UNCERTAINTY in GAMES

A. Subjective uncertainty.

		Player 2	
		C	*D*
Player 1	*A*	z_1	z_2
	B	z_3	z_4

Suppose that **Player 1's ranking** of the outcomes is:

$$z_1 \succ z_4 \succ z_3 \succ z_2$$

If Player 1 believes that Player 2 will play C

If Player 1 believes that Player 2 will play D

If Player 1 believes that Player 2 is equally likely to play C or D

B. Objective uncertainty.

Player 1 has to choose between *A* and *B* knowing that she is playing a simultaneous game against another player, but not knowing whether it is Player 2 or Player 3. She is in front of a computer terminal. The computer will “toss a coin” and choose the opponent (and inform the opponent, but not Player 1).

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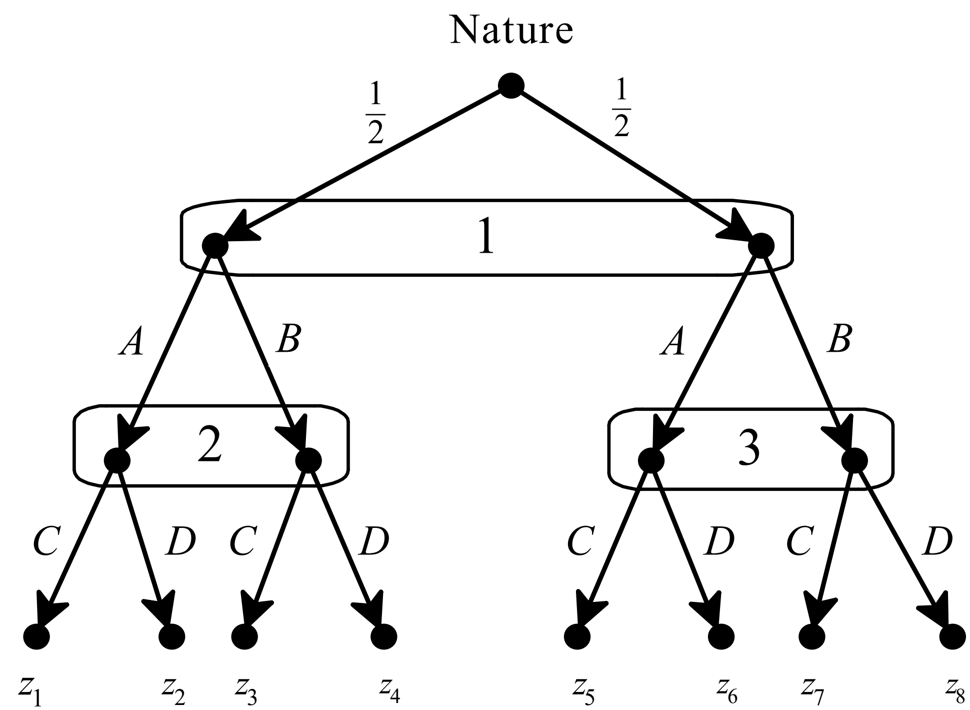

Suppose that Player 1 knows that

- For Player 2: $z_1 \succ z_2$ and $z_3 \succ z_4$
- For Player 3: $z_6 \succ z_5$ and $z_8 \succ z_7$

Then $A \rightarrow \begin{pmatrix} & \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ and $B \rightarrow \begin{pmatrix} & \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

Theorem 1 Let $Z = \{z_1, z_2, ..., z_m\}$ be a set of basic outcomes and $\mathcal{L}$ the set of lotteries over *Z*. If $\succsim$ is a von Neumann-Morgenstern ranking of the elements of $\mathcal{L}$ then there exists a function U, called a *von Neumann-Morgenstern utility function*, that assigns a number (called utility) to every basic outcome and is such that, for any two lotteries $L = \begin{pmatrix} z_1 & z_2 & ... & z_m \\ p_1 & p_2 & ... & p_m \end{pmatrix}$ and $M = \begin{pmatrix} z_1 & z_2 & ... & z_m \\ q_1 & q_2 & ... & q_m \end{pmatrix}$,

$$L \succ M \quad \text{if and only if} \quad \underbrace{p_1 U(z_1) + p_2 U(z_2) + ... + p_m U(z_m)}_{\text{expected utility of lottery } L} > \underbrace{q_1 U(z_1) + q_2 U(z_2) + ... + q_m U(z_m)}_{\text{expected utility of lottery } M}$$

and

$$L \sim M \quad \text{if and only if} \quad \underbrace{p_1 U(z_1) + p_2 U(z_2) + ... + p_m U(z_m)}_{\text{expected utility of lottery } L} = \underbrace{q_1 U(z_1) + q_2 U(z_2) + ... + q_m U(z_m)}_{\text{expected utility of lottery } M}$$

2.25 3.33

EXAMPLE 1. $Z = \{z_1, z_2, z_3, z_4\}$ $\quad L = \begin{pmatrix} z_1 & z_2 & z_3 & z_4 \\ \frac{1}{8} & \frac{5}{8} & 0 & \frac{2}{8} \end{pmatrix}$ $\quad M = \begin{pmatrix} z_1 & z_2 & z_3 & z_4 \\ \frac{1}{6} & \frac{2}{6} & \frac{1}{6} & \frac{2}{6} \end{pmatrix}$

Suppose we know that $U = \begin{cases} z_1 & z_2 & z_3 & z_4 \\ 6 & 2 & 8 & 1 \end{cases}$

EXAMPLE 2.

$$A = \begin{pmatrix} \text{paid 3-week vacation} & \text{no vacation} \\ 50\% & 50\% \end{pmatrix} \qquad B = \begin{pmatrix} \text{paid 1-week vacation} \\ 100\% \end{pmatrix}$$

Suppose Ann says $\boxed{B \succ A}$ How would she rank

$$C = \begin{pmatrix} \text{paid 3-week vacation} & \text{no vacation} \\ 5\% & 95\% \end{pmatrix} \text{ and } D = \begin{pmatrix} \text{paid 1-week vacation} & \text{no vacation} \\ 10\% & 90\% \end{pmatrix} ?$$

Theorem 2. Let $\succsim$ be a von Neumann-Morgenstern ranking of the set of basic lotteries $\mathcal{L}$. Then the following are true.

(A) If $U : Z \to \mathbb{R}$ is a von Neumann-Morgenstern utility function that represents $\succsim$, then, for any two real numbers a and b with $a > 0$, the function $V : Z \to \mathbb{R}$ defined by $V(z_i) = aU(z_i) + b$ ($i = 1,2,\ldots,m$) is also a von Neumann-Morgenstern utility function that represents $\succsim$.

(B) If $U : Z \to \mathbb{R}$ and $V : Z \to \mathbb{R}$ are two von Neumann-Morgenstern utility functions that represent $\succsim$, then there exist two real numbers a and b with $a > 0$ such that $V(z_i) = aU(z_i) + b$ ($i = 1,2,\ldots,m$).

$$U = \begin{cases} z_1 & z_2 & z_3 & z_4 & z_5 & z_6 \\ 10 & 6 & 16 & 8 & 6 & 14 \end{cases}$$

(10,6,2,0)->A,
(10,8,4,0)->B

Player 2

Player 1		*C*	*D*
	A	z_1	z_2
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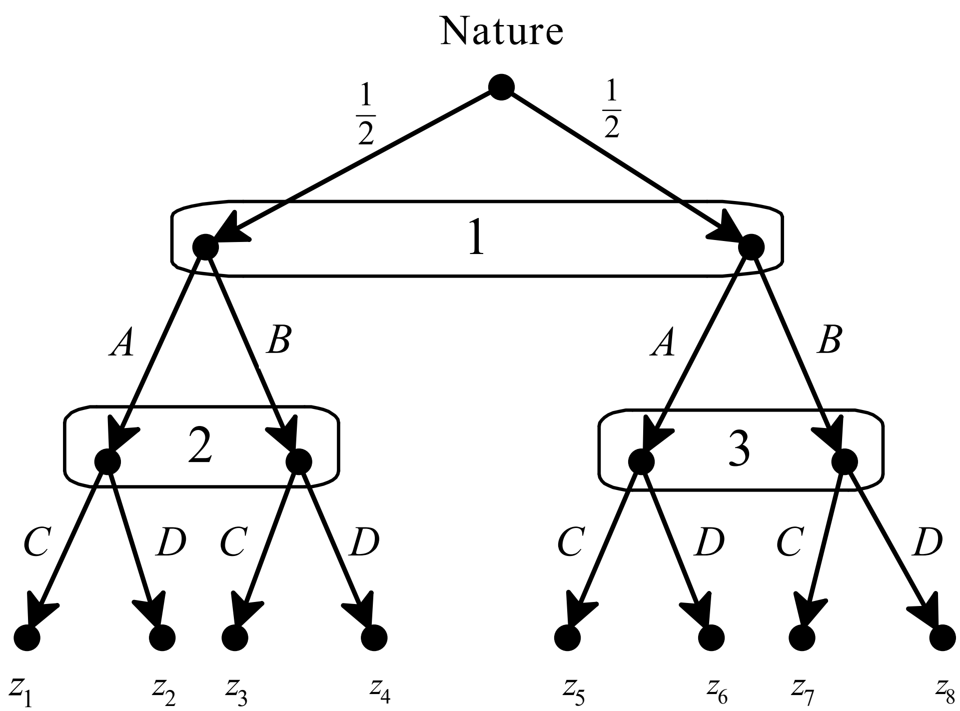

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