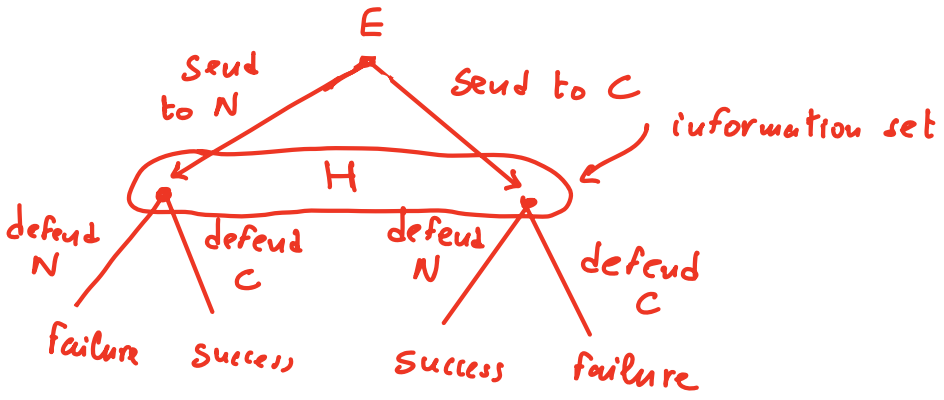
June 1944. Eisenhower has to decide whether to land his troops in Calais or in Normandy. **Hitler** has to decide whether to amass his troops near Calais or Normandy, not knowing Eisenhower's decision:



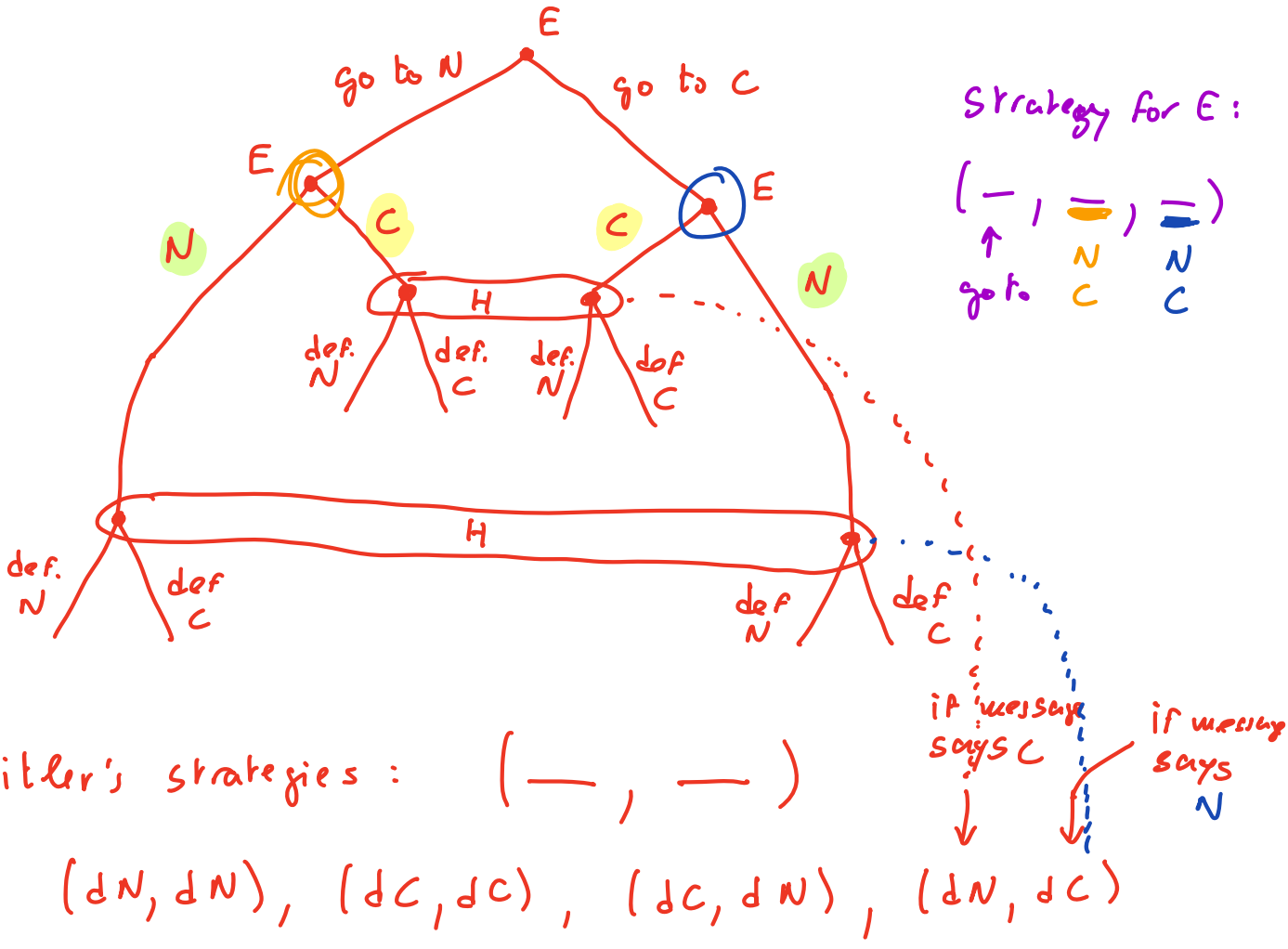
Strategy: a strategy for a player is a list of choices, one for every ~~node~~ information set of that player

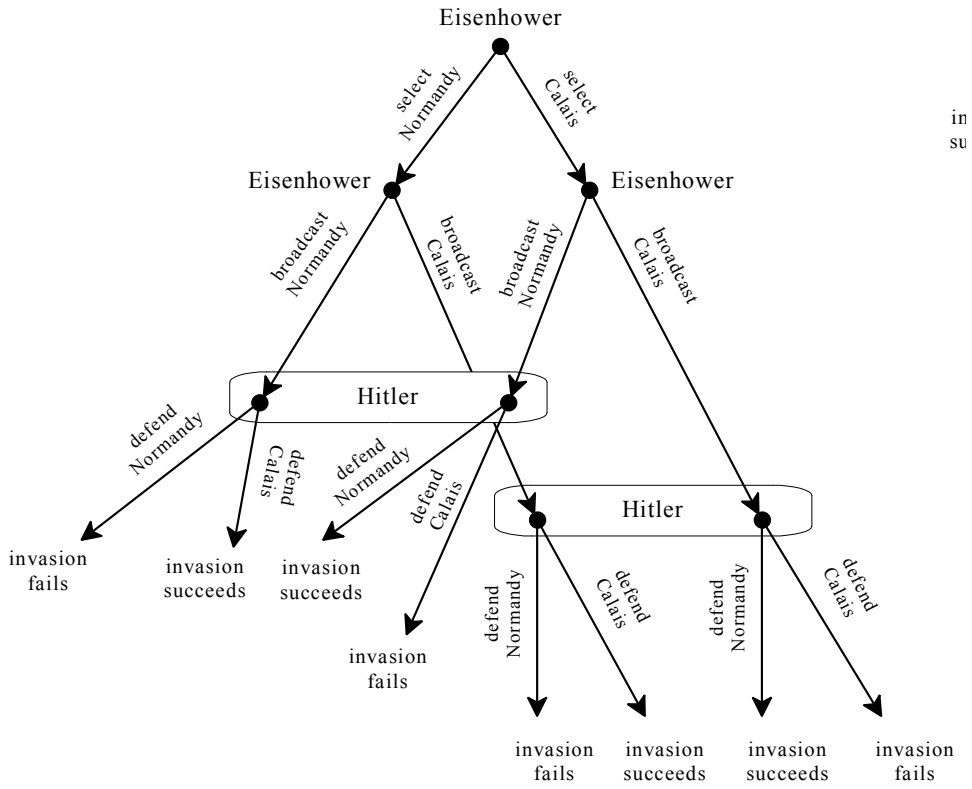
H's possible strategies: defend N
defend C

A game has perfect information if every information set consists of a single node.

GARBO THE SPY

Eisenhower sends a communication to general Patton, stating the chosen location for the landing (knowing that it is likely to be intercepted by the Germans and thus he might tell the truth or lie), the Germans intercept the communication and then Hitler has to decide whether to amass his troops near Calais or Normandy.





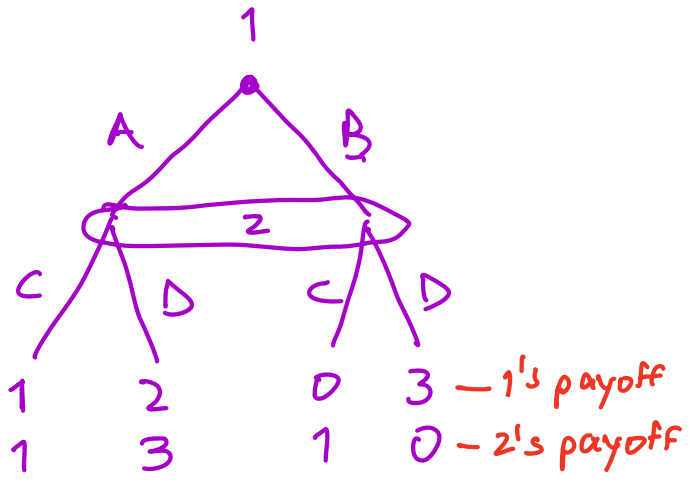
Hitler

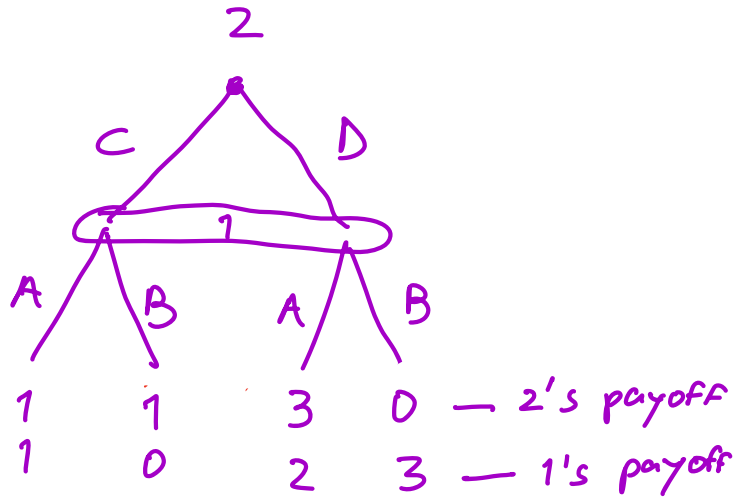
Eisenhower

Note: the same normal form may be associated with different extensive forms.

Example:

1 \ 2	C	D
A	1, 1	2, 3
B	0, 1	3, 0





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2 red cards, 1 black

top	R	R	B
middle	R	B	R
bottom	B	R	R
	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

$$\text{Prob}\{\text{top card is R}\} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

A game with chance moves.

A coin is tossed twice. If the outcome is HH then Player 1 is informed that it was HH and if the outcome is any other outcome then Player 1 is only told that it was **not** HH. Then Player 1 chooses between *A* and *B*. Player 2 is not told what the outcome was, nor is she told what Player 1 chose and she has to choose between *C* and *D*. The outcomes are sums of money: the first is what Player 1 gets and the second what Player 2 gets:

If the outcome is HH:

	C		*D*	
A	**$4**	**$0**	**$8**	**$4**
B	**$0**	**$8**	**$12**	**$4**

If the outcome is HT or TH or TT:

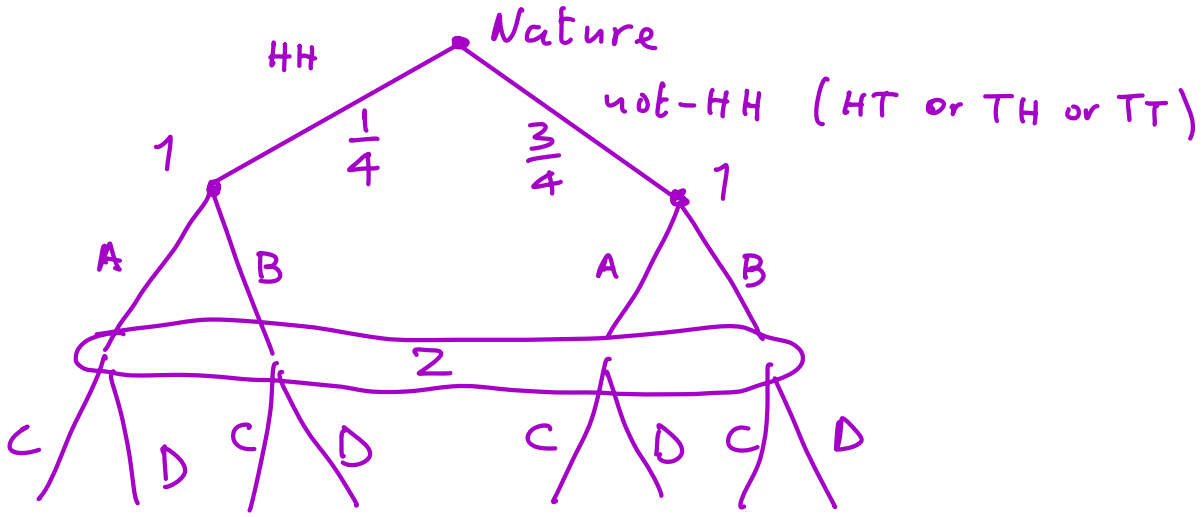
	C		*D*	
A	**$0**	**$8**	**$8**	**$0**
B	**$12**	**$0**	**$0**	**$8**

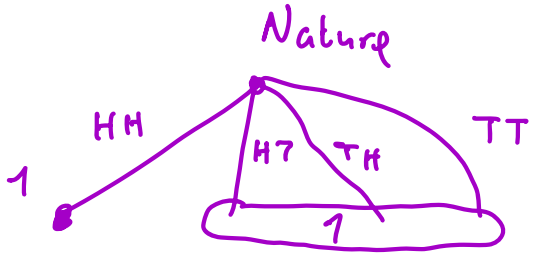
If the outcome is HH:

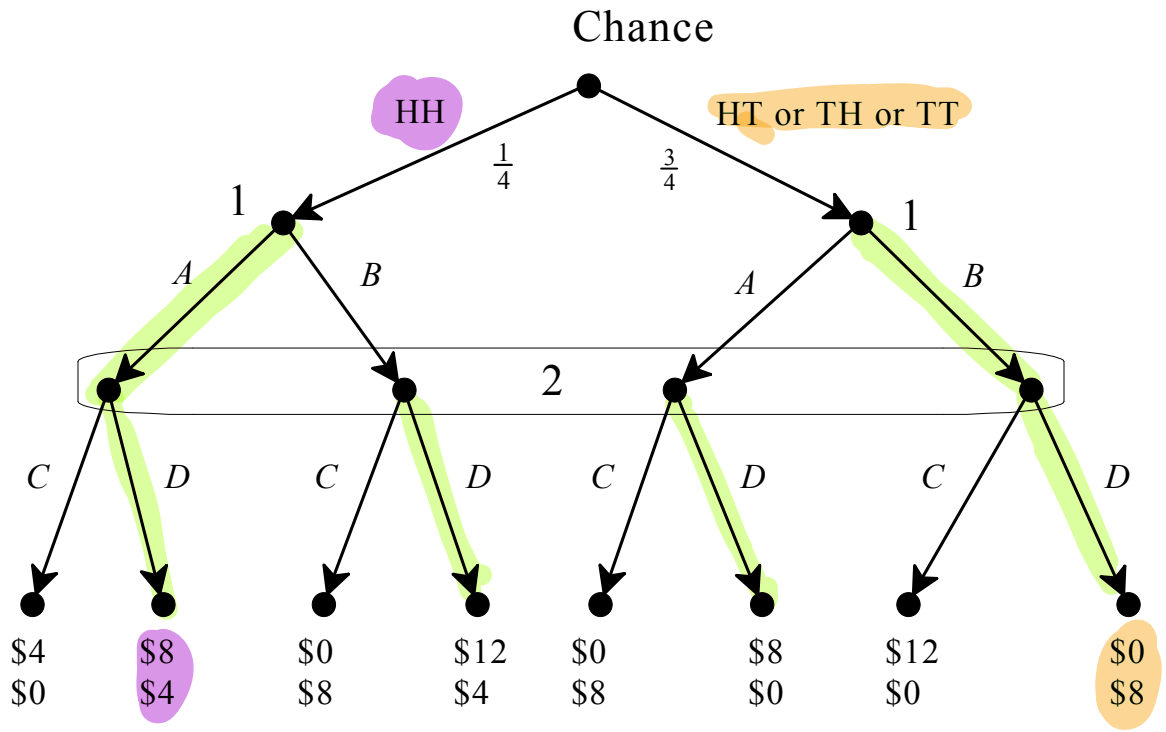
	C		D	
A	$4	$0	$8	$4
B	$0	$8	$12	$4

If the outcome is HT or TH or TT:

	C		D	
A	$0	$8	$8	$0
B	$12	$0	$0	$8







If each player is selfish and greedy then the associated strategic form is as follows:

1 \ 2	*C*	*D*
AA		
AB		($8, $4, 1/4) ($0, $8, 3/4)
BA		
BB		

A player is risk neutral if he considers a money lottery to be equivalent to its expected value

e.g.

$\begin{pmatrix} \$12 & \$8 \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$

expected value

$12\frac{1}{4} + 8\frac{3}{4} =$

$3 + 6 = 9$

1

	C	D
AA	$\begin{pmatrix} \$4 & \$0 \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$	$\begin{pmatrix} \$0 & \$8 \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$	\$8	$\begin{pmatrix} \$4 & \$0 \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$
AB	$\begin{pmatrix} \$4 & \$12 \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$	\$0	$\begin{pmatrix} \$8 & \$0 \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$	$\begin{pmatrix} \$4 & \$8 \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$
BA	\$0	\$8	$\begin{pmatrix} \$12 & \$8 \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$	$\begin{pmatrix} \$4 & \$0 \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$
BB	$\begin{pmatrix} \$0 & \$12 \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$	$\begin{pmatrix} \$8 & \$0 \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$	$\begin{pmatrix} \$12 & \$0 \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$	$\begin{pmatrix} \$4 & \$8 \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$

2

1

7

10

7

9

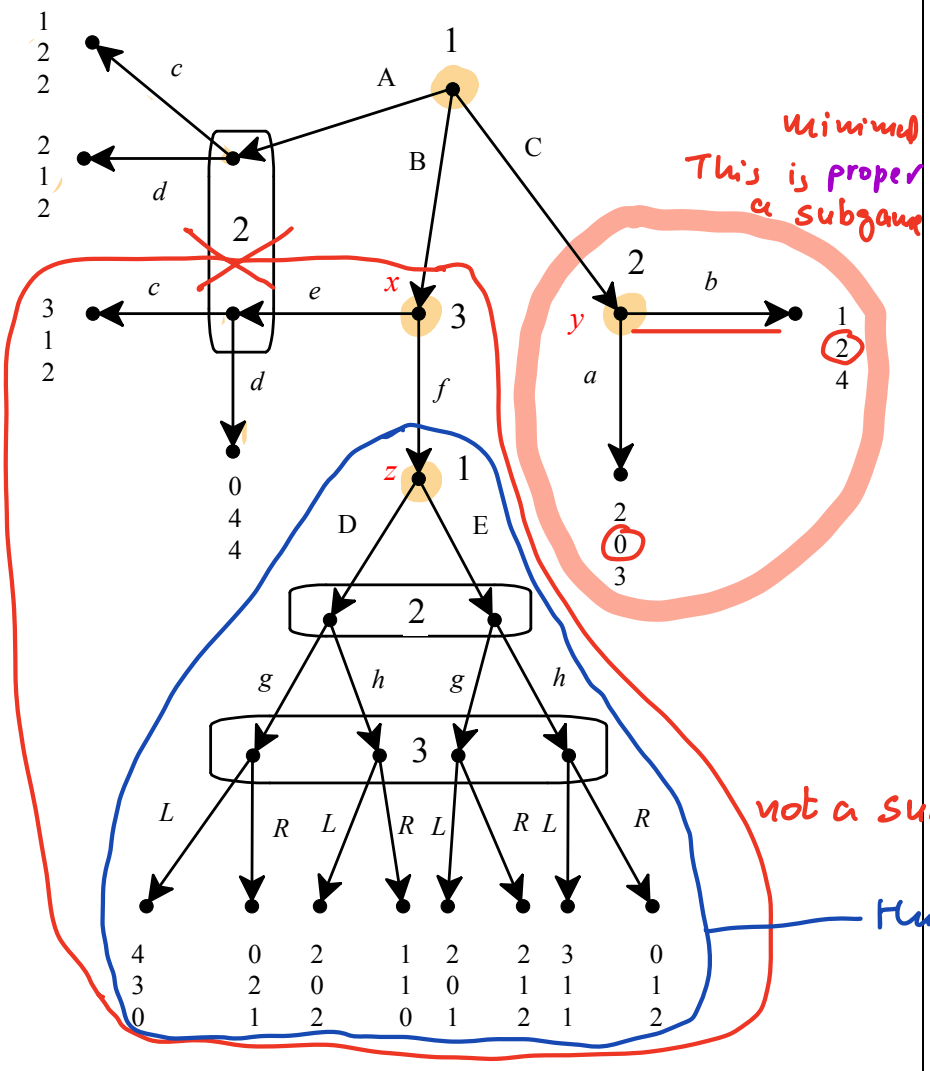
9

Now if we add the assumption that the players are risk neutral then the above strategic form can be simplified to the following:

		Player 2 C	Player 2 D
Player 1	AA	7, 6	8, 1
	AB	10, 0	2, 7
	BA	0, 8	9, 1
	BB	9, 2	3, 7

$\begin{pmatrix} \$50,000 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} \$0 & \$100,000 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

expected value is \$50,000

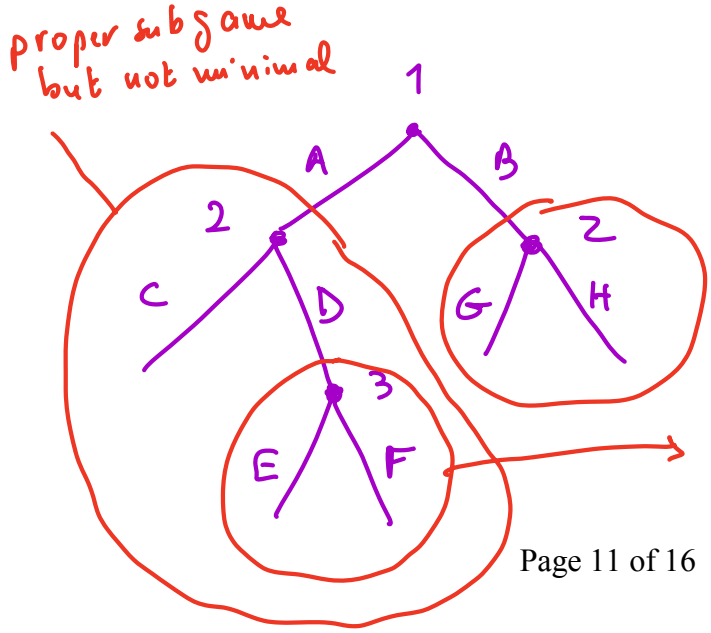


Definition of **SUBGAME**.

- Start from a singleton information set
- Take the entire sub-tree from there
- If you don't cut any information sets then you have a subgame otherwise you don't

minimal
This is proper a subgame
not a subgame
proper
This is a subgame
minimal

SUBGAME-PERFECT EQUILIBRIUM



MINIMAL PROPER SUBGAME