HOMEWORK 1 **(for due date see the web page)**

Consider a second-price auction with three bidders, numbered 1, 2 and 3. The rules are:

(1) bids must belong to the set {1, 2, 3, …, 100},

(2) the object is assigned to the highest bidder and, in case of ties, to the bidder with the smallest index among those who submitted the highest bid,

(3) the winner pays the second-highest bid.

For example, if the bids are ($9, $12, $7) then the winner is bidder 2 and she pays $9, while if the bids are ($10, $10, $7) then the winner is bidder 1 and he pays $10.

Denote an outcome as a pair (i, p) where $i \in \{1,2,3\}$ is the winner and p is the price paid by the winner. The bidders have the following preferences, where, for each $i \in \{1,2,3\}$, v_i is an integer in the set {1, 2, 3, …, 50}.

Bidder 1:

- for all p, p' and for $i, j \in \{2,3\}$, $(i,p) \sim_1 (j,p') \sim_1 (1,v_1)$,
- for all p, p' and for $j \in \{2,3\}$,
  $(1,p) \succ_1 (j,p')$ if $p < v_1$
  $(j,p') \succ_1 (1,p)$ if $p > v_1$,
- for all p, p', $(1,p) \succ_1 (1,p')$ if and only if $p < p'$
- and everything that follows from the above and transitivity.

Bidder 2:

- for all p, p', $(1,p) \sim_2 (1,p') \sim_2 (3,1) \sim_2 (3,2) \sim_2 (3,3) \sim_2 (3,4) \sim_2 (3,5) \sim_2 (2,v_2)$,
- for all p, p',
  $(2,p) \succ_2 (1,p')$ if $p < v_2$
  $(1,p') \succ_2 (2,p)$ if $p > v_2$,
- for all $p, p' \in \{5,6,7,...,100\}$, $(3,p) \succ_2 (3,p')$ if and only if $p < p'$,
- for all p, p', $(2,p) \succ_2 (2,p')$ if and only if $p < p'$,
- and everything that follows from the above and transitivity.

Bidder 3:

- for all p, p', $(1, p) \sim_3 (1, p')$
- for all p, $(1, p) \sim_3 (3, v_3)$ and $(2, p) \sim_3 (3, v_3 + 5)$
- for all p, p', $(3, p) \succ_3 (3, p')$ if $p < p'$
- and everything that follows from the above and transitivity.

(a) Write a utility function that represents the preferences of Bidder 1.

(b) Does Bidder 1 have a dominants strategy? If your answer is Yes, then state what that strategy is and whether it is weakly or strictly dominant. If your answer is No then justify your claim.

(c) Optional: you can skip this question if you wish. Write a utility function that is consistent with the preferences of Bidder 2. [Note: the preferences given above are incomplete, so you would have to provide a consistent completion of them.]

(d) Suppose that $v_2 = 10$. Is $b_2 = 10$ a dominant strategy for Bidder 2? Prove your claim.

(e) Continue to suppose that $v_2 = 10$. Is any bid different from 10 a dominant strategy for Bidder 2? Prove your claim.

(f) Optional: you can skip this question if you wish. Write a utility function that is consistent with the preferences of Bidder 3. [Note: the preferences given above are incomplete, so you would have to provide a consistent completion of them.] Give an intuitive description of Bidder 3's preferences.

(g) Suppose that $v_3 = 10$ and suppose that Bidder 3 expects Bidder 1 to bid 12 and Bidder 2 to bid 16. Explain why Bidder 3 should not choose a bid $b_3 > 16$.

(h) Suppose that $v_3 = 10$ and suppose that Bidder 3 expects Bidder 1 to bid 11 and Bidder 2 to bid 14. Explain why Bidder 3 should not choose a bid $b_3 \leq 11$.