

PRACTICE PROBLEMS 11

Topic: Applications of extensive games to imperfect competition

VERY IMPORTANT: do **not** look at the answers until you have made a VERY serious effort to solve the problem. If you turn to the answers to get clues or help, you are wasting a chance to test how well you are prepared for the exams. I will **not** give you more practice problems later on.



- 1.** In 2004 both firm 1 and firm 2 had a constant average cost of 2 per unit. They simultaneously chose output levels (Cournot competition) for the whole year at the end of January 2004. Inverse industry demand is

$$P(Q) = 14 - Q.$$

- (a) Calculate the Cournot equilibrium.

On January 1st, 2005 firm 1 has the option of installing a new technology that has an average cost of zero. Installing the new technology costs \$F. If firm 1 installs the new technology, firm 2 will be aware of it. Installation will be completed by January 15 and production levels will be chosen simultaneously at the end of January 2005.

- (b) For what values of F should firm 1 make the investment in the new technology?

- 2.** Consider the following game. An incumbent monopolist (firm 1) can either be passive or take a specific action which costs K dollars. A potential entrant (firm 2) observes this and decides whether to enter or not. If she stays out her profits are zero, while the incumbent's profits are the monopoly profits π_M (minus the cost of the action, if such action was taken). If the incumbent took the action and the potential entrant comes in, the incumbent has a choice between undoing the action (thereby recovering K dollars) or making no change. A duopoly game follows the three situations: (no-action, in), (action, in, undo), (action, in, no-change). Let π_i^* be firm i's profit (i=1,2) at the unique Nash equilibrium of the game where no action was taken or, if it was taken, it was subsequently undone. Let π_i^0 be firm i's profit at the unique Nash equilibrium of the remaining duopoly game. Assume that $\pi_2^* > 0$ and $\pi_i^* > \pi_i^0$ for i=1,2.

- (a) Draw the extensive game described above.

- (b) Find the subgame-perfect equilibria of this game. Is there a subgame-perfect equilibrium characterized by entry deterrence?

- 3.** The production of gasoline goes through two stages: (i) extraction of oil from the ground and (ii) refining. Oil extraction costs \$2 per gallon, while refining costs \$1 per gallon. The owner of the well sets the price (per gallon) w to the owner of the refinery, who in turn sets the price p to consumers. Consumers' demand for gasoline is known to both firms and given by

$$Q = 100 - 4p$$

(Q is the quantity measured in gallons and p is the price measured in dollars).

Calculate the backward induction solution of this perfect information game.