MIDTERM EXAM
ANSWER ALL QUESTIONS (total 100 points)

1. [52 points] A travel agent is auctioning two one-week vacations for the first week of July. The first (call it prize *A*) is a vacation in Hawaii and the second (call it prize *B*) a vacation in Texas. There are three bidders, imaginatively called 1, 2 and 3. All bidders consider prize *A* more desirable than prize *B* and none of the bidders is interested in winning both prizes (they find it hard to be in two places at the same time). Let a_i be the value of prize *A* to bidder *i* and b_i the value of prize *B* to bidder *i*. Thus $a_i > b_i$ for all $i \in \{1,2,3\}$. Having read about the good properties of Vickrey's second-price auction, the travel agent has decided to organize the following auction: a sealed bid auction in which each bidder submits a non-negative price p_i; the highest bidder wins prize *A* and pays the second-highest price; the second-highest bidder wins prize *B* and pays the lowest price; in case of ties, the player with the lower index prevails over the bidder(s) who have submitted the same bid as she did. For example, if $p_1 = \$10, p_2 = \$12, p_3 = \$10$ then bidder 2 wins prize *A* and pays \$10, bidder 1 wins prize *B* and pays \$10 and bidder 3 pays nothing and wins nothing. [Note that the second-highest price is the higher of the two prices remaining after removing the bid of the highest bidder: for example if the bids are 15, 15 and 8 then the highest bid is 15, the second highest is also 15 and the lowest is 8; if all the players bid \$23 then the highest, second highest and lowest are all the same and equal to 23.] Each player is **selfish and greedy** (that is, each player cares only about his/her own net gain).

(a) [10 points] For the case where $a_1 = 100, b_1 = 70, a_2 = 90, b_2 = 60, a_3 = 75, b_3 = 40$ and the possible bids are \$100 and \$50, write the strategic-form of the game.

From now on assume that bids can be any non-negative numbers and that all you know about the a_is and b_is is that $a_i > b_i > 0$ for all $i \in \{1,2,3\}$.

(b) [8 points] Write the payoff function of Player 1.

(c) [10 points] Does Player 1 have a dominant strategy? Prove your claim.

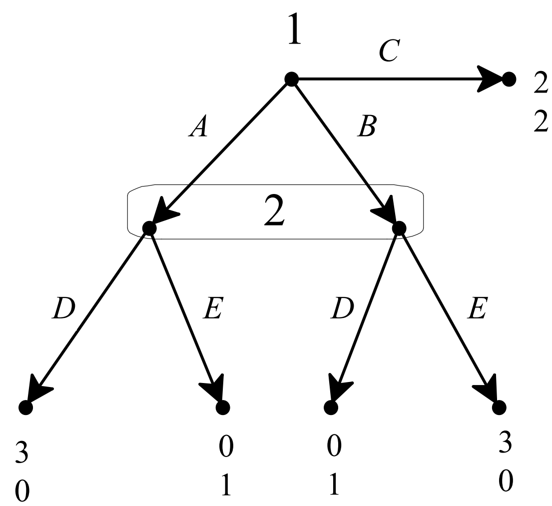
(d) [2 points] Is there a dominant-strategy equilibrium?

(e) [10 points] Assume that $a_1 > a_2 > a_3$. What further restrictions on the parameters a_i and b_i ($i \in \{1,2,3\}$) are **necessary and sufficient** for (a_1, a_2, a_3) to be a Nash equilibrium? Prove your claim (prove both sufficiency and necessity).

(f) [6 points] Suppose that $a_1 = 180, b_1 = 95, a_2 = 120, b_2 = 88, a_3 = 80, b_3 = 68$. Is $(180,120,80)$ a Nash equilibrium?

(g) [6 points] Suppose that $a_1 = 180, b_1 = 95, a_2 = 120, b_2 = 78, a_3 = 80, b_3 = 68$. Is $(180,120,80)$ a Nash equilibrium?

2. [36 points]. Consider the following game, where the payoffs are von Neumann-Morgenstern payoffs:



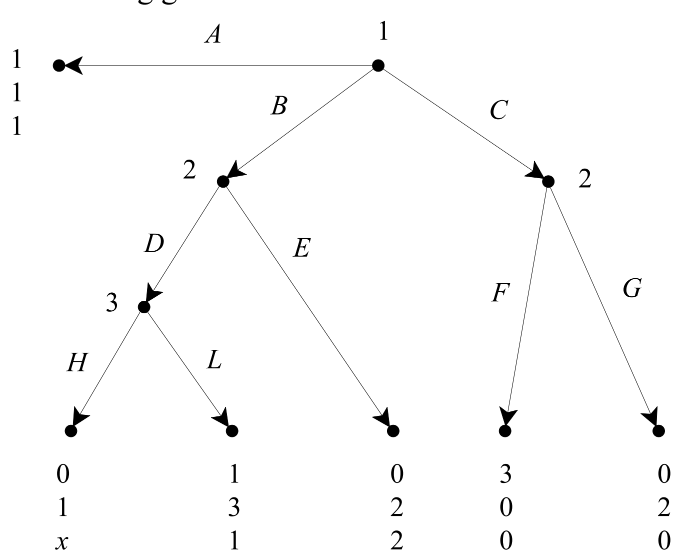
(a) [10 points] Find the best reply function of Player 1 (against every possible mixed strategy of Player 2).

(b) [8 points] Explain why there are no pure-strategy weak sequential equilibria.

(c) [10 points] Find a weak sequential equilibrium. For the strategy profile σ that you suggest find all the assessments that contain σ and are weak sequential equilibria.

(d) [8 points] Find a sequential equilibrium (one is enough; prove your claim).

3. [12 points] Consider the following game:



(a) [3 points] Are there values of x for which Player 3 has a **strictly** dominant strategy? Explain your answer.

(b) [2 points] Does Player 1 have a weakly dominated strategy? (If your answer is Yes, name the strategy; if your answer is No prove your claim.)

(c) [4 points] What strategy profiles are Nash equilibria irrespective of the value of x?

(d) [3 points] Find all the backward induction solutions for the cases where $x = 0$ and $x = 10$.

Midterm Exam ANSWERS

1. **(a)** The strategic form is as follows:

		Player 2: 50			Player 2: 100		
Player 1	50	50	10	0	20	40	0
	100	50	10	0	0	10	0

Player 3 bids 50

		Player 2: 50			Player 2: 100		
Player 1	50	20	0	25	0	−10	−10
	100	0	0	−10	0	−40	0

Player 3 bids 100

(b) The payoff function of player 1 is as follows:

$$\pi_1 = \begin{cases} a_1 - \max\{p_2, p_3\} & \text{if } p_1 \geq \max\{p_2, p_3\} \\ b_1 - p_3 & \text{if } p_1 < p_2 \text{ and } p_1 \geq p_3 \\ b_1 - p_2 & \text{if } p_1 < p_3 \text{ and } p_1 \geq p_2 \\ 0 & \text{if } p_1 < p_2 \text{ and } p_1 < p_3 \end{cases}$$

(c) Player 1 does **not** have a dominant strategy.
Proof: suppose that bidding $\hat{p}$ were a dominant strategy for player 1. It cannot be $\hat{p} \geq a_1$ because in the case where $p_2 = \hat{p}$ and $p_3 = 0$ we have that $\pi_1(\hat{p}, \hat{p}, 0) = a_1 - \hat{p} \leq 0$ while $\pi_1(b_1, \hat{p}, 0) = b_1 > 0$. It cannot be $b_1 \leq \hat{p} < a_1$ because in the case where $p_2 = \hat{p} + \varepsilon < a_1$ and $p_3 = b_1$ (where $\varepsilon > 0$) we have that $\pi_1(\hat{p}, \hat{p}+\varepsilon, b_1) = b_1 - b_1 = 0$ while $\pi_1(a_1, \hat{p}+\varepsilon, b_1) = a_1 - (\hat{p}+\varepsilon) > 0$. Finally, it cannot be $\hat{p} < b_1$ because in the case where $p_2 = p_3 = b_1$ we have that $\pi_1(\hat{p}, b_1, b_1) = 0$ while $\pi_1(a_1, b_1, b_1) = a_1 - b_1 > 0$.

(d) Since there is at least one player who does not have a dominant strategy, there is no dominant strategy equilibrium.

(e) The restrictions are: (1) $b_1 - a_3 \leq a_1 - a_2$ and (2) $b_2 - a_3 \geq 0$.
Proof that these restrictions are sufficient for (a_1, a_2, a_3) to be a Nash equilibrium:

(i) $\pi_1(a_1,a_2,a_3) = a_1 - a_2 > 0$ (since $a_1 > a_2$); for every $p_1 \geq a_2$, $\pi_1(p_1,a_2,a_3) = a_1 - a_2$; for $p_1 < a_2$, either $\pi_1(p_1,a_2,a_3) = b_1 - a_3$ (if $p_1 \geq a_3$) or $\pi_1(p_1,a_2,a_3) = 0$ (if $p_1 < a_3$), so that, by restriction (1), player 1 cannot increase his payoff.

(ii) $\pi_2(a_1,a_2,a_3) = b_2 - a_3$. For $p_2 > a_1$, $\pi_2(a_1,p_2,a_3) = a_2 - a_1 < 0$. For $a_3 \leq p_2 \leq a_1$, $\pi_2(a_1,p_2,a_3) = b_2 - a_3$. For $p_2 < a_3$, $\pi_2(a_1,p_2,a_3) = 0$. Thus player 2 cannot increase her payoff if constraint (2) is satisfied.

(iii) $\pi_3(a_1,a_2,a_3) = 0$. For $p_3 \leq a_2$, $\pi_3(a_1,a_2,p_3) = 0$. For $a_2 < p_3 \leq a_1$, $\pi_3(a_1,a_2,p_3) = b_3 - a_2$. For $p_3 > a_1$, $\pi_3(a_1,a_2,p_3) = a_3 - a_1$. Since $b_3 < a_3 < a_2 < a_1$, player 3 cannot increase his payoff.

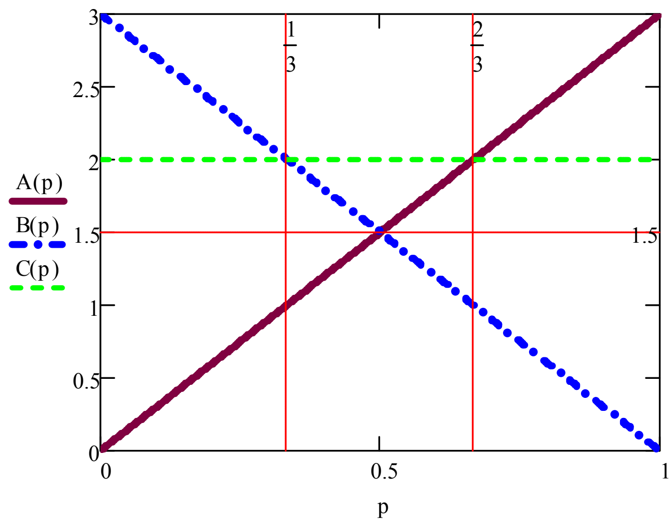
Proof that they are necessary: (1) if $b_1 - a_3 > a_1 - a_2$ then player 1 can increase his payoff by switching to any $p_1 = a_3$; (2) if $b_2 - a_3 < 0$ then player 2 can increase her payoff by switching to $p_2 = 0$.

(f) Yes because $b_1 - a_3 = 15 < a_1 - a_2 = 60$ and $b_2 - a_3 = 8 > 0$.

(g) No, because $b_2 - a_3 = -2 < 0$.

2.

(a) Let p be the probability with which Player 2 plays D. The following diagram shows the Player 1's expected payoff from each of her pure strategies:



From the diagram it is clear that Player 1's best reply function is as follows:

$$BR_1(p) = \begin{cases} B & \text{if } p < \frac{1}{3} \\ \begin{pmatrix} A & B & C \\ 0 & q & 1-q \end{pmatrix} \text{ with any } q \in [0,1] & \text{if } p = \frac{1}{3} \\ C & \text{if } \frac{1}{3} < p < \frac{2}{3} \\ \begin{pmatrix} A & B & C \\ q & 0 & 1-q \end{pmatrix} \text{ with any } q \in [0,1] & \text{if } p = \frac{2}{3} \\ A & \text{if } \frac{2}{3} < p \leq 1 \end{cases}$$

(b) Since (the strategy component of) a weak sequential equilibrium is a Nash equilibrium, it is sufficient to show that there are no pure-strategy Nash equilibria.

- (A,D) is not a Nash equilibrium because D is not a best reply to A.
- (A,E) is not a Nash equilibrium because A is not a best reply to E.
- (B,D) is not a Nash equilibrium because B is not a best reply to D.
- (B,E) is not a Nash equilibrium because E is not a best reply to B.
- (C,D) is not a Nash equilibrium because C is not a best reply to D.
- (C,E) is not a Nash equilibrium because C is not a best reply to E.

(c) Let x denote the left node of Player 2's information set and y the right node. Consider the assessment (σ,μ), where $\sigma = \left(\begin{pmatrix} A & B & C \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} D & E \\ p & 1-p \end{pmatrix} \right)$ and $\mu = \begin{pmatrix} x & y \\ q & 1-q \end{pmatrix}$. For C to be sequentially rational it is necessary and sufficient that $\frac{1}{3} \leq p \leq \frac{2}{3}$. For $\begin{pmatrix} D & E \\ p & 1-p \end{pmatrix}$ to be sequentially rational with $0 < p < 1$ it must be that Player 2 is indifferent between playing D and E; a necessary and sufficient condition for this to be the case is that $q = \frac{1}{2}$. Thus any assessment of the form $\sigma = \left(\begin{pmatrix} A & B & C \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} D & E \\ p & 1-p \end{pmatrix} \right)$, $\mu = \begin{pmatrix} x & y \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ with $\frac{1}{3} \leq p \leq \frac{2}{3}$ is a weak sequential equilibrium.

(d) $\sigma = \left(\begin{pmatrix} A & B & C \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} D & E \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \right)$, $\mu = \begin{pmatrix} x & y \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ is a sequential equilibrium. To see this, consider the completely mixed strategy profile $\sigma_n = \left(\begin{pmatrix} A & B & C \\ \frac{1}{n} & \frac{1}{n} & 1-\frac{2}{n} \end{pmatrix}, \begin{pmatrix} D & E \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \right)$. Then $\sigma_n \to \sigma$ as $n \to \infty$. The beliefs obtained from σ_n using Bayes' rule are $\mu_n = \begin{pmatrix} x & y \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$, so convergence to μ is trivial. Sequential rationality is satisfied because the given assessment is a weak sequential equilibrium.

3.

(a) For no values of x does Player 3 have a strictly dominant strategy: if Player 1 plays A then Player 3's two strategies yield the same payoff, namely 1.

(b) Yes: B is weakly dominated by A.

(c) The following 4 strategy profiles: (A,DG,H), (A,EG,H), (A,DG,L) and (A,EG,L).

(d) When $x = 0$ there are two: (A,DG,L) and (B,DG,L). When $x = 10$ there is only one: (A,EG,H).

FINAL EXAM

ANSWER ALL QUESTIONS (total 100 points)

1. [30 points] There are N individuals who are identical in terms of initial wealth W, potential loss x and von Neumann-Morgenstern utility-of-wealth function $U(m) = \sqrt{m}$. Let N = 8,000, $W = \$3,600$, $x = \$2,700$. The proportion q_H of these N consumers ($0 < q_H < 1$) have a probability of incurring the loss equal to p_H (call them the H type) and the remaining proportion $(1-q_H)$ have a probability of incurring the loss equal to p_L (call them the L type). Let $p_L = \frac{1}{10}$, $p_H = \frac{1}{4}$, $q_H = \frac{1}{8}$. The insurance industry is a monopoly.

(a) [4 points] Find the maximum premium that each type of consumer is willing to pay for full insurance.

(b) [5 points] Suppose that the monopolist decides to offer only a partial insurance contract with premium $h = \$575$ and deductible $D = \$800$. Calculate the total profit of the monopolist.

(c) [3 points] Suppose that the monopolist decides to offer only a full insurance contract with premium $h = \$350$. Calculate the total profit of the monopolist.

(d) Suppose now that the monopolist decides to offer a menu consisting of two contracts: A and B, where A is the contract $(h = 684, D = 416)$ and B is a contract such that (1) the L people prefer B to NI (NI means no insurance) and (2) the H people prefer A to B.
(d.1) Represent these two contracts in a diagram where wealth in the bad state is measured on the horizontal axis and wealth in the good state on the vertical axis. In particular,

(d.1.a) [4 points] draw the indifference curve of the H type that goes through contract A and the indifference curve of the L type that goes through NI and clearly show where contracts A and B are situated and

(d.1.b) [2 points] give the coordinates of contract A.

(d.2) [4 points] Write an expression that gives the monopolist's profits when it offers the pair of contracts $\{A,B\}$. [There will be some numbers and some unknowns in this expression, since the details of contract B are not given.]

(d.3) [8 points] Find another contract C such that the pair $\{B,C\}$ yields higher profits for the monopolist and **compute the increase in profits** relative to the profits from the pair $\{A,B\}$.

2. [20 points] Let y denote the amount of education. There are three types of potential workers: those (Group I) with productivity **36** (a constant, thus independent of education), those (Group II) with productivity ($\mathbf{60 + 3}\boldsymbol{y}$) and those (Group III) with productivity ($\mathbf{72 + 2}\boldsymbol{y}$). Each worker knows whether she belongs to Group I or Group II or Group III, while the potential employer does not. The cost of acquiring y units of education is **24y** for Group I, **12y** for Group II and **6*y*** for Group III. The potential employer believes that those applicants with education **less than *a*** belong to Group I, those with education **at least *a* but less than *b*** belong to Group II and those with education **at least *b*** belong to Group III and offers each applicant a wage equal to the applicant's estimated productivity, given the applicant's level of education (the level of education can be verified by the employer during the job interview).

- **(a)** [10 points] Write down a list of inequalities (involving the parameters a and b) that are necessary and sufficient for the existence of a signaling equilibrium.
- **(b)** [5 points] Explain why $a = 2$ and $b = 2.5$ is not a signaling equilibrium.
- **(c)** [5 points] Is $a = 2.5$ and $b = 4$ a signaling equilibrium? [Explain your answer.]

3. [50 points] Players 1 and 2 are involved in a dispute over which of them owns an item, which is currently in the possession of Player 1; Player 2 may be able to sue Player 1 in an attempt to take possession of the item. Player l's legal claim to the item may be strong (type S) or weak (type W), and Player 2 may be informed (type i) or uninformed (type u) about the strength of Player l's legal claim. Each type of Player 1 assigns probability $\frac{4}{5}$ to Player 2 being uninformed. An uninformed Player 2 assigns probability $\frac{1}{4}$ to Player 1 having a weak legal claim, while an informed Player 2 knows whether Player 1 has a strong or weak legal claim. Each player knows his own type. The interaction between the players proceeds as follows: first, Player 1 decides whether to trade the item to a third party (T) or to keep the item (K). If he trades the item, the game ends, and the payoffs are (3, 0) (the first number is Player 1's payoff and the second number Player 2's payoff). If Player 1 keeps the item, then Player 2 has three options: give up (g), sue cautiously (c) or act boldly (b). If Player 2 gives up, Player 1 will be able to spend more time searching for a buyer, and so payoffs are (5, 0). If Player 2 sues cautiously, much of the surplus generated by the item is used up in court fees: in this event, payoffs are (–1, 2) if Player 1 has a weak legal claim and (2, –1) if Player 1 has a strong legal claim. Finally, "acting boldly" means that Player 2 attempts to obtain the item by quasi-legal means. This attempt succeeds if Player 1 has a weak legal claim – in which case payoffs are (0, 5) – but leads to Player 2's imprisonment if Player 1 has a strong legal claim – in which case payoffs are (5, –10). All these payoffs are von Neumann-Morgenstern payoffs.

- **(a)** [10 points] Describe this situation using states and information partitions. For at least one state write the game associated with that state.
- **(b)** [6 points] Find the common prior.
- **(c)** [10 points] Use the Harsanyi transformation to convert the representation of part (a) into an extensive-form game.
- **(d)** [4 points] How many pure strategies does Player 2 have?
- **(e)** [20 points] Find all the (pure- and mixed-strategy) weak sequential equilibria of the game of part (c) in which Player 1 uses a "pooling" pure strategy, that is, he chooses the same action at all his information sets.

Final Exam $\boxed{\text{ANSWERS}}$

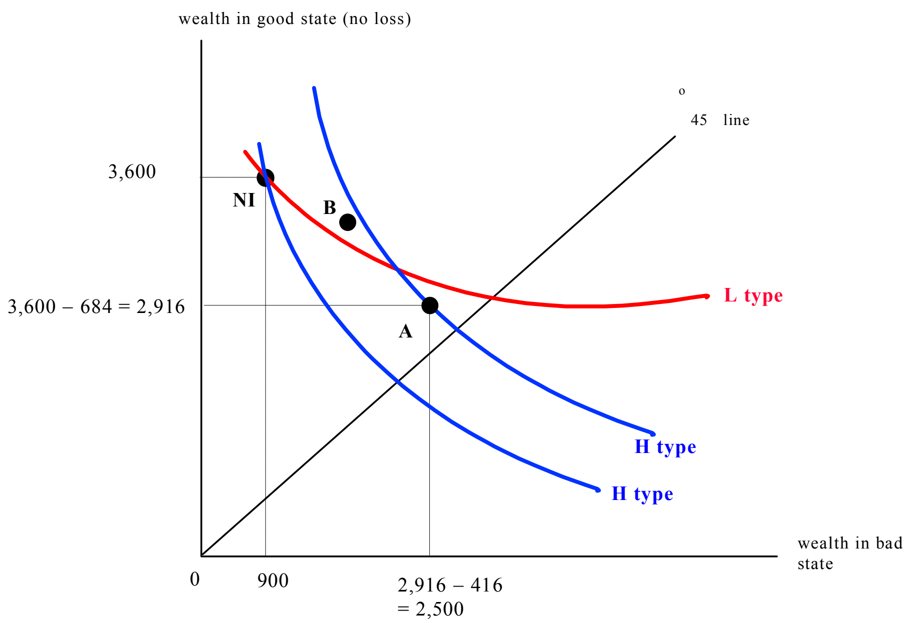
1.

(a) Let h_H^* be the maximum premium that the H type is willing to pay for full insurance. Then h_H^* is given by the solution to the equation $p_H\sqrt{W-x}+(1-p_H)\sqrt{W}=\sqrt{W-h}$, which is $h_H^*=843.75$. Let h_L^* be the maximum premium that the L type is willing to pay for full insurance. Then h_L^* is given by the solution to the equation $p_L\sqrt{W-x}+(1-p_L)\sqrt{W}=\sqrt{W-h}$, which is $h_L^*=351$.

(b) The expected utility of NI (No Insurance) is 52.5 for the H people and 57 for the L people. For the H people the expected utility of contract $(h=575, D=800)$ is 53.04 and thus they will purchase the contract. For the L people the expected utility of contract $(h=575, D=800)$ is 54.22 and thus they will **not** purchase the contract. Hence the monopolists profits are
$\frac{1}{8}8{,}000\left(575-\frac{1}{4}(2{,}700-800)\right)=\$100{,}000$.

(c) From part (a) we know that everybody will buy and thus the monopolist's profits will be
$\frac{1}{8}8{,}000\left(350-\frac{1}{4}2{,}700\right)+\frac{7}{8}8{,}000\left(350-\frac{1}{10}2{,}700\right)=\$235{,}000$.

(d) **(d.1)**



(d.2) The H type will buy A (with an expected utility of $\frac{1}{4}\sqrt{2{,}500}+\frac{3}{4}\sqrt{2{,}916}=53$) and the L type will buy B. Let h_B be the premium of contract B and D_B be the deductible. Then the monopolist's profits will be

$$\frac{1}{8}8{,}000\left[684-\frac{1}{4}(2{,}700-416)\right]+\frac{7}{8}8{,}000\left[h_B-\frac{1}{10}(2{,}700-D_B)\right]=113{,}000+7{,}000\left[h_B-\frac{1}{10}(2{,}700-D_B)\right]$$

(d.3) If the monopolist replaces contract A with contract C, given by the intersection of the 45^{o} line and the H-indifference curve that goes through A, then the L people will still buy contact B while the H

people will buy contract C, which they prefer to B. Contract C is a full-insurance contract whose premium is given by the solution to $\sqrt{3{,}600-h} = \underbrace{\frac{1}{4}\sqrt{2{,}500}+\frac{3}{4}\sqrt{2{,}916}}_{EU_H(A)}$, which is $h = 791$. With the pair $\{B,C\}$ the monopolist's profits would be

$\frac{1}{8}8{,}000\left[791-\frac{1}{4}2{,}700\right]+\frac{7}{8}8{,}000\left[h_B-\frac{1}{10}(2{,}700-D_B)\right]=116{,}000+7{,}000\left[h_B-\frac{1}{10}(2{,}700-D_B)\right]$ thus an increase of \$3,000.

2. **(a)** First of all, since for every group the marginal cost of one extra unit of education exceeds the marginal benefit (in terms of increased salary), everybody will consider only $y = 0$, $y = a$ and $y = b$. The inequalities are as follows.

For Group I:

(I.1) $36 > 60 + 3a - 24a$, that is, $a > \frac{8}{7}$

(I.2) $36 > 72 + 2b - 24b$, that is, $b > \frac{18}{11}$

For Group II:

(II.1) $60 + 3a - 12a > 36$, that is, $a < \frac{8}{3}$

(II.2) $60 + 3a - 12a > 72 + 2b - 12b$, that is, $b > \frac{6}{5}+\frac{9}{10}a$

For Group III:

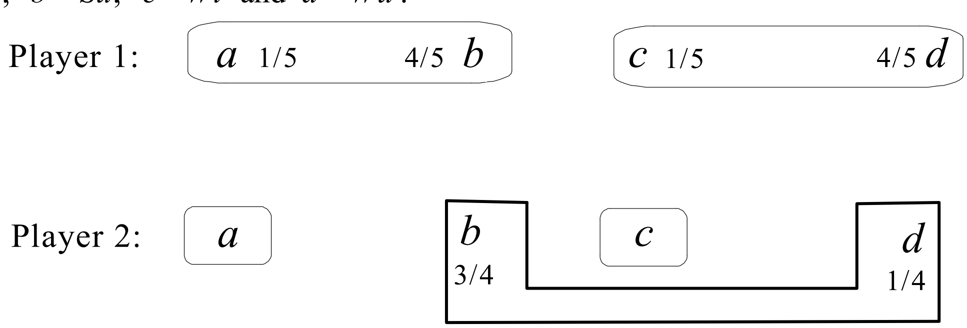
(III.1) $72 + 2b - 6b > 36$, that is, $b < 9$

(III.2) $72 + 2b - 6b > 60 + 3a - 6a$, that is, $b < 3+\frac{3}{4}a$

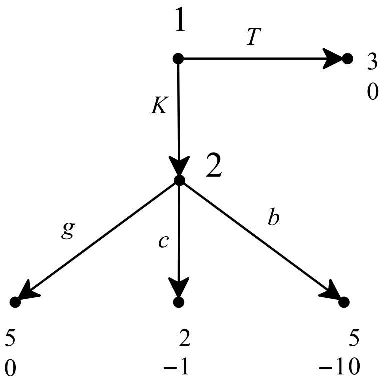
(b) When $a = 2$ and $b = 2.5$, inequality (II.2) is violated. Thus Group II individuals would be better off pretending to be Group III by choosing $y = 2.5$.

(c) Yes, when $a = 2.5$ and $b = 4$, all the above inequalities are satisfied.

3. **(a)** Let $a = Si$, $b = Su$, $c = Wi$ and $d = Wu$:

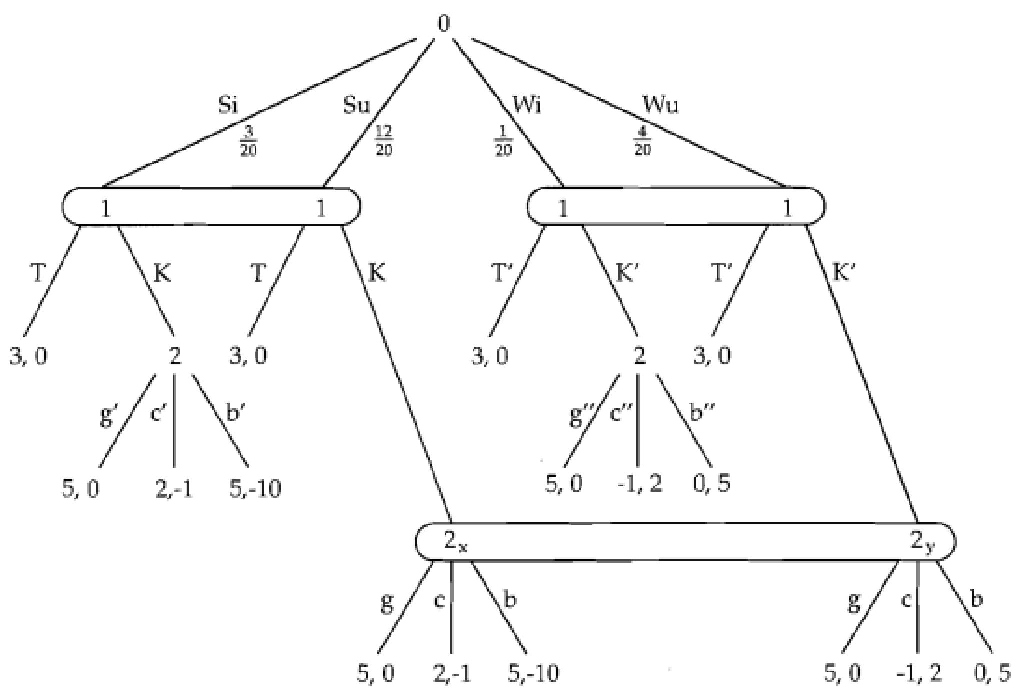


The game associated with states a and b is the same, namely:



(a) The common prior is $\begin{pmatrix} a & b & c & d \\ \frac{3}{20} & \frac{12}{20} & \frac{1}{20} & \frac{4}{20} \end{pmatrix}$.

(b) The game is as follows, where Nature is denoted as Player 0:



(c) Player 2 has $3^3 = 27$ pure strategies.

(d) Clearly 2 plays g' and b'' in any weak sequential equilibrium. Furthermore, by Bayes' rule, Player 1 must assign probability $\frac{1}{5}$ to the left node at each of his information sets. We take all this as given in the rest of the analysis. We have to consider the pure strategies (K, K') and (T, T') of Player 1.

1. (K, K'). In this case Player 2 must assign probability $\frac{3}{4}$ to node x so that the expected payoffs are: 0 from g, $-\frac{1}{4}$ from c and $-\frac{25}{4}$ from b, so that g is uniquely sequentially rational. Given that Player 2 plays g then at the left information set Player 1 (who must assign probability $\frac{1}{5}$ to the left node) gets 3 with T and 5 with K, so that K is indeed sequentially rational; at the right information set Player 1 (who must assign probability $\frac{1}{5}$ to the left node) gets 3 with T and $\frac{1}{5}0 + \frac{4}{5}5 = 4$, so that K' is indeed sequentially rational. Thus we have a WSE given by $((K, K'), (g', b'', g))$ with $\mu(x) = \frac{3}{4}$.

2. (T,T'). In this case any beliefs are allowed at Player 2's information set. At that information set g is optimal when $\mu(x)\in\left[\frac{2}{3},1\right]$, c is optimal when $\mu(x)\in\left[\frac{1}{4},\frac{2}{3}\right]$ and b is optimal when $\mu(x)\in\left[0,\frac{1}{4}\right]$. Thus g and b are never simultaneously optimal. At the left information set Player 1 weakly prefers T to K if and only if $3\geq\frac{1}{5}5+\frac{4}{5}\left(2\sigma_2(c)+5(1-\sigma_2(c))\right)$, that is, if and only if $\sigma_2(c)\geq\frac{5}{6}$. At the right information set Player 1 weakly prefers T' to K' if and only if $3\geq\frac{1}{5}0+\frac{4}{5}\left(5\sigma_2(\mathrm{g})+-\sigma_2(c)\right)$, that is, if and only if $15+4\sigma_2(c)\geq 20\sigma_2(g)$. If $\sigma_2(c)\geq\frac{5}{6}$ then both conditions are satisfied and thus we have the following WSEs:

(1) $\left((T,T'),(g',b'',c)\right)$ and $\mu(x)\in\left[\frac{1}{4},\frac{2}{3}\right]$,

(2) $\left((T,T'),\left(g',b'',\begin{pmatrix} g & c & b \\ 1-p & p & 0\end{pmatrix}\right)\right)$ with $p\geq\frac{5}{6}$ and $\mu(x)=\frac{2}{3}$,

(3) $\left((T,T'),\left(g',b'',\begin{pmatrix} g & c & b \\ 0 & p & 1-p\end{pmatrix}\right)\right)$ with $p\geq\frac{5}{6}$ and $\mu(x)=\frac{1}{4}$.