SECOND-DEGREE PRICE DISCRIMINATION

FIRST Degree:	The firm knows that it faces different individuals with different demand functions and furthermore the firm can tell who is who. In this case the firm extracts all the consumer surplus, usually with a **two-part tariff** (with P = MC, thus the same price for everybody, but with different tariffs for different individuals).
SECOND Degree:	The firm knows that it faces different individuals with different demand functions but it cannot tell who is who. In this case the firm offers a menu of different packages or options designed in such a way that consumers sort themselves out (self-select) by choosing different packages. With second degree price discrimination the firm is not able to extract all the consumer surplus.

Consider the case where the firm knows that it faces consumers with different willingness to pay, but it cannot tell which consumer is which. For example, the firm knows that higher-income people are willing to pay more than lower-income people, but unfortunately consumers don't have their annual incomes printed on their foreheads. Similarly, airlines know that business travelers are less flexible in their travel plans and are willing to pay more to fly than vacationers, but when somebody calls to make a reservation the airline cannot tell whether the caller is a businessperson or a vacationer. Is the firm then forced to charge the same price to everybody? The answer is no. What firms do in these cases is to offer different packages and then let consumers **self-select** into the different price categories. This type of price discrimination is called second-degree price discrimination.

Consider a monopolist who faces N identical high-income consumers, each with inverse demand

$$P_H = A - Q$$

and n identical low-income consumers, each with inverse demand

$$P_L = a - Q$$

with $A > a > 0$. The monopolist has the following cost function:

$$C = cQ$$

with $0 < c < a$.

Let $W_H(Q)$ denote the willingness to pay of a high-income consumer for Q units. Then $W_H(Q) = \int_0^Q P_H(x)dx = AQ - \frac{Q^2}{2}$. Similarly, let $W_L(Q)$ denote the willingness to pay of a low-income consumer for Q units. Then $W_L(Q) = \int_0^Q P_L(x)dx = aQ - \frac{Q^2}{2}$. Suppose that the monopolist decides to sell packages (Q,V) consisting of Q units at a package price of V (so that the implied per unit price is $\frac{V}{Q}$). Then the monopolist has three options.

$\boxed{\textbf{OPTION 1.}}$ Offer only one type of package (Q,V) which will be bought only by high-income people, because $V > W_L(Q)$ but $V \le W_H(Q)$. Then the monopolist might as well charge $V = W_H(Q)$ and its profits will be

$$\pi_1 = N[W_H(Q) - cQ] = N\left(AQ - \frac{Q^2}{2} - cQ\right).$$

Solving $\frac{d\pi_1}{dQ} = 0$ gives $\boxed{Q_1^* = A - c}$ with $\boxed{V_1^* = W_H(A-c) = \frac{A^2 - c^2}{2}}$ and corresponding profit of $\boxed{\pi_1^* = N\frac{(A-c)^2}{2}}$.

$\boxed{\textbf{OPTION 2}}$ **.** Offer only one type of package (Q,V) which will be bought by both high-income and low-income people, because $V \le W_L(Q)$ (which implies that $V < W_H(Q)$ since $W_L(Q) < W_H(Q)$). In this case the monopolist might as well charge $V = W_L(Q)$ and its profits will be

$$\pi_2 = (N+n)[W_L(Q) - cQ] = (N+n)\left(aQ - \frac{Q^2}{2} - cQ\right).$$

Solving $\frac{d\pi_2}{dQ} = 0$ gives $\boxed{Q_2^* = a - c}$ with $\boxed{V_2^* = W_L(a-c) = \frac{a^2 - c^2}{2}}$ and corresponding profit of $\boxed{\pi_2^* = (N+n)\frac{(a-c)^2}{2}}$.

OPTION 3 . Offer two types of packages: a package (Q_H,V_H) targeted to high-income consumers and (Q_L,V_L) targeted to low-income consumers. Then it must be that

(1) $V_L \leq W_L(Q_L)$ so that L-consumers are willing to buy "their" package

(2) $W_L(Q_L) - V_L \geq W_L(Q_H) - V_H$ incentive compatibility constraint for L-consumers (they do not prefer the H package to the L package)

(3) $V_H \leq W_H(Q_H)$ so that H-consumers are willing to buy "their" package

(4) $W_H(Q_H) - V_H \geq W_H(Q_L) - V_L$ incentive compatibility constraint for H-consumers (they do not prefer the L package to the H package)

Note that, since, for every $Q > 0$, $W_H(Q) > W_L(Q)$, **(3) follows from (1) and (4)**: from (1) we get $W_L(Q_L) - V_L \geq 0$ and using the fact that $W_H(Q_L) > W_L(Q_L)$ we get that $W_H(Q_L) - V_L > 0$, which, by (4), gives $W_H(Q_H) - V_H > 0$, i.e. $V_H < W_H(Q_H)$. **Thus from the incentive compatibility constraint for *H*-consumers we get that *H*-consumers must be getting a positive surplus** (the total price of their package is less than what they are willing to pay for it).

Ignore for the moment constraint (2). We'll reason as if (2) were not a constraint and then show that it will be satisfied.

Profit maximization requires that constraints (1) and (4) be satisfied as equalities: $V_L = W(Q_L)$ and $V_H = \mathrm{W_H}(Q_H) - \mathrm{W_H}(Q_L) + \mathrm{W_L}(Q_L) = AQ_H - \frac{(Q_H)^2}{2} - (A-a)Q_L$

Thus the profit function becomes

$$\pi = N\left(V_H - cQ_H\right) + n\left(V_L - cQ_L\right) =$$
$$N\left[AQ_H - \frac{(Q_H)^2}{2} - (A-a)Q_L - cQ_H\right] + n\left[aQ_L - \frac{(Q_L)^2}{2} - cQ_L\right]$$

Solving $\frac{\partial \pi}{\partial Q_H} = N(A - Q_H - c) = 0$ and $\frac{\partial \pi}{\partial Q_L} = -N(A-a) + n(a - Q_L - c) = 0$ we get

$$\boxed{Q_H^* = A - c \quad \text{and} \quad Q_L^* = a - c - \frac{N}{n}(A-a)}\ .$$

Note that, since $A > a$, $Q_H^* > Q_L^*$. Note also that this solution is acceptable if and only if $Q_L^* > 0$, that is, if and only if

$$\boxed{n(a-c) > N(A-a)}\ .$$

Now let us make sure that constraint (2) is also satisfied. Since (1) is satisfied as an equality the LHS of (2) is zero. The RHS of (2) is $W_L(Q_H^*) - V_H^* = -(A-a)(Q_H^* - Q_L^*)$ which is negative since $Q_H^* > Q_L^*$.

Substituting the optimal quantities in π we get that the maximum profit with second-degree price discrimination is

$$\boxed{\pi_3^* = \frac{(N+n)(a-c)^2}{2} + \frac{(N+n)N(A-a)^2}{2n} = \pi_2^* + \frac{(N+n)N(A-a)^2}{2n}}$$

Thus Option 2 is inferior to Option 3. The reason is that, starting from the package of Option 2 (Q = $a-c$), if the monopolist prepares another package for *H*-consumers by adding one unit and increasing the price from V_2^* to V_2^* + Marginal Willingness to Pay of *H*-consumers at $Q_2^* = a-c$, that is, to $V_2^* + \frac{dW_H}{dQ}(a-c) = V_2^* + A - (a-c)$ the firm's profit from the new package increases by $A-(a-c)-c = A-a$ and the new package will be bought by the *H*-consumers because it yields the same surplus as the initial package.

Thus the firm will only compare Options 1 and 3.

Now, if the constraint that is required for the solution to option 3, namely $\boxed{n(a-c) > N(A-a)}$, is satisfied, then $\pi_3^* > \pi_1^*$. This can be seen intuitively as follows. Start with the optimal Option 1 package: $Q = A-c$ and $V = W_H(A-c)$. Now introduce a new package with a very small quantity and charge for it a price equal to the willingness to pay for it of an *L*-customer. That is, increase the quantity of the package offered to *L*-consumers from zero to a small positive amount. Then the Marginal Willingness to pay of an *L*-costumer is $\left.\frac{dW_L}{dQ}\right|_{Q=0} = a$ and by charging this price, the monopolist will increase its profit by ***n(a −c)***. On the other hand, this package will be available also to the *H*-consumers and give them a surplus of *(A−a*), while the current package gives them zero surplus. Thus to prevent them from switching, the price of the current package must be reduced by *(A−a)* leading to a loss in revenue of ***N(A−a)***. Hence introducing the new package is profitable if and only if $\boxed{n(a-c) > N(A-a)}$.

Algebraically, it can be shown that, within the relevant range of parameter space (where the above constraint is satisfied)

$$\pi_3^* - \pi_1^* = \frac{1}{2n}\left[n(a-c) - N(A-a)\right]^2 > 0$$

> In conclusion, **the monopolist will use option 3 if** $n(a-c) > N(A-a)$
> **and option 1 if** $n(a-c) < N(A-a)$

Example 1: $N = n$, $A = 10$, $a = 4$, $c = 2$. Then $n(a-c) = 2n < N(A-a) = 4n$ and the monopolist will choose **option 1**. In fact, $\pi_1^* = 32n$, $\pi_2^* = 4n$ (and π_3^* is not defined because $Q_L^* = -4$).

Example 2: $N = n$, $A = 10$, $a = 8$, $c = 2$. Then $n(a-c) = 6n > N(A-a) = 2n$ and the monopolist will choose **option 3**. In fact, $\pi_1^* = 32n$, $\pi_2^* = 36n$ and $\pi_3^* = 40n$

Note that in Example 2, $Q_H^* = 8$, $Q_L^* = 4$, $V_H^* = 40$ *and* $V_L^* = 24$. Thus the **effective prices per unit** are $P_H = \frac{V_H}{Q_H} = \frac{40}{8} = 5$ and $P_L = \frac{V_L}{Q_L} = \frac{24}{4} = 6$. Thus the package designed for the *H*-consumers incorporates a **quantity discount** or lower effective price per unit. This is always true when the firm chooses second-degree price discrimination, because of the incentive compatibility constraints for the High-demand consumers. Notice that there is **no cost-based justification for the quantity discount** (average cost is constant, it does not decrease with quantity).

Welfare implications of second-degree price discrimination

Is second-degree price discrimination Pareto efficient? The answer (just like in the case of third-degree price discrimination) is that it may efficient and it may be inefficient.

Example where it is efficient. Suppose that United offers the following fares for flights from LA to NY: unrestricted round-trip for $800 and a "vacation special" of $400 requiring a stay of 2 weeks. A business traveler is willing to pay up to $800 for an unrestricted flight and is not interested in a flight that requires a minimum stay. A vacation traveler is willing to pay up to $400 for a round-trip with or without restrictions. If the cost of transporting one passenger is $100 then with both fares United makes a profit of (800 + 400) - 200 = $1,000. Since consumer surplus is zero, social welfare is $1,000. If it were illegal to charge different prices then United would have to choose between charging $800 with a profit of 800 - 100 = 700 or charging $400 with a profit of 2(400) - 200 = 600. Thus it would choose to sell only one ticket for $800 with a total social surplus of 700 (zero consumer surplus). Thus in this example second-degree price discrimination is efficient.

Example where it is inefficient. Same as before but now the vacationer is willing to pay $650 for an unrestricted ticket and $600 for one with restrictions. Then

option 1: charge $800 for an unrestricted fare and $600 for one that requires a two-week stay. Profit is 800 + 600 - 200 = 1200, zero consumer surplus, hence social welfare $1,200

option 2: charge $650 for an unrestricted fare. Profit is 2(650) - 200 = 1100 and consumer surplus is 150 + 0 = 150. Social welfare is $1,250.

The firm would choose option 1 (it maximizes profits) which is Pareto inferior to option 2 which would be the outcome if price discrimination were not allowed.