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HOMEWORK 6 (for due date see the web page)

Let e denote the level of education. There are three types of potential workers: those (type L) with productivity θ_L, those (type M) with productivity θ_M and those (type H) with productivity θ_H, with $\theta_H > \theta_M > \theta_L > 0$. For each type $i \in \{L, M, H\}$ the fraction of type i in the population is $\frac{1}{3}$. Each potential worker knows her own type, while the potential employer cannot tell the type of any potential worker, although he knows the distribution of types in the population. The employer observes the education level of each potential worker (but not her type) and offers a wage which depends on the applicant's level of education. For every type $i \in \{L, M, H\}$ the cost of acquiring e units of education is $\dfrac{e}{\theta_i}$. Each worker's utility is given by the difference between the wage she is paid and the cost of education.

(a) [Note: for this part do **not** assume that each worker must be paid a wage equal to her productivity.] Is there an incentive-compatible situation where (1) the employer offers two wages, depending on the education level: wage w^* to those whose education level is e^* and wage $w_M \neq w^*$ to those whose education level is $e_M \neq e^*$ and refuses to hire anybody with education $e \notin \{e^*, e_M\}$, (2) both types θ_L and θ_H choose education level e^*, while type θ_M choose education e_M? [Note that you should make no assumptions about whether $e_M < e^*$ or $e^* > e_M$ and similarly for w_M and w^*.] If there is such an incentive-compatible situation, please describe it in detail. If your claim is that it does not exist, please prove it.

For parts (b) and (c) assume that the employer pays each worker a wage equal to the worker's expected productivity (as computed by the employer, who is risk neutral).

(b) Define and describe in detail a pooling equilibrium, that is, a signaling equilibrium where all three types make the same choice of education level, call it $\bar{e}$. [Assume that the employer believes that anybody who shows up with education level $e \neq \bar{e}$ must be of type L.]
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(c) Find all the pooling equilibria when $\theta_L = 1, \theta_M = 2, \theta_H = 6$.

Now let us change the situation as follows. There are only two types of potential workers: those with productivity θ_L and those with productivity θ_H, with $\theta_H > \theta_L > 0$. The fraction of type θ_L in the population is equal to the fraction of type θ_H. Assume that the cost of education is the *same for both types* and is given by $c(e) = e$. Suppose that the utility of worker of type $\theta \in \{\theta_L, \theta_H\}$ who is paid wage w and chooses education level e is $U(w,e,\theta) = \theta w - e$. Assume also that $e \in [a,b]$ with $0 < a < b$, that is, there is a minimum level of education a that every worker must have (it is mandated by the government) and a maximum level of education b (e.g. corresponding to a PhD). As before, each potential worker knows her own type, while the potential employer cannot tell the type of any potential worker, although he knows the distribution of types in the population. The employer observes the education level of each potential worker (but not her type) and offers a wage which depends on the applicant's level of education.

(d) (d.1) Are there separating signaling equilibria (where different types of workers choose different education levels)? If there are, please describe such equilibria (note that you have to specify the wage that the employer offers for every possible level of education and you cannot assume that it is zero). If not, please prove your claim. [Recall that part of the definition of a signaling equilibrium is that each worker is paid a wage equal to her true productivity]

(d.2) Is there a separating equilibrium when $a = 6, b = 14, \theta_L = 3, \theta_H = 5$? If yes, please describe it. If not, please explain why not.