ANSWERS TO PRACTICE PROBLEMS 4

1. It is not strategy proof.

Example with n = 2 and m = 3: Suppose that 1's *true* preferences are $\begin{matrix} x \\ y \\ z \end{matrix}$ and 2's *stated*

preferences are $\begin{matrix} x \\ y \\ z \end{matrix}$. Then if 1 tells the truth the chosen outcome is x, while if she strategically

reports the ranking $\begin{matrix} y \\ z \\ x \end{matrix}$ the chosen outcome is y, her most preferred.

2. **(i)**

Individual 2's reported ranking

		x, y, z	x, z, y	y, x, z	y, z, x	z, x, y	z, y, x
individual	x, y, z	**x**	**x**	**y**	**y**	**x**	**y**
1's	x, z, y	**x**	**x**	**x**	**z**	**z**	**z**
reported	y, x, z	**x**	**x**	**y**	**y**	**x**	**y**
ranking	y, z, x	**y**	**z**	**y**	**y**	**z**	**z**
	z, x, y	**x**	**x**	**x**	**z**	**z**	**z**
	z, y, x	**y**	**z**	**y**	**y**	**z**	**z**

(ii) **1 is not a dictator:** row 2, column 4 chosen outcome is z, not the top-ranked of 1.

2 is not a dictator: column 1, row 4 chosen outcome is y, not the top-ranked of 2.

Manipulable: 1 with true preferences y, x z (row 3) has an incentive to state y, z, x (row 4) if she expects 2 to state x, y, z (column 1).

3. (a)

2's →	abc	acb	bac	bca	cab	cba
1's ↓						
abc	a	a	a	a	a	a
acb	a	a	a	a	a	a
bac	a	a	b	b	a	b
bca	a	a	b	b	c	b
cab	a	a	a	c	c	c
cba	a	a	b	b	c	c

3 reports abc

2's →	abc	acb	bac	bca	cab	cba
1's ↓						
abc	a	a	a	a	a	a
acb	a	a	a	a	a	a
bac	a	a	b	b	a	c
bca	a	a	b	b	c	c
cab	a	a	a	c	c	c
cba	a	a	c	c	c	c

3 reports acb

2's →	abc	acb	bac	bca	cab	cba
1's ↓						
abc	a	a	b	b	a	b
acb	a	a	b	b	a	c
bac	b	b	b	b	b	b
bca	b	b	b	b	b	b
cab	a	a	b	b	c	c
cba	b	c	b	b	c	c

3 reports bac

2's →	abc	acb	bac	bca	cab	cba
1's ↓						
abc	a	a	b	b	c	b
acb	a	a	b	b	c	c
bac	b	b	b	b	b	b
bca	b	b	b	b	b	b
cab	c	c	b	b	c	c
cba	b	c	b	b	c	c

3 reports bca

`	abc	acb	bac	bca	cab	cba
1's ↓						
abc	a	a	a	c	c	c
acb	a	a	a	c	c	c
bac	a	a	b	b	c	c
bca	c	c	b	b	c	c
cab	c	c	c	c	c	c
cba	c	c	c	c	c	c

3 reports cab

`	abc	acb	bac	bca	cab	cba
1's ↓						
abc	a	a	b	b	c	c
acb	a	a	c	c	c	c
bac	b	c	b	b	c	c
bca	b	c	b	b	c	c
cab	c	c	c	c	c	c
cba	c	c	c	c	c	c

3 reports cba

(b) Suppose that 2 reports *cab* and 3 reports *bca* and that 1's true ranking is *abc*. Then if 1 reports sincerely, the outcome is *c* (4th table, 1st row and 4th column), which is the worst outcome according to his true ranking. If, instead, he reports *bac* then the outcome is *b*, which − according to his true ranking *abc* − is better than *c*.