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Ecn 200C – Micro Theory – Professor Giacomo Bonanno

ANSWERS TO PRACTICE PROBLEMS $\boxed{7}$

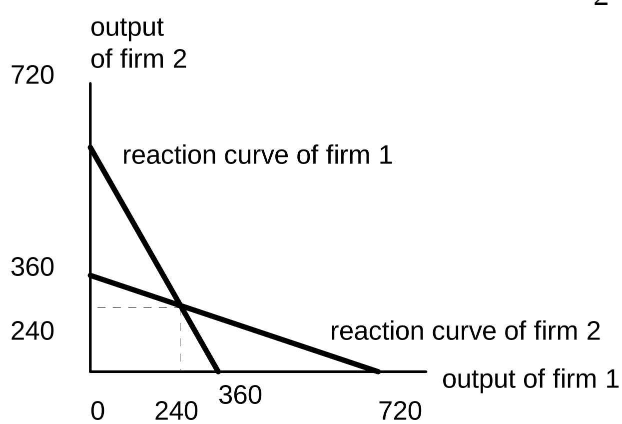
1. **(a)** The profit function of firm 1 is given by

$$\pi_1(q_1,q_2) = q_1\,[2000 - 2(q_1+q_2)] - 560\,q_1 - 80{,}000$$

The reaction function of firm 1 is obtained by solving the equation $\frac{\partial \pi_1}{\partial q_1} = 0$, which gives

$$q_1 = 360 - \frac{q_2}{2}.$$

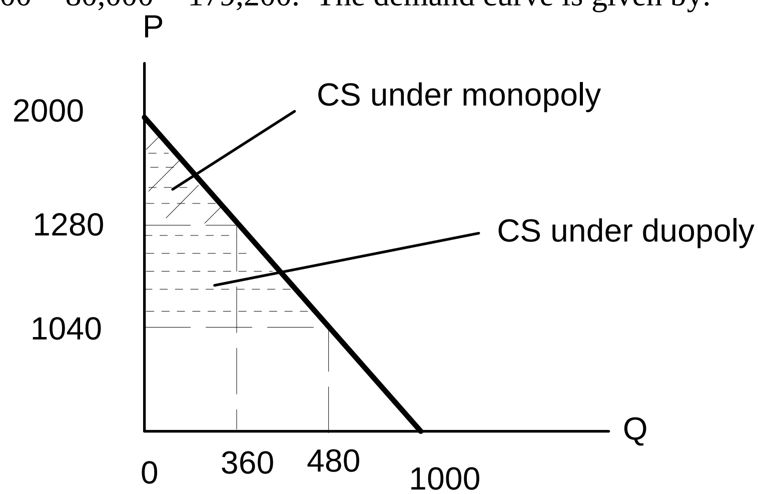
Similarly, the reaction function of firm 2 is given by $q_2 = 360 - \frac{q_1}{2}$.



(b) The Cournot-Nash equilibrium is given by:

$$q_1 = q_2 = 240$$
$$Q = 480$$
$$P = 1040$$
$$\pi_1 = \pi_2 = 115{,}200 - 80{,}000 = 35{,}200.$$

(c) A monopolist would set $Q = 360$ and $P = 1280$. Her profits would be $\pi = 259{,}200 - 80{,}000 = 179{,}200$. The demand curve is given by:



Thus consumer surplus under monopoly would be (2000 – 1280)(360)/2 = 129,600, while consumer surplus under duopoly is (2000 – 1040)(480)/2 = 230,400. Hence social welfare under monopoly is 129,600 + 179,200 = 308,800, and social welfare under duopoly is 230,400 + 2(35,200) = 300,800. Thus social welfare is **higher** under monopoly than under duopoly. The reason is that under duopoly the fixed cost has to be paid twice and in this case the fixed cost is larger than the gain in consumer surplus obtained by switching from monopoly to duopoly.

2. Inverse demand is $P = 300 - 3Q$.

(a) $\pi_1(q_1,q_2) = q_1[300 - 3(q_1+q_2)] - 2q_1 - 150.$

$\pi_2(q_1,q_2) = q_2[300 - 3(q_1+q_2)] - 2q_2 - 150$

(b) $\frac{\partial \pi_1}{\partial q_1} = 300 - 6q_1 - 3q_2 - 2$. Setting it equal to zero and solving for q_1 gives: $R_1(q_2) = \frac{298 - 3q_2}{6}$.

(c) $R_1(20) = \frac{238}{6} = 39.67.$

(d) The Cournot equilibrium is given by the solution to the system of equations $\frac{\partial \pi_1}{\partial q_1} = 0$ and $\frac{\partial \pi_2}{\partial q_2} = 0$. The solution is: $q_1 = q_2 = 33.11$, $P = 101.33$, $\pi_1 = \pi_2 = 3139$.

3. The profit (revenue) function of firm i is given by

$$\pi_i(q_1,\dots,q_n) = q_i\, e^{-(q_1+\dots+q_n)}.$$

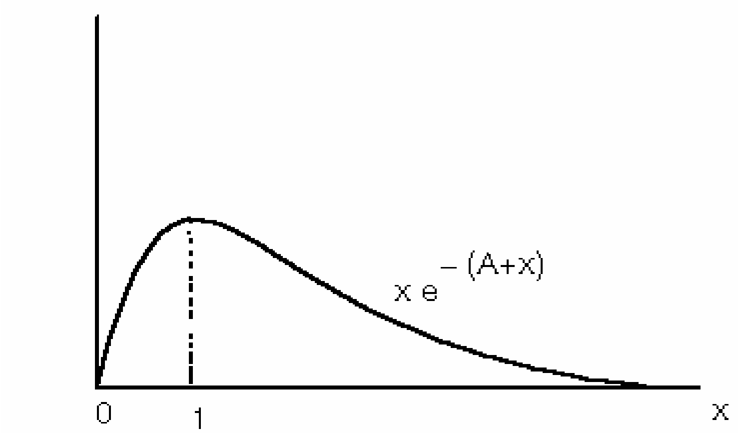
For an arbitrary number A, consider the function

$$f(x) = x\, e^{-(A+x)}.$$

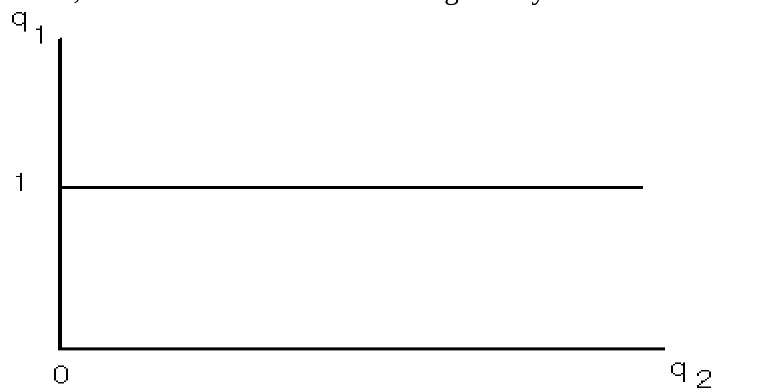
We have:

$$\frac{df}{dx} = (1-x)\, e^{-(A+x)}$$

$$\frac{d^2 f}{dx^2} = -2\, e^{-(A+x)} + x\, e^{-(A+x)} < 0 \quad \text{if} \quad x < 2.$$



Thus for every A, $f(x)$ is maximized at $x = 1$. Think of x as q_i and A as $\sum_{j \neq i} q_j$. Thus $q_i = 1$ is a dominant strategy for firm i. If $n = 2$, the reaction curve of firm 1 is given by



Similarly for firm 2.

For arbitrary n there is a unique Cournot-Nash equilibrium where each firm produces 1 unit. Equilibrium price is e^{-n} and equilibrium output is n. As $n \to \infty$, industry output goes to infinity, while price goes to zero (= marginal cost).

4. The only candidates for Nash equilibria are the pairs (p_1,p_2) with $p_1 = p_2 = p \geq 0.75$. Suppose this is an equilibrium. Then there should be no incentive to undercut your rival. If you undercut [as little as possible, i.e from p to (p–0.05)] your profits are:

$$(p - 0.05 - 0.75)\,[100 - (p - 0.05)] \qquad (1).$$

If you don't undercut, your profits are:

$$(p - 0.75)\,\frac{1}{2}\,(100 - p) \qquad (2).$$

Thus necessary and sufficient condition for p to be a Nash equilibrium is that (2) ≥ (1). That is,

$$-p^2 + 100.95\,p - 85.08 \leq 0.$$

The roots of the equation $-p^2 + 100.95\,p - 85.08 = 0$ are $p = 0.8499$ and $p = 100.10$. Thus either $p \leq 0.8499$ or $p \geq 100.10$. However, when $p \geq 100$ demand is zero and this cannot be a Nash equilibrium. Hence there are **two** Nash equilibria: $p_1 = p_2 = 0.75$, $p_1 = p_2 = 0.80$.