji	0.092	0.115	0.115	0.115	0.113	0.116	0.121	0.114	0.123
Bulgaria	0.091	0.239	0.240	0.244	0.235	0.230	0.242	0.232	0.244
Serbia	0.090	0.217	0.212	0.214	0.208	0.199	0.210	0.201	0.213
Venezuela, RB	0.089	0.163	0.167	0.170	0.164	0.159	0.167	0.158	0.169
Montenegro	0.087	0.174	0.174	0.175	0.170	0.165	0.173	0.164	0.175
Malaysia	0.084	0.177	0.176	0.180	0.174	0.169	0.177	0.168	0.180
Macedonia, FYR	**0.076**	**0.192**	**0.192**	**0.195**	**0.189**	**0.184**	**0.193**	**0.184**	**0.196**
Jordan	0.070	0.128	0.122	0.125	0.122	0.119	0.125	0.117	0.127
Tunisia	0.068	0.140	0.140	0.144	0.138	0.132	0.139	0.131	0.141
Albania	0.068	0.137	0.140	0.143	0.138	0.134	0.140	0.132	0.142
Gabon	0.067	0.103	0.098	0.098	0.096	0.089	0.092	0.085	0.093
Kazakhstan	0.066	0.210	0.195	0.195	0.190	0.186	0.195	0.186	0.197
Namibia	0.064	0.095	0.094	0.095	0.092	0.090	0.094	0.087	0.095
Peru	0.063	0.136	0.141	0.145	0.140	0.136	0.143	0.135	0.145
Colombia	0.063	0.136	0.137	0.139	0.135	0.131	0.137	0.129	0.139
Equatorial Guinea	0.062	0.090	0.089	0.090	0.088	0.084	0.086	0.080	0.088
Ecuador	0.061	0.135	0.140	0.142	0.138	0.134	0.140	0.132	0.142
Belarus	0.061	0.233	0.220	0.222	0.216	0.207	0.218	0.208	0.220
Cape Verde	0.060	0.080	0.082	0.082	0.080	0.080	0.083	0.077	0.084
Botswana	0.057	0.100	0.098	0.096	0.096	0.092	0.095	0.089	0.097
Thailand	0.054	0.146	0.147	0.151	0.145	0.138	0.145	0.137	0.147
Swaziland	0.050	0.096	0.093	0.092	0.091	0.086	0.090	0.083	0.091
Morocco	0.044	0.072	0.073	0.075	0.072	0.071	0.074	0.069	0.076
Ukraine	0.040	0.167	0.155	0.158	0.153	0.149	0.157	0.148	0.159
Armenia	0.038	0.121	0.119	0.121	0.118	0.115	0.120	0.113	0.122
Syrian Arab Rep	0.036	0.095	0.094	0.089	0.093	0.093	0.097	0.090	0.099
Egypt, Arab Rep	0.035	0.120	0.119	0.121	0.118	0.115	0.120	0.113	0.122
Georgia	0.033	0.107	0.103	0.103	0.101	0.098	0.102	0.095	0.104
Paraguay	0.031	0.101	0.105	0.107	0.104	0.105	0.110	0.103	0.112
Sri Lanka	0.029	0.082	0.087	0.088	0.086	0.084	0.088	0.081	0.089
Indonesia	0.028	0.072	0.073	0.074	0.072	0.071	0.074	0.068	0.075
Philippines	0.026	0.069	0.070	0.071	0.069	0.067	0.070	0.064	0.071

Moldova	0.026	0.107	0.097	0.097	0.094	0.090	0.093	0.087	0.095
Lesotho	0.026	0.058	0.055	0.054	0.054	0.051	0.053	0.048	0.054
Azerbaijan	0.025	0.095	0.094	0.095	0.092	0.091	0.094	0.088	0.096
Sudan	0.025	0.050	0.054	0.051	0.053	0.051	0.052	0.047	0.053
Bolivia	0.024	0.090	0.089	0.089	0.087	0.083	0.087	0.081	0.089
China	**0.023**	**0.059**	**0.057**	**0.057**	**0.055**	**0.052**	**0.053**	**0.049**	**0.054**
São Tomé and Principe	0.023	0.042	0.044	0.044	0.043	0.041	0.042	0.038	0.043
Cameroon	0.022	0.043	0.045	0.045	0.044	0.043	0.044	0.040	0.045
Iraq	0.021	0.067	0.060	0.059	0.060	0.056	0.059	0.054	0.060
Senegal	0.021	0.039	0.041	0.042	0.040	0.038	0.038	0.034	0.039
Nigeria	0.019	0.039	0.040	0.039	0.038	0.035	0.034	0.030	0.034
Côte d'Ivoire	0.019	0.033	0.034	0.035	0.034	0.033	0.033	0.030	0.034
Congo, Rep.	0.018	0.034	0.032	0.032	0.031	0.028	0.028	0.025	0.029
Mongolia	0.018	0.059	0.052	0.051	0.051	0.048	0.050	0.045	0.051
Angola	0.016	0.022	0.022	0.022	0.021	0.019	0.019	0.017	0.020
Kenya	0.015	0.038	0.038	0.039	0.038	0.037	0.037	0.034	0.038
Benin	0.015	0.033	0.033	0.034	0.033	0.030	0.030	0.027	0.031
Kyrgyz Republic	0.014	0.068	0.063	0.064	0.061	0.059	0.060	0.055	0.061
India	**0.014**	**0.048**	**0.046**	**0.047**	**0.045**	**0.044**	**0.046**	**0.042**	**0.047**
Chad	0.013	0.030	0.029	0.027	0.028	0.028	0.026	0.024	0.027
Togo	0.013	0.027	0.027	0.028	0.027	0.026	0.026	0.023	0.027
Vietnam	0.013	0.050	0.043	0.042	0.041	0.038	0.039	0.035	0.040
Cambodia	0.012	0.043	0.039	0.039	0.038	0.036	0.036	0.033	0.037
Mali	0.011	0.022	0.023	0.023	0.023	0.022	0.021	0.019	0.022
Central African Rep	0.010	0.019	0.020	0.021	0.020	0.019	0.018	0.016	0.019
Burkina Faso	0.010	0.025	0.025	0.026	0.025	0.024	0.025	0.022	0.026
Lao PDR	0.010	0.037	0.035	0.035	0.034	0.032	0.032	0.029	0.033
Nepal	0.009	0.031	0.029	0.029	0.028	0.028	0.029	0.026	0.030
Sierra Leone	0.009	0.025	0.023	0.023	0.022	0.019	0.018	0.016	0.019
Tajikistan	0.009	0.060	0.047	0.045	0.044	0.040	0.039	0.036	0.040
Madagascar	0.008	0.025	0.024	0.023	0.023	0.021	0.021	0.019	0.022
Rwanda	0.007	0.019	0.018	0.019	0.018	0.018	0.019	0.017	0.019
Guinea	0.007	0.021	0.020	0.020	0.020	0.018	0.016	0.014	0.017
Malawi	0.006	0.017	0.016	0.016	0.016	0.014	0.014	0.012	0.014
Niger	0.006	0.013	0.014	0.014	0.014	0.013	0.013	0.011	0.013
Guinea-Bissau	0.006	0.012	0.013	0.012	0.013	0.012	0.012	0.011	0.013
Liberia	0.004	0.010	0.008	0.008	0.008	0.007	0.007	0.006	0.007
Congo, Dem. Rep.	0.003	0.004	0.004	0.005	0.004	0.004	0.004	0.003	0.004

Data source: 2005 ICP benchmark prices and ADB (2007).

Notes: See Table 1 which reports results for selected countries, shown here in bold.

Table A2: Parameters Estimates for AIDS

Category	R^2	alpha	beta	gamma											
Food and non-alcoholic beverages	0.736	0.183	-0.193	0.050	-0.035	-0.012	-0.047	0.001	-0.021	-0.023	0.013	0.034	-0.062	0.079	0.024
Alcoholic beverages and tobacco	0.172	0.033	0.004	-0.035	-0.017	0.011	-0.030	0.003	-0.016	0.037	-0.008	0.025	0.009	0.012	0.010
Clothing and footwear	0.075	0.050	-0.005	-0.012	0.011	0.009	-0.007	-0.010	-0.011	0.011	0.025	-0.029	0.016	0.020	-0.022
Gross Rent, water, fuel and power	0.407	0.158	0.017	-0.047	-0.030	-0.007	0.108	0.005	-0.002	0.011	-0.003	-0.016	-0.005	0.004	-0.017
Household furnishings	0.123	0.053	0.007	0.001	0.003	-0.010	0.005	-0.027	-0.008	-0.003	0.004	0.043	0.009	0.017	-0.033
Medical and health services	0.407	0.093	0.029	-0.021	-0.016	-0.011	-0.002	-0.008	0.021	0.029	0.001	-0.007	0.008	-0.002	0.010
Transport	0.218	0.104	0.035	-0.023	0.037	0.011	0.011	-0.003	0.029	-0.041	0.001	0.051	-0.066	-0.047	0.041
Communication	0.167	0.027	0.008	0.013	-0.008	0.025	-0.003	0.004	0.001	0.001	-0.009	0.006	-0.001	-0.006	-0.023
Recreation	0.641	0.070	0.044	0.034	0.025	-0.029	-0.016	0.043	-0.007	0.051	0.006	-0.061	0.027	-0.025	-0.046
Education	0.136	0.078	-0.001	-0.062	0.009	0.016	-0.005	0.009	0.008	-0.066	-0.001	0.027	0.005	0.025	0.035
Restaurants	0.325	0.057	0.017	0.079	0.012	0.020	0.004	0.017	-0.002	-0.047	-0.006	-0.025	0.025	-0.111	0.035
Other goods and services	0.507	0.093	0.038	0.024	0.010	-0.022	-0.017	-0.033	0.010	0.041	-0.023	-0.046	0.035	0.035	-0.014

Table A3: Chinese Prices and Consumption Shares

	Chinese prices		Chinese Consumption Shares		
Category	Actual	Predicted	Actual	Estimated with actual prices	Estimated with predicted prices
Food and non-alcoholic beverages	0.742	0.557	0.241	0.333	0.318
Alcoholic beverages and tobacco	0.976	0.976	0.020	0.030	0.031
Clothing and footwear	0.950	0.573	0.064	0.052	0.052
Gross Rent, water, fuel and power	0.775	0.840	0.146	0.147	0.148
Household furnishings	0.837	0.837	0.039	0.044	0.044
Medical and health services	0.252	0.399	0.062	0.072	0.075
Transport	0.692	0.781	0.041	0.072	0.074
Communication	0.435	0.435	0.042	0.018	0.018
Recreation	0.521	0.606	0.046	0.035	0.039
Education	0.424	0.295	0.097	0.081	0.081
Restaurants	0.850	0.758	0.053	0.046	0.049
Other goods and services	0.821	0.743	0.149	0.068	0.070

Appendix B: Proofs of Propositions

Proof of Lemma 1:

Lemma 1 of Feenstra, Hong and Prasada Rao (2009) establishes that for the AIDS expenditure function, $e(\overline{p}_{jk},u_j)/e(\overline{p}_{jk},u_k)$ equals by the ratio of expenditures $e(p_j,u_j)/e(p_k,u_k)$ deflated by a Könus cost of living index, evaluated at a reference utility u^* lying in-between u_j and u_k. Feenstra and Reinsdorf (2000) provide a formula for the Könus cost of living index in the AIDS case evaluated with a reference utility level u_r, and it can be verified that $u_r = u^*$, so this is the formula that we need. The Feenstra-Reinsdorf formula differs from the Törnqvist price index by using shares evaluated at the *median* prices on a log-linear path in-between p_j and p_k, which will differ only slightly from the arithmetic average shares used in the Törnqvist formula. QED

Proof of Proposition 1:

We first derive a reduced system either in the international prices vector, *Π*, or in the output-based real exchange rates, *RO*. Following Neary (2004), we start with an international price vector Π^0 which is positive. Then using Π^0 in (19), we get a value for RO_j for j=1,2,…,C. Substituting these RO_j*'s* into (18) yields a new price vector Π^1. Thus we have a function that maps Π^0 to Π^1. We represent this relationship by:

$$\Pi^1 = H(\Pi^o) \text{ or in general, } \Pi^t = H(\Pi^{t-1}) \tag{A2}$$

Equation (A2) represents a system of non-linear simultaneous equations. We can state the following general properties of H: (i) H is positive for all $\Pi^0 > 0$; (ii) H is linearly homogeneous; and (iii) H is a continuous function. These properties follow from the structure of the equations (18) and (19) and the assumed properties of the revenue function.

Further we observe that if Π is a solution to equations (18) and (19), then kΠ is also a solution for any k>0. This means that we can have a solution that is unique up to a factor of proportionality. Thus we can restrict ourselves to solutions of (A2) up to a linear restriction $\sum_{i=1}^{M+2N} \pi_i = 1$ with $\pi_i > 0$. This basically means the mapping H in (A2) is a mapping from the unit simplex into itself. Further H is continuous. From Brower's fixed point theorem there exists a fixed point Π^* such that $\Pi^* = H(\Pi^*)$. We note here that uniqueness of the fixed point is not guaranteed by this theorem.[14]

Proof of Proposition 2:

Because the outputs y_k are feasible for country *k,* but not optimal at the prices P_j, it follows that $r_k(P_j, v_k) \geq P_j' y_k$. This establishes the first inequality below and the second is established similarly:

$$\frac{r_j(P_j, v_j)}{r_k(P_j, v_k)} \leq \left(\frac{P_j' y_j}{P_j' y_k} \right) \quad \text{and} \quad \frac{r_j(P_k, v_j)}{r_k(P_k, v_k)} \geq \left(\frac{P_k' y_j}{P_k' y_k} \right),$$

Using the Gerschenkron effect it follows that:

$$\frac{r_j(P_j, v_j)}{r_k(P_j, v_k)} \leq \left(\frac{P_j' y_j}{P_j' y_k} \right) < \left[\left(\frac{P_j' y_j}{P_j' y_k} \right) \left(\frac{P_k' y_j}{P_k' y_k} \right) \right]^{0.5} < \left(\frac{P_k' y_j}{P_k' y_k} \right) \leq \frac{r_j(P_k, v_j)}{r_k(P_k, v_k)}. \qquad \text{(A3)}$$

Now consider the first condition in (22), which states that $y_k \delta_k$ is feasible using the technology of county *j.* Because these outputs are not optimally chosen for the prices P_j it is immediate that:

$$P_j'(y_k \delta_k) \leq P_j' y_j .$$

It follows that $\delta_k \leq P_j' y_j / P_j' y_k$. Then consider the second condition in (22), which states that

[14] Likewise, Neary (2004) proves the existence of a unique positive solution to the Geary system but only shows the existence of a positive solution to the GAIA system.

(y_j / δ_j) is feasible using the technology of county *k*. Because these outputs are not optimally chosen for the prices P_k it is immediate that:

$$P_k'(y_j / \delta_j) \le P_k' y_k .$$

It follows that $\delta_j \ge P_k' y_j / P_k' y_k$. Using the Gerschenkron effect again we have therefore shown:

$$\delta_k \le \left(\frac{P_j' y_j}{P_j' y_k} \right) < \left[\left(\frac{P_j' y_j}{P_j' y_k} \right) \left(\frac{P_k' y_j}{P_k' y_k} \right) \right]^{0.5} < \left(\frac{P_k' y_j}{P_k' y_k} \right) \le \delta_j . \tag{A4}$$

Using (A3) and (A4), and by continuity of the function $r_j(\Pi, v_j) / r_k(\Pi, v_k)$, there exists a value for Π lying in-between P_j and P_k such that (23) holds. QED

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Table 1: Comparisons of Real Consumption per Capita, 2005

Countries	Nominal Consump.	Geary-Khamis	EKS	CCD direct	CCD indirect	Using expenditure function and reference prices: $\overline{p}_{j,us}$	Geary-Khamis	GAIA	Diewert
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Iceland	1.248	0.767	0.794	0.795	0.784	0.768	0.790	0.784	0.791
United States	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
United Kingdom	0.895	0.772	0.793	0.807	0.796	0.767	0.778	0.772	0.779
Netherlands	0.756	0.696	0.720	0.736	0.722	0.707	0.697	0.689	0.699
Canada	0.731	0.722	0.735	0.733	0.729	0.715	0.739	0.733	0.741
South Korea	0.297	0.375	0.375	0.378	0.366	0.350	0.381	0.370	0.383
Macedonia	0.076	0.192	0.192	0.195	0.189	0.184	0.193	0.184	0.196
China	0.023	0.059	0.057	0.057	0.055	0.052	0.053	0.049	0.054
India	0.014	0.048	0.046	0.047	0.045	0.044	0.046	0.042	0.047
With adjusted Chinese prices:									
China	0.023	0.064	0.062	0.063	0.060	0.061	0.064	0.059	0.066
% Difference from China above		9.0	8.8	9.5	9.6	16.3	21.1	21.0	20.9

Data source: 2005 ICP benchmark prices and ADB (2007); see Appendix Table A1 for all countries.

Notes: Column (1) gives the nominal consumption expenditure per capita in 2005, relative to the US level. Column (2) gives the Geary-Khamis calculation of real consumption. Columns (3)-(5) present flexible weight measures of real consumption expenditures per capita, using respectively the EKS, CCD, and indirect CCD methods, as defined in the text in equations (4)-(6). Column (6) to (9) show measures of real consumption based on the estimated AIDS expenditure function, using particular reference prices: column (6) uses the geometric mean of each country's prices with the United States; column (7) uses the GK reference prices; column (8) uses the GAIA reference prices; and column (9) uses every country's prices as references prices and then takes the geometric mean of the results, as in Diewert (2009). All calculations are transitive. The results in the second-last row use adjusted prices for China as described in Appendix A.

Table 2: Actual and Reference Prices

Category	Sample Mean	United States	Geary-Khamis	GAIA	South Korea
Food and non-alcoholic beverages	1	1.100	1.319	1.134	1.627
Alcoholic beverages and tobacco	1	1.395	1.257	1.343	1.162
Clothing & footwear	1	1.136	1.270	1.113	1.257
Gross rent, water, fuel and power	1	1.889	1.389	1.316	1.446
Household furnishings	1	1.302	1.309	1.271	0.988
Medical and health services	1	2.995	1.494	1.554	1.047
Transport	1	0.950	1.231	1.291	1.023
Communication	1	1.128	1.015	0.921	0.641
Recreation	1	1.231	1.182	1.169	1.121
Education	1	3.412	1.395	1.595	1.602
Restaurants	1	1.025	1.239	1.182	1.310
Other goods and services	1	1.632	1.400	1.400	1.215
$b(p)$	1	1.054	0.996	1.026	0.930

Notes. Listed are the reference prices used in the calculations for Table 1 and Appendix Table A1. Since all prices have been normalized by the sample mean (over 124 countries), the prices $p = 1$ represent the sample mean. The second set of prices considered are those for the U.S. The third set of reference prices considered are the GK prices computed as in equations (1)-(2), the fourth set of are the GAIA reference prices computed as in (11)-(12), and the final set of prices are for South Korea. The final row is computed using as $b(p) = \prod_i p_i^{\beta_i}$ using the β estimates from Appendix Table A2.