200C – Micro Theory – Professor Giacomo Bonanno

PRACTICE PROBLEMS 8

Topic: Hotelling's model and product differentiation

VERY IMPORTANT: do **not** look at the answers until you have made a VERY serious effort to solve the problem. If you turn to the answers to get clues or help, you are wasting a chance to test how well you are prepared for the exams. I will **not** give you more practice problems later on.

1. Consider Hotelling's model (a street of length one, consumers uniformly distributed along the street, each consumer has a transportation cost equal to 2d, where d is the distance traveled). Suppose there are two gas stations, one located at $\frac{1}{4}$ and the other located at 1.

(a) Calculate the demand functions for the two firms.

(b) If the two gas stations compete in prices and settle at a Nash equilibrium, will they charge the same price for gasoline? (assume that production costs are zero, that is, firms maximize revenue).

2. Consider Hotelling's model (consumers uniformly distributed over a street of length 1, linear transportation cost, infinite reservation price). Suppose there are two firms and the price of the product (e.g. bread) is fixed by the government and firms can only choose where to locate. Assume that if many consumers are indifferent between the two firms then half of them will go to firm 1 and the other half to firm 2. Assume zero production costs. Find a Nash equilibrium of this game. Are there many Nash equilibria?

3. Consider Hotelling's model (street of length one, consumers uniformly distributed along the street, linear transportation cost, infinite reservation price). Assume that there are two firms and that they choose both location and price at the same time (rather than location first and then price). The firms act independently, that is, when firm 1 chooses its price and location it does not know what price and location were chosen by firm 2 and vice versa. Find the Nash equilibria of this game.

4. There are two consumers, and two firms producing a homogeneous product. Both consumers buy one and only one unit provided the price is not greater than 10. Fixed costs of production are zero for both firms. Firm 1's marginal and average production cost is 4, while that of firm 2 is 6. If both firms charge the same price, both consumers go to firm 1, otherwise each consumer will prefer the firm with the lower price. The law requires a firm that announces a price to sell to all consumers who apply.

Find all the Nash equilibria in prices (show that they are Nash equilibria and that there are no other Nash equilibria).

5. There are two firms, denoted by 1 and 2, producing differentiated products. Costs of production are zero for both firms. The direct demand functions are given by:

$$q_1 = 1 - \frac{a}{a-b} p_1 + \frac{b}{a-b} p_2$$

$$q_2 = \frac{a}{a-b} p_1 - \frac{b(a-1)}{(a-b)(b-1)} p_2$$

where p_i is the price of firm i, q_i is the output of firm i (i = 1,2) and $a > b > 1$.

The corresponding inverse demand functions are given by:

$$p_1 = \frac{a-1}{a}(1-q_1) - \frac{b-1}{a} q_2$$

$$p_2 = \frac{b-1}{b}(1-q_1-q_2)$$

Compute the Bertrand-Nash (competition in prices) and the Cournot-Nash (competition in output levels) equilibria and compare each firm's price, output and profits in the two equilibria (e.g. is firm 1's price at the Bertrand equilibrium higher or lower than its price at the Cournot equilibrium?).