SPRING 2014

ECN/ARE 200C: MICROECONOMIC THEORY Professor Giacomo Bonanno

MIDTERM EXAM ANSWER ALL QUESTIONS (total 100 points)

1. [38 points] Player 1 can take action C or L and player 2 can take action c or f. The von Neumann-Morgenstern payoffs are as follows

		Player 2		
	-	С	f	
Player 1	С	4,4	6,3	
	L	3 , 1	5,2	

The game, however, is more complex than the above strategic form. Player 1 moves first and chooses between *C* and *L*. He then sends an e-mail to player 2 telling her truthfully what choice he made. However, it is commonly known between the two that a hacker likes to intercept e-mails and change the text. The hacker is a computer program that, with probability $(1-\varepsilon)$, leaves the text unchanged and, with probability ε , it changes the sentence "I chose *C*" into the sentence "I chose *L*" and the sentence "I chose *L*" into the sentence "I chose *C*". This is commonly known. The value of ε is also commonly known. Assume that $\varepsilon \in (0, \frac{1}{4})$.

- (a) [10 points] Draw the extensive game.
- (b) [9 points] Find all the pure-strategy weak sequential equilibria.
- (c) [9 points] Are all (pure and mixed) weak sequential equilibria of this game sequential equilibria?
- (d) [10 points] Are there any sequential equilibria (pure or mixed) in which player 2, when he receives a message from player 1 saying "I chose L" plays *f* with probability 1?

2. [37 points] There are two firms producing a homogeneous product whose inverse demand function is P = 12 - Q (where Q is industry output). Firm 1 acts as Stackelberg leader: it chooses its output first; firm 2 chooses its output after observing the output of firm 1. Firm 2 decides to enter (i.e. chooses a positive level of output) if and only if it expects to make positive profits. Consider two scenarios.

(a) [8 points] Both firms have zero costs. Find the backward-induction solution of this perfectinformation game.

From now on assume that both firms have zero *marginal* cost, but a positive fixed cost F (which is incurred by a firm if and only if the firm produces a positive level of output).

- (b) [9 points] Calculate and draw the reaction curve of firm 2 when F = 4.
- (c) [10 points] What is the backward-induction solution when F = 4?
- (d) [10 points] What is the backward-induction solution when F = 0.25?

- 3. [25 points] An electric circuit connects two switches and a light. One switch is in Room 1, the second switch is in Room 2 and the light is in Room 3. Player 1 is in Room 1, Player 2 in Room 2 and Player 3 in Room 3. The two switches are now in the Off position. The light in Room 3 comes on if and only if both switches are in the On position. Players 1 and 2 act simultaneously and independently: each decides whether to leave her switch in the Off position or turn it to the On position. If the light comes on in Room 3 then the game ends and Players 1 and 2 get \$100 each while Player 3 gets \$300. If the light in Room 3 stays off, then Player 3 (not knowing what the other players did) has to make a guess as to what Players 1 and 2 did (thus, for example, one possible guess is "both players left their respective switches in the Off position"). The payoffs are as follows: (i) if Player 3's guess turns out to be correct then each player gets \$200, (ii) if Player 3 gets one correct guess but the other wrong (e.g. he guesses that both Player 1 and Player 2 chose "Off" and, as a matter of fact, Player 1 chose "Off" while Player 2 chose "On"), then Player 3 gets \$100, the player whose action was guessed correctly gets \$200 and the remaining player gets nothing (in the previous example, Player 1 gets \$100 Player 2 gets nothing and Player 3 gets \$100) and (iii) if Player 3's guess is entirely wrong then all the players get nothing. It is common knowledge among all the players that all the players are selfish and greedy, that is, each player only cares about how much money he/she gets and prefers more money to less.
 - (a) [12 points] Represent this situation as a strategic-form game, assigning the rows to Player 1, the columns to Player 2, etc.
 - (b) [6 points] For each player state whether he/she has a weakly or strictly dominated strategy and prove your claim.
 - (c) [7 points] Find all the pure-strategy Nash equilibria.

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University of California, Davis -- Department of Economics **ECON. 200C: MICROECONOMIC THEORY**

Midterm Exam **ANSWERS**

Giacomo Bonanno

1. (a) The extensive form is as follows:



- (b) To find the pure-strategy sequential equilibria of this game, consider the two possible pure strategies of player 1. If player 1 plays L then player 2 assigns probability 1 to the bottom node of each information set and responds with f with probability 1. This makes player 1 want to deviate to C. Thus there is no pure-strategy sequential equilibrium where player 1 plays L. On the other hand, if player 1 chooses C with probability 1, then player 2 assigns probability 1 to the top node of each information set and responds with c, which makes playing C optimal for player 1. Thus (C,(c,c)) is the only pure-strategy weak sequential equilibrium.
- (c) All information sets are reached with positive probability, whatever (pure or mixed) strategies the players choose. Thus consistency reduces to the requirement that beliefs be Bayesian, and so all weak sequential equilibria are sequential equilibria.
- (d) Suppose that player 2 plays f after reading the message "I chose L". We know from the argument in part (b) that there are no equilibria of this kind in which player 1 chooses a pure strategy, so player 1 must be mixing. For him to be willing to do so, he must receive the same payoff from playing C or L. If we let p be the probability that 2 plays c if she receives the message "I chose C", then 1 is indifferent when

$$\pi_1(C) = \pi_1(L) \iff 4(1-\varepsilon)p + 6[1-(1-\varepsilon)p] = 3\varepsilon p + 5(1-\varepsilon p)$$
$$\iff p = \frac{1}{2-4\varepsilon}$$

Since $\varepsilon \in (0, \frac{1}{4})$, $p \in (\frac{1}{2}, 1)$, so player 2 randomizes after reading "I chose C". For 2 to be willing to do this, she must be indifferent between *c* and *f* in this event. This is true when

$$\pi_2(c \mid \text{"I chose C"}) = \pi_2(f \mid \text{"I chose C"})$$

$$\Leftrightarrow 4q(1-\varepsilon) + 1(1-q)\varepsilon = 3q(1-\varepsilon) + 2(1-q)\varepsilon$$

$$\Leftrightarrow q = \varepsilon$$

where q is the probability with which c is played. We have now specified behavior at all information sets. To ensure that the specified behavior is an equilibrium, we need to check that f is optimal for player 2 if she receives the message "I chose L". This will be true if

$$\pi_2(c \mid \text{"I chose L"}) \le \pi_2(f \mid \text{"I chose L"})$$

$$\Leftrightarrow 4q\varepsilon + 1(1-q)(1-\varepsilon) \le 3q\varepsilon + 2(1-q)(1-\varepsilon)$$

$$\Leftrightarrow 4\varepsilon^2 + (1-\varepsilon)^2 \le 3\varepsilon^2 + 2(1-\varepsilon)^2$$

$$\Leftrightarrow \varepsilon \le \frac{1}{2}.$$

Since $\varepsilon < \frac{1}{4}$, player 2 strictly prefers to play *f* after receiving the message "I chose L". Thus the strategy profile we have constructed is a sequential equilibrium:

Behavior strategy of player 1: $\begin{pmatrix} C & L \\ \varepsilon & 1-\varepsilon \end{pmatrix}$,

behavior strategy of player 2 at the information set on the left (where she receives the message "I chose C"): $\begin{pmatrix} c & f \\ \frac{1}{2-4\varepsilon} & \frac{1-4\varepsilon}{2-4\varepsilon} \end{pmatrix}$, behavior strategy of player 2 at the information set on the right (where she receives the message

"I chose L"): $\begin{pmatrix} c & f \\ 0 & 1 \end{pmatrix}$.

2. (a) The profit function of firm 2 is given by $\Pi_2(q_1, q_2) = q_2(12 - q_1 - q_2)$. The reaction function of firm 2 is given by the solution, w.r.t. q_2 , of $\frac{\partial \Pi_2}{\partial q_2} = 0$. Thus $R_2(q_1) = 6 - \frac{q_1}{2}$. Hence the profit function of firm 1 (using backward induction) is $\Pi_1(q_1) = q_1(12 - q_1 - R_2(q)) = \frac{1}{2}(12q_1 - q_1^2)$. Taking $\frac{\partial \Pi_1}{\partial q_1} = 0$ gives $q_1 = 6$. The output of firm 2 is thus $R_2(6) = 3$.

(**b**) When there is a fixed cost F > 0, the reaction function of firm 2 is still $R_2(q_1) = 6 - \frac{q_1}{2}$ provided that $\Pi_2(q_1, R_2(q_1)) > F$ where the function $\Pi_2(q_1, q_2)$ is as given in part (a). Solving

 $\Pi_{2}(q_{1}, R_{2}(q_{1})) = F \text{ when } F = 4 \text{ gives } q_{1} = 8. \text{ Thus the reaction function of firm 2 is}$ $R_{2}(q_{1}) = \begin{cases} 6 - \frac{q_{1}}{2} & \text{if } q_{1} < 8 \\ 0 & \text{if } q_{1} \ge 8 \end{cases}$ It is shown in the following diagram: q_{2} q_{1}

(c) When F = 4, the profit function of firm 1 becomes:

$$\Pi_{1}(q_{1}) = q_{1}(12 - q_{1} - R_{2}(q)) - 4 = \begin{cases} \frac{1}{2}(12q_{1} - q_{1}^{2}) - 4 & \text{if } q_{1} < 8 \\ q_{1}(12 - q_{1}) - 4 & \text{if } q_{1} \ge 8 \end{cases}$$

it is sketched below:



As is clear from the picture, the backward induction outcome is $q_1 = 8$ (and $q_2 = 0$). Alternatively, this can be established by comparing the maximum value of the function $\frac{1}{2}(12q_1 - q_1^2) - 4$ which is 14 (achieved at $q_1 = 6$) with the maximum value of the function $q_1(12 - q_1) - 4$ in the range $q_1 \ge 8$, which is 28 and is achieved at $q_1 = 8$.

(d) When F = 0.25, the profit function of firm 1 becomes:

$$\Pi_{1}(q_{1}) = q_{1}(12 - q_{1} - R_{2}(q)) - 0.25 = \begin{cases} \frac{1}{2}(12q_{1} - q_{1}^{2}) - 0.25 & \text{if } q_{1} < 11 \\ q_{1}(12 - q_{1}) - 0.25 & \text{if } q_{1} \ge 11 \end{cases}$$

it is sketched below:



As is clear from the picture, the backward induction outcome is $q_1 = 6$ (and $q_2 = 3$). Alternatively, this can be established by comparing the maximum value of the function $\frac{1}{2}(12q_1 - q_1^2) - 0.25$ which is 17.75 (achieved at $q_1 = 6$) with the maximum value of the function $q_1(12-q_1) - 0.25$ in the range $q_1 \ge 11$, which is 10.75 and is achieved at $q_1 = 11$.

3. (a) The game is as follows:

		Player 2			
	_	On	Off		
Player 1	On	100 , 100 , 300	200, 200, 200		
	O ff	0 , 0 , 0	0 , 200 , 100		
		Player 3: 10n-20ff			
		Player 2			
Player 1	-	On	Off		
	On	100 , 100 , 3 00	0,0,0		
	Off	200, 200, 200	200 , 0 , 100		
		Player 3: 10ff-20n			
		Player 2			
Player 1	_	On	Off		
	On	100,100,300	0 , 200 , 100		
	Off	200 , 0 , 100	200 , 200 , 200		
		Player 3: both Off			

- (b) None of the players have a dominated strategy. For Player 1 On is better than Off against (On, 1On-2Off) but worse than Off against (On,1Off-2On). Similarly for Player 2. For Player 3, 1On-2Off is better than the other two strategies against (On,Off), 1Off-2On is better than the other two strategies against (Off,On) and both-Off is better than the other two strategies against (Off,Off).
- (c) There are three pure-strategy Nash equilibria, which are highlighted above.

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FINAL EXAM ANSWER ALL QUESTIONS (total 100 points)

[40 points] There is a single seller of a used car and two identical risk-neutral buyers. The buyers believe that the seller's car is of low quality (L) with probability λ∈ (0,1) and of high quality with probability (1-λ). All of the following is common knowledge among the three: (1) the above beliefs of the buyers, (2) the fact that the seller knows the value of her car, (3) the cost of certification explained below and (4) the following valuations:

	Н	L
value to seller	150	50
value to buyer	200	100

The seller can obtain a "certificate" for the car from an independent mechanic at a cost of 25 if it is an H car and a cost of 175 if it is an L car (it costs more to certify an L car because the mechanic has to be bribed to lie). Consider the following game: first the seller decides whether or not to obtain a certificate, then the buyers observe whether or not the certificate was obtained and then simultaneously bid for the car, buyer b offering some price $p \in [0, \infty)$ and buyer b' offering some price $p' \in [0, \infty)$; finally the seller either accepts one of the two price offers or rejects both.

- (a) [10 points] Sketch the extensive-form game that is obtained by applying the Harsanyi transformation to this situation of incomplete information.
- (b) [10 points] Find a weak sequential equilibrium that is separating. Prove that it is a weak sequential equilibrium.
- (c) [10 points] Suppose that $\lambda = \frac{3}{4}$. Explain why the possibility of certifying the car leads to a Pareto improvement relative to the situation where certification is not possible.
- (d) [10 points] If there were perfect information about the quality of the car, would the car be sold and, if so, at what price?
- **2.** [25 points] A monopolist has two customers: a rich customer with inverse demand $P_H = 20 q$ and a poorer customer with inverse demand $P_L = 15 q$. The monopolist has zero production costs. A contract is a pair C = (q, v) specifying a quantity q and a total payment of v.
- (a) [6 points] What contracts would the monopolist offer if he could distinguish the two types?

From now on assume that the monopolist cannot distinguish the two types (that is, he does not know which customer is the rich one which is the poorer one). Thus he will offer a menu of contracts and let the customers choose.

- (b) [6 points] Briefly explain why the monopolist will "undersupply" the poorer consumer.
- (c) [13 points] Find the profit-maximizing menu of contracts.

- **3.** [30 points] In the insurance industry there are two types of potential customers, *L* and *H*. Both types have the same initial wealth of \$16,000, face a potential loss of \$7,000 and have the von Neumann-Morgenstern utility-of-wealth function $U(m) = \sqrt{m}$. There are a total of 4,200 potential customers, of which 700 are of type *H*. The probability of loss for the *H* type is 20%, while the probability of loss for the *L* type is 10%. Assume that, if indifferent between not insuring and insuring the consumer decides to insure and that, if indifferent between two contracts, the consumer will choose the one that favors the insurance company (provided that it is not worse than not insuring). For parts (a)-(d) assume that the insurance industry is a monopoly.
 - (a) [5 points] Calculate the monopolist's profits if it decides to offer only one contract and chooses the contract that extracts the maximum surplus from the H type.
 - (b) [5 points] (b.1) If the monopolist decided to offer only one contract that would attract both types, what would be the best such contract (best in the sense that it maximizes profits)? You don't need to calculate the premium and deductible: just write the relevant equations. Does such a contract exist?
 [5 points] (b.2) Find a two-contract menu that yields higher profits than the contract of part (b.1). Again, no need to calculate: just write the relevant equations.

For questions (c)-(d) assume that the monopolist decides to offer a menu of contracts (C_H, C_L) where $C_H = (h_H = 1,500, D_H = 0)$ is the contract targeted to the *H* people and $C_L = (h_L, D_L)$ is the contract targeted to the *L* people.

- (c) [5 points] Write down the individual rationality constraint for the *L* people and the incentive-compatibility constraints for both types that make the pair (C_H, C_L) a "separating pair". Why don't we need to worry about the individual rationality constraint for the *H* people?
- (d) [5 points] Of all the contracts C_L that, together with $C_H = (1,500, 0)$, satisfy the constraints of part (c), find the profit-maximizing one. Again, no need to calculate: just write the relevant equations.
- (e) [5 points] Find the pair of contracts that is the only candidate for equilibrium in a competitive industry (where profits are zero and no profitable entry is possible).
- **4.** [5 points] Why is Akerlof's "lemon problem" unlikely to arise in the market for real lemons (that is, edible lemons)?

Final Exam **ANSWERS**

- 1.
- (a) The game is as follows. The proper subgames start at the nodes after both buyers have made an offer (4 are shown in the figure, but there are infinitely many), where the seller must respond to the offers.



(b) The strategies are as follows: (1) the seller certifies her car if and only if it is of high quality, (2) buyer *b* offers p = 200 if the car is certified and p = 100 if it is not. Buyer *b'* does the same, (3) after being presented with any pair of offers, the seller says yes to the (or one of the) highest offer if it is greater than or equal to her valuation and if both offers are below her valuation then she rejects both and keeps her car.

The buyers' beliefs are that the car is H with probability 1 if it is certified and L with probability 1 if it is not. Checking sequential rationality: (3) is a best response in the subgames since any cost of certification is a sunk cost at this stage; (1) is a best response given the buyers' strategies (if she has an L car then 100 > 50 and if she has an H car then 175 > 150); (2) is the result of Bertrand competition between the buyers

- (c) Without certification, the price of the car would be 100 and the seller would sell only if her car is of low quality (if both cars were offered for sale the price would be 125, which is less than the value to the seller of an H car: $EV_b = 125 < V_s^H = 150$). So the separating equilibrium involves a Pareto improvement: no buyer is worse off or better off than without signaling, nor is the seller affected if she has a low-quality car, but the seller of an H car is better off with signaling: her net payoff is (200 25) = 175 > 150.
- (d) With perfect information, there would be no need for certification and the H car would be sold for 200 and the L car for 100.

2.

- (a) He would act as a perfectly discriminating monopoly and offer the rich customer contract $C_H = (20, \$200)$ and the other customer contract $C_L = (15, \$112.50)$
- (b) When the poorer consumer is efficiently supplied, the last infinitesimal unit sold to her yields zero marginal profit from that customer (since marginal price = marginal cost = 0). But if the monopolist sold an infinitesimal unit less to that customer, he could raise the price to the rich customer for the last infinitesimal unit by \$5 (the difference in height between the two demand curves) since the IC constraint of the rich customer would loosen when the poorer customer is sold an infinitesimal unit less.

(c) Let q be the quantity in the contract targeted to the poorer customer. We know that the quantity in the contract targeted to the rich consumer is 20 (given by the intersection of her demand curve with marginal cost: the solution to 20 - Q = 0). Then the monopolist's problem is to choose q to maximize the following expression, where the first bracketed term is the profit from the poorer customer (using the fact that the IR constraint is binding for her) and the second bracketed term measures the profit from the rich customer (using the fact that the IR constraint is binding for her) and the second bracketed term

$$\pi(q) = \left[\int_0^q (15-x)dx\right] + \left[\int_0^{20} (20-x)dx - \int_0^q (20-x)dx + \int_0^q (15-x)dx\right] = 10q - \frac{q^2}{2} + 200$$

The solution is q = 10. Note that this occurs where the marginal decrease in profits from the poorer consumer by selling one less infinitesimal unit (namely 5 at q = 10) equals the marginal increase in profit from the richer consumer. Thus the profit-maximizing menu is contract (10,100) targeted to the poorer consumer and contract (20,150) targeted to the richer consumer. The richer consumer gets a surplus.

3.

(a) The monopolist would offer full insurance at the maximum premium that the *H* type are willing to pay, given by the solution to √16,000 - h = 1/5 √9,000 + 4/5 √16,000 , which is h^{*}_H = 1,560. The corresponding profits are π_a = 700 (1,560 - 1/5 7,000) = 112,000.
(b) (b.1)The average probability of loss is p
= (700)/(4,200) ⋅ 1/5 + (3,500)/(4,200) ⋅ 1/10 = (7/60) = 0.1167. The profit-

maximizing contract is the point on the *L*-reservation indifference curve at which the slope is equal to $-\frac{\overline{p}}{1-\overline{p}}$. Thus the contract must satisfy the following equations:

(1)
$$\frac{\sqrt{16,000-h}}{\underbrace{9\sqrt{16,000-h-D}}_{\frac{p_L}{1-p_L},\frac{U'(16,000-h-D)}{U'(16,000-h)}} = \frac{7}{\underbrace{60}_{\frac{\overline{p}}{1-\overline{p}}}} \text{ and}$$

(2)
$$\frac{1}{10}\sqrt{9,000} + \frac{9}{10}\sqrt{16,000} = \frac{1}{10}\sqrt{16,000-h-D} + \frac{9}{10}\sqrt{16,000-h}$$

Denote the solution by (\hat{h}, \hat{D}) . Denote the solution by (\hat{h}, \hat{D}) . A solution exists if and only if the slope of the *L*-reservation indifference curve at the no insurance (NI) point is greater than $\frac{\overline{p}}{1-\overline{p}} = \frac{7}{60}$ in absolute value (since it is less than that amount at the point where it intersects the 45° line). And indeed the slope at the NI point is $\frac{\sqrt{16,000}}{9\sqrt{9,000}} = \frac{4}{27} > \frac{7}{60}$.

(b.2) If the monopolist keeps the contract of part (b.1) but adds a second contract, given by the

intersection of the 45° line and the *H*-indifference curve through the contract of part (b.1) then its profits will increase. The additional contract solves the equation

$$\sqrt{16,000 - h} = \frac{1}{5}\sqrt{16,000 - \hat{h} - \hat{D}} + \frac{4}{5}\sqrt{16,000 - \hat{h}}.$$
(c) (IR_L) $\frac{1}{10}\sqrt{9,000} + \frac{9}{10}\sqrt{16,000} \le \frac{1}{10}\sqrt{16,000 - h_L - D_L} + \frac{9}{10}\sqrt{16,000 - h_L}$
(IC_L) $\sqrt{14,500} \le \frac{1}{10}\sqrt{16,000 - h_L - D_L} + \frac{9}{10}\sqrt{16,000 - h_L}$
(IC_H) $\frac{1}{5}\sqrt{16,000 - h_L - D_L} + \frac{4}{5}\sqrt{16,000 - h_L} \le \sqrt{14,500}$

 IR_H is satisfied because 1,500 < 1,560.

- (d) It is the one that satisfies the equations $\frac{1}{10}\sqrt{9,000} + \frac{9}{10}\sqrt{16,000} = \frac{1}{10}\sqrt{16,000 - h_L - D_L} + \frac{9}{10}\sqrt{16,000 - h_L} \text{ and}$ $\frac{1}{5}\sqrt{16,000 - h_L - D_L} + \frac{4}{5}\sqrt{16,000 - h_L} = \sqrt{14,500}.$
- (e) The full insurance contract with premium $p_H x = 1,400$ and the partial insurance contract (\tilde{h}, \tilde{D}) that satisfies the following equations: $\tilde{h} = \frac{1}{10}(7,000 - \tilde{D})$ and $\sqrt{16,000 - 1,400} = \frac{1}{5}\sqrt{16,000 - \tilde{h} - \tilde{D}} + \frac{4}{5}\sqrt{16,000 - \tilde{h}}$ which reduces to the equation $\sqrt{14,600} = \frac{1}{5}\sqrt{16,000 - 700 + \frac{1}{10}\tilde{D} - \tilde{D}} + \frac{4}{5}\sqrt{16,000 - 700 + \frac{1}{10}\tilde{D}}$ whose solution is $\tilde{D} = 5,906.81057$ with corresponding premium $\tilde{h} = 109.31894$.
- **4.** It is true that also in the market for edible lemons buyers are uncertain about the quality of any particular edible lemon that they buy, but the key feature of the "lemon problem" is not that information is imperfect but that it is asymmetric: sellers know more than buyers about the quality of used cars. The market for edible lemons is more like the market for new cars in that sellers know no more than buyers. Edible lemons are therefore likely to be sold at a price which reflects their average quality