HOMEWORK 2 **(for due date see the web page)**

There are two firms that produce a homogeneous product. Let p_i be the price of firm i (i = 1,2). Assume for simplicity that the firms have **zero costs**. There is a large number N of potential consumers, each with the same reservation price for the product, equal to \$60, that is, each consumer buys one unit from one of the two firms if and only if at least one of the prices is less than or equal to 60. When $p_1 = p_2 \leq 60$, 50% of the consumers go to Firm 1 and 50% to Firm 2. What happens when the prices are different? Some consumers are very sensitive to price differences, while others are not. For example, if $p_1 = p_2 + 0.25$ then some consumers might prefer Firm 2 because they save 25 cents, but probably many consumers would be indifferent between the two firms. Let $f : [0,60] \to \mathbb{R}^+$ be the density function (thus $f(x) \geq 0$, for all $x \in [0,60]$, and $\int_0^{60} f(x)dx = 1$)) that measures the price-difference sensitivity of consumers. For example, suppose that $p_1 < p_2 < 60$. Then $\int_0^{p_2 - p_1} f(x)dx$ gives the *fraction* of consumers who *prefer the cheaper firm* (Firm 1), while the others are indifferent between the two firms. Assume that 50% of the indifferent consumers go to one firm and 50% go to the other firm. Consider three cases:

1. f is constant (uniform distribution)
2. $f(x) = \begin{cases} \frac{1}{1000}(25 - 0.5x) & \text{if } 0 \leq x \leq 20 \\ 0.015 & \text{if } 20 < x \leq 60 \end{cases}$
3. $f(x) = \begin{cases} \frac{1}{1000}(68 - 5.6x) & \text{if } 0 \leq x \leq 10 \\ 0.012 & \text{if } 10 < x \leq 60 \end{cases}$

(a) For each of the three cases above, write the demand function of each firm (cover *all* the possibilities, that is, all pairs (p_1, p_2) with $p_i \in [0, \infty)$ for every i = 1,2).

(b) In each of the three cases, determine whether $p_1 = p_2 = 60$ is a Nash equilibrium.

(c) In each of the three cases, determine whether $p_1 = p_2 = 0$ is a Nash equilibrium.