(10,6,2,0)->A,
(10,8,4,0)->B

Player 2

		C	D
Player 1	A	z_1	z_2
	B	z_3	z_4

Suppose that **Player 1's ranking** of the outcomes is:

$$z_1 \succ z_4 \succ z_3 \succ z_2$$

best ... worst

← suppose answer is

Q. 1 : how do rank the outcomes $z_1, \dots, z_4$?

best	z_1	1
	z_4	0.6
	z_3	
worst	z_2	0

[Continuity axiom]

Q. 2 : if you have to choose between $\begin{pmatrix} z_4 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} z_1 & z_2 \\ p & 1-p \end{pmatrix}$

what value of p would make you indifferent?

Suppose answer is : $p = 0.6$

What if we want to contruct a utility fiunction which is not the normalized one?

Money lotteries and attitudes to risk

A money lottery is a lottery whose outcomes are sums of money.

$$L = \begin{pmatrix} \$4 & \$12 & \$24 & \$36 \\ \frac{1}{4} & \frac{1}{12} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$$

The expected value of L is …

We use the expected value to define the **risk-attitude** of an agent: offer the agent a choice between $\$\mathbb{E}[L]$ for sure, that is, the lottery and the lottery L. If the agent says that

-
-
-

In the above example, ….

		Player 2	
		c	*d*
Player 1	*a*	\$20 \$0	\$60 \$50
	b	\$30 \$80	\$50 \$30

Assume that each player is selfish and greedy (only cares about how much money she herself gets.

CASE 1: Suppose that Player 1 is **risk neutral** and thinks that Player 2 is **equally likely** to play *c* and *d*.

Then

NOTE: when a player is risk neutral we can take as von Neumann-Morgenstern utility function:

because with such a function expected utility = expected value and, by risk neutrality, the player ranks money lotteries according to their expected values.

If both players are risk neutral, the above game becomes:

		Player 2	
		c	*d*
Player 1	*a*		
	b		

Player 2

		c	d
Player 1	a	\$20 \$0	\$60 \$50
	b	\$30 \$80	\$50 \$30

Assume that each player is selfish and greedy (only cares about how much money she herself gets.

CASE 2: Suppose that Player 1 has the following von Neumann-Morgenstern utility function:

$$U(\$x) = \sqrt{x}$$

and thinks that Player 2 is **equally likely** to play *c* and *d*.

Then

If both players have the above von Neumann-Morgenstern utility function then the game becomes

Player 2

		c	d
Player 1	a		
	b		

Player 2

		c	d
Player 1	a		
	b		

Player 2

Player 1

	C	D
A	$\begin{pmatrix} z_1 & z_2 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$	z_3
B	$\begin{pmatrix} z_1 & z_3 \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$	z_4

	z_1	z_2	z_3	z_4
vNM utility function of Player 1:	30	0	36	30
vNM utility function of Player 2:	0	6	0	2

Player 2

Player 1

	C	D
A		
B		

Player 2

		C	D
Player 1	A	**15** **3**	**36** **0**
	B	**34** **0**	**30** **2**

MIXED STRATEGIES

$$\left(\begin{pmatrix} A & B \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix}, \begin{pmatrix} C & D \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}\right)$$

$$\begin{pmatrix} \text{strategy profile} & (A,C) & (A,D) & (B,C) & (B,D) \\ \text{probability} & & & & \end{pmatrix}$$

next page →

30.95 < 31=B

Is $\left(\begin{pmatrix} A & B \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix}, \begin{pmatrix} C & D \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}\right)$ a Nash equilibrium?

Player 2

Player 1		*C*		*D*	
	A	**15**	**3**	**36**	**0**
	B	**34**	**0**	**30**	**2**

$$\left(\begin{pmatrix} A & B \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix}, \begin{pmatrix} C & D \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}\right) \quad \begin{pmatrix} \text{strategy profile} & (A,C) & (A,D) & (B,C) & (B,D) \\ \text{probability} & \frac{1}{20} & \frac{3}{20} & \frac{4}{20} & \frac{12}{20} \end{pmatrix}$$

expected utility (or payoff) of Player 1 is

30.96

Player 2

		C	D
Player 1	A	**15**, **3**	**36**, **0**
	B	**34**, **0**	**30**, **2**

$$\left(\begin{pmatrix} A & B \\ \frac{2}{5} & \frac{3}{5} \end{pmatrix}, \begin{pmatrix} C & D \\ \frac{6}{25} & \frac{19}{25} \end{pmatrix}\right)$$ **is** a Nash equilibrium $\begin{pmatrix} \text{strategy profile} & (A,C) & (A,D) & (B,C) & (B,D) \\ \text{probability} & & & & \end{pmatrix}$

expected utility (or payoff) of Player 1 is

Computing the mixed-strategy Nash equilibria

Theorem. At a Nash equilibrium in mixed strategies, a player must be indifferent between any two PURE strategies that she plays with positive probability.

Player 2

Player 1		*C*		*D*	
	A	**15**	**3**	**36**	**0**
	B	**34**	**0**	**30**	**2**