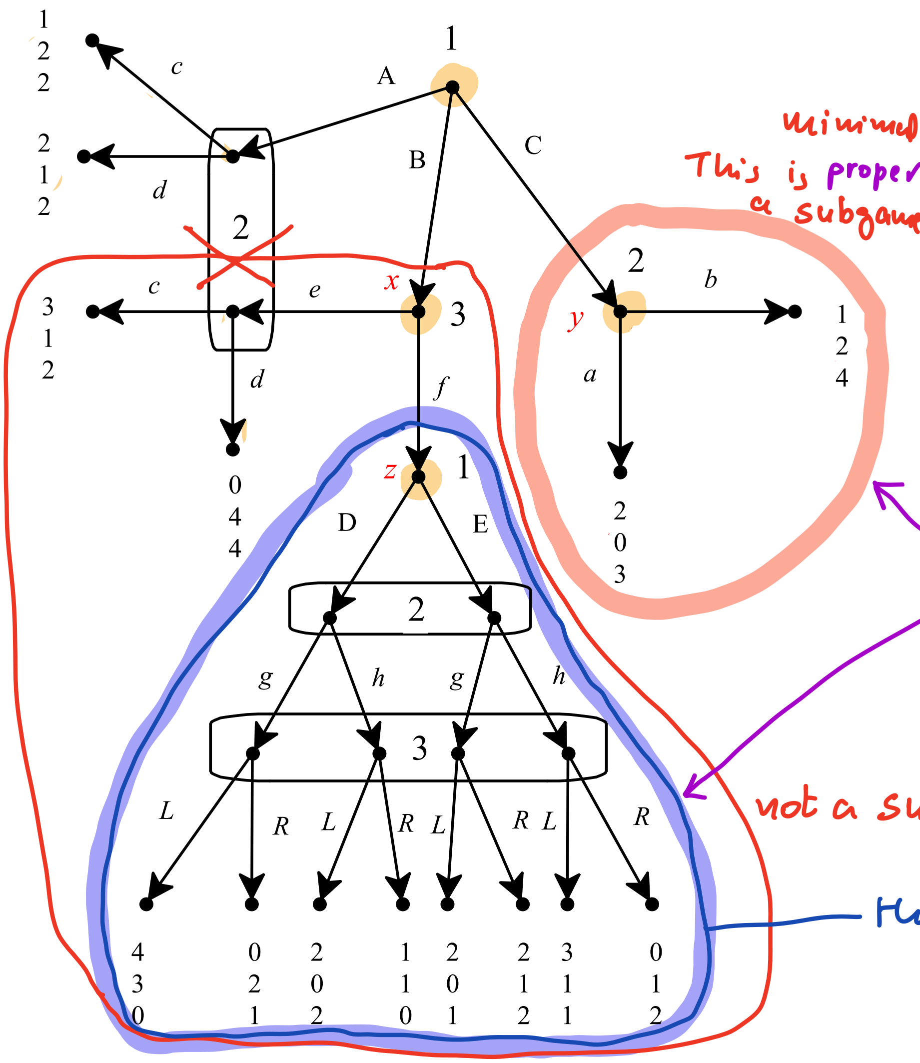


Minimal
This is a proper subgame

Two proper Subgames both minimal

not a subgame

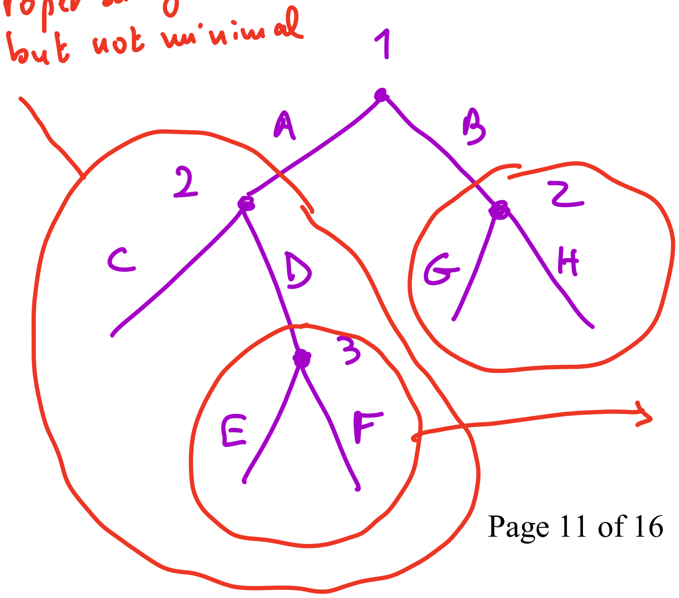
This is a proper subgame minimal

Definition of **SUBGAME**.

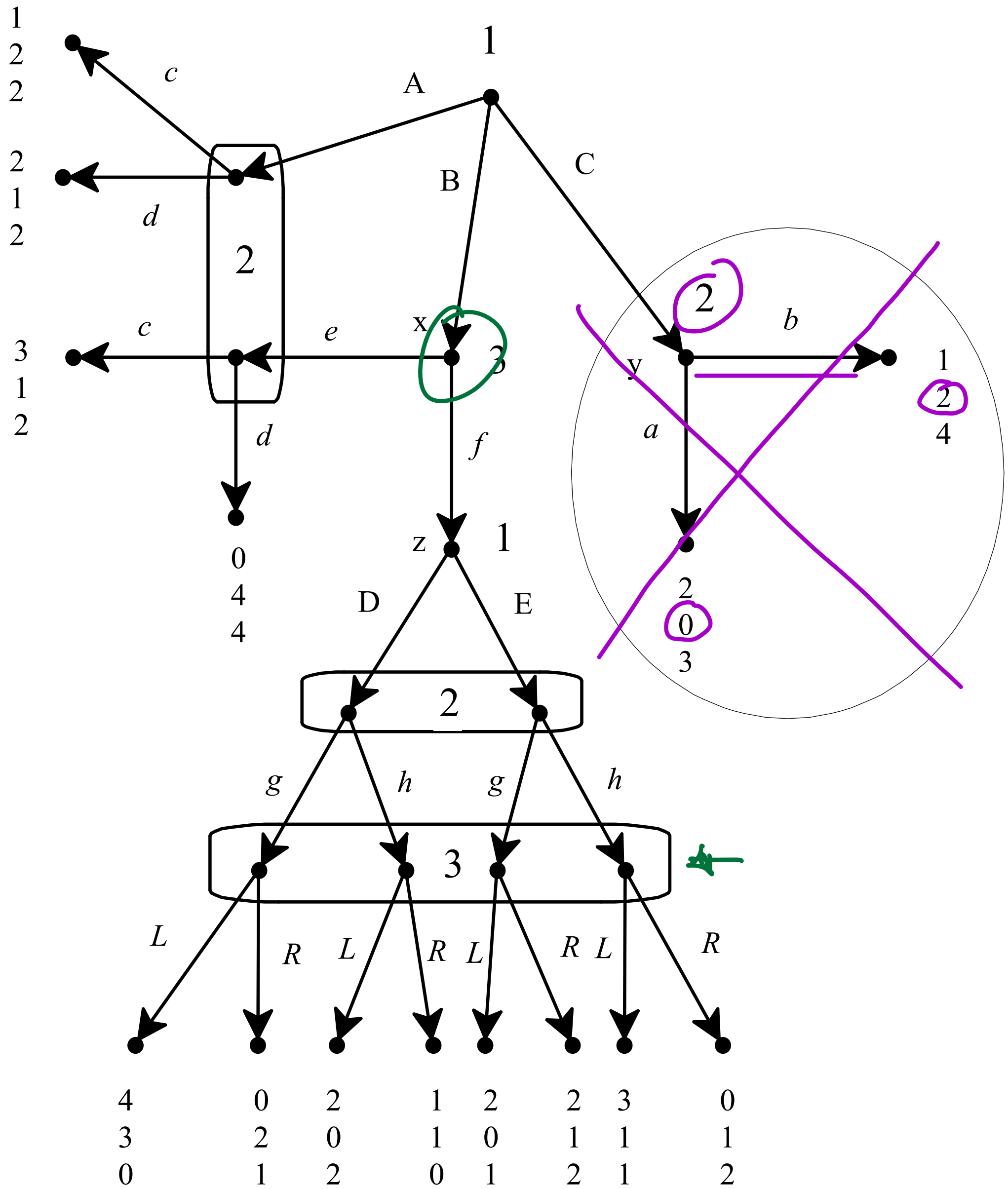
- Start from a singleton information set
- Take the entire sub-tree from there
- If you don't cut any information sets then you have a subgame otherwise you don't

SUBGAME-PERFECT EQUILIBRIUM

proper subgame but not minimal

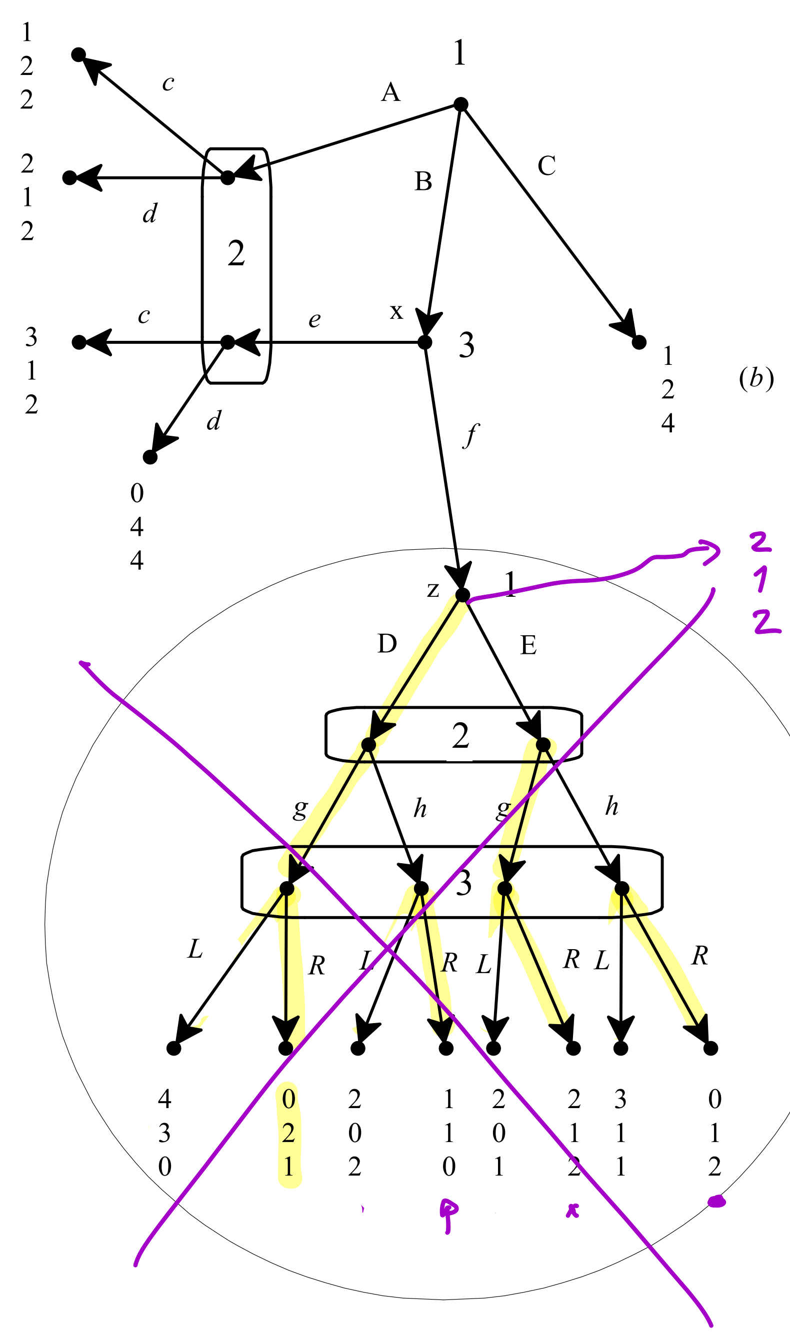


MINIMAL PROPER SUBGAME


1
A
B
C
2
c
d
c
d
e
x
3
f
y
a
b
z
1
D
E
g
h
g
h
L
R
L
R
L
R
L
R
1 2 2
2 1 2
3 1 2
0 4 4
1 2 4
2 0 3
4 3 0
0 2 1
2 0 2
1 1 0
2 0 1
2 1 2
3 1 1
0 1 2

1
2
4

(b)



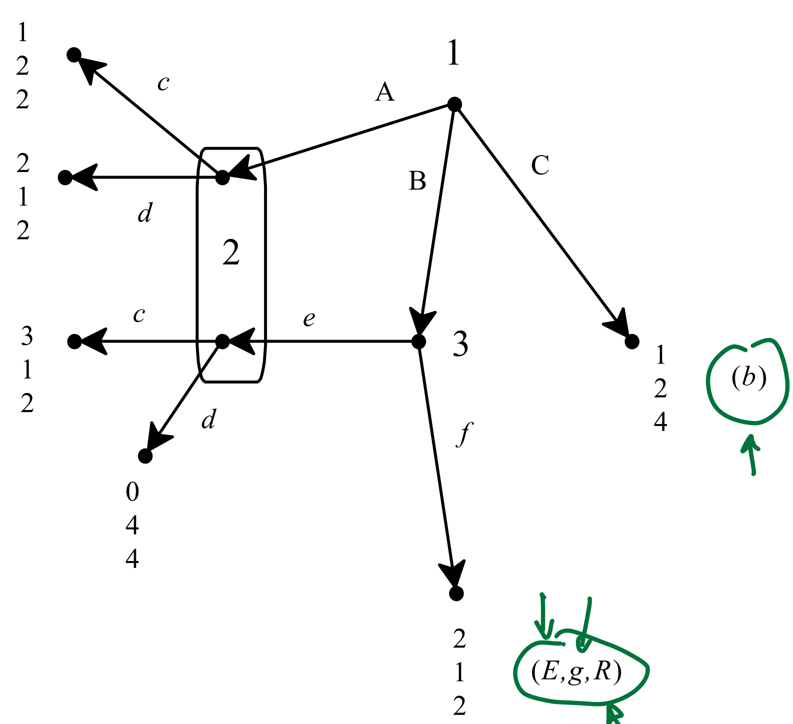
		2					
		g			*h*		
1	***D***	4	3	0	2	0	2
	E	2	0	1	3	1	1

Player 3: L

		2					
		g			*h*		
1	***D***	0	2	1	1	1	0
	E	2	1	2	0	1	2

Player 3: R

Nash equilibrium: (E, g, R)



No more subgames

		2: c			2: d		
1	A	1	2	2	2	1	2
	B	3	1	2	0	4	4
	C	1	2	4	1	2	4

Player 3: e

		2: c			2: d		
1	A	1	2	2	2	1	2
	B	2	1	2	2	1	2
	C	1	2	4	1	2	4

Player 3: f

Nash equilibrium: (B, c, f)

Subgame-Perfect Equilibrium of the original game:

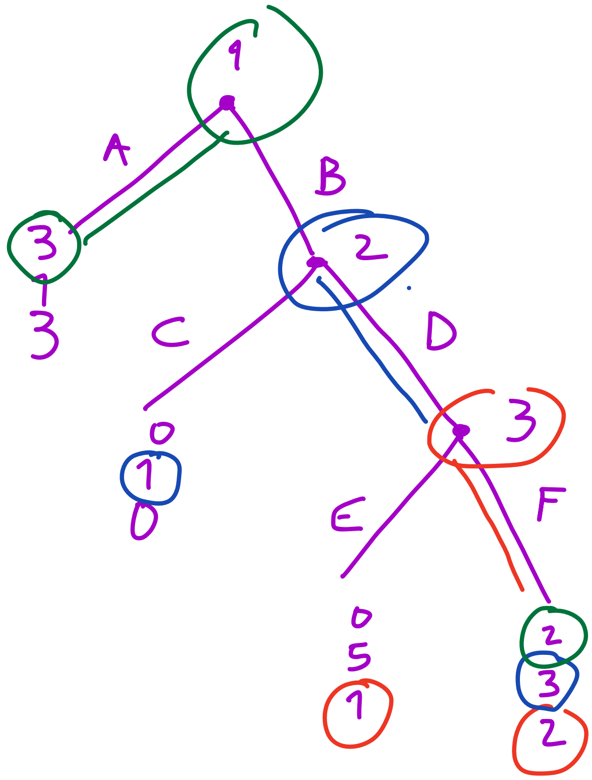
In the original game:

Strategy of Player 1 : BE

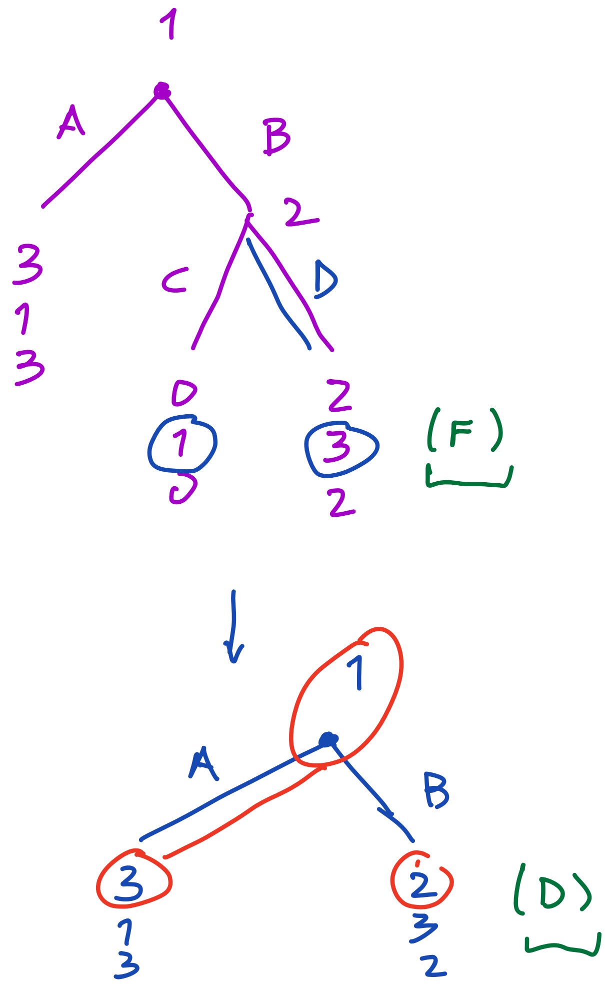
" Player 2 : bcg

" Player 3 : fR

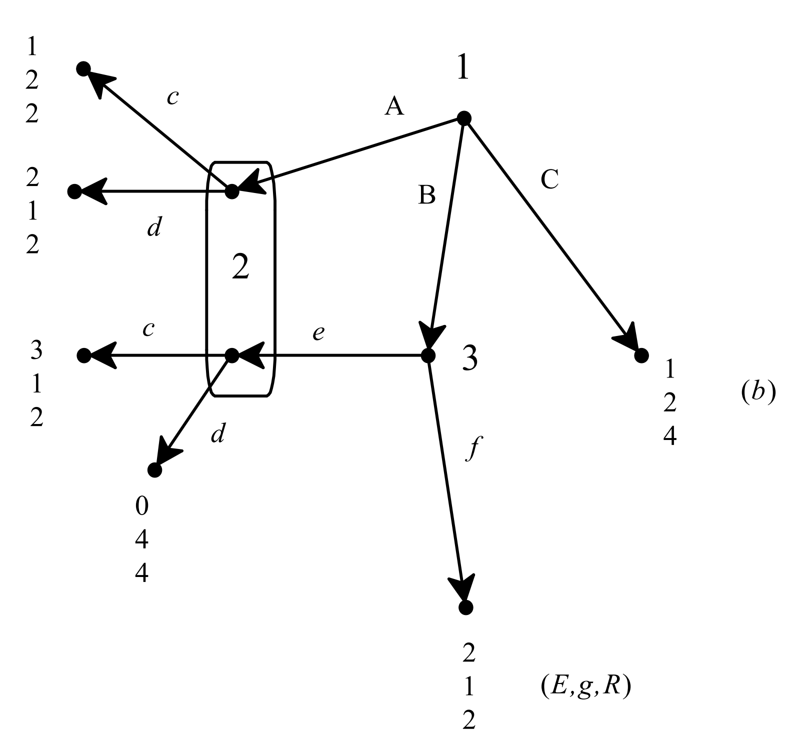
Subgame-perfect equilibrium $\underbrace{(BE,\ bcg,\ fR)}_{\text{strategy profile for original game}}$



BI: (A, D, F)



(A, D, F)



No more subgames

2

		c			d		
	A						
1	B						
	C						

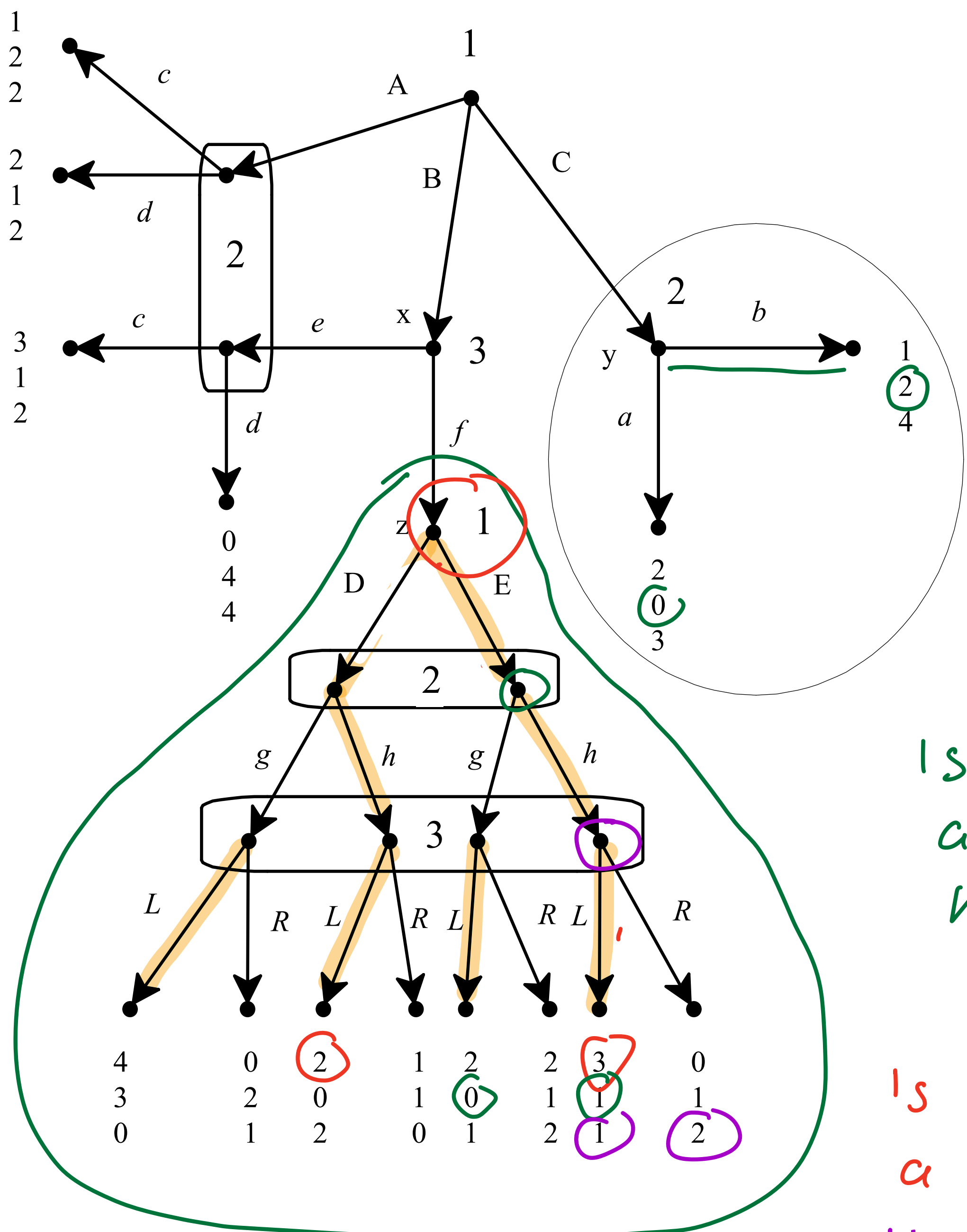
Player 3: e

2

		c			d		
	A						
1	B						
	C						

Player 3: f

Subgame-Perfect Equilibrium of the original game:



Is (D, h, L) a N.E. of this subgame?

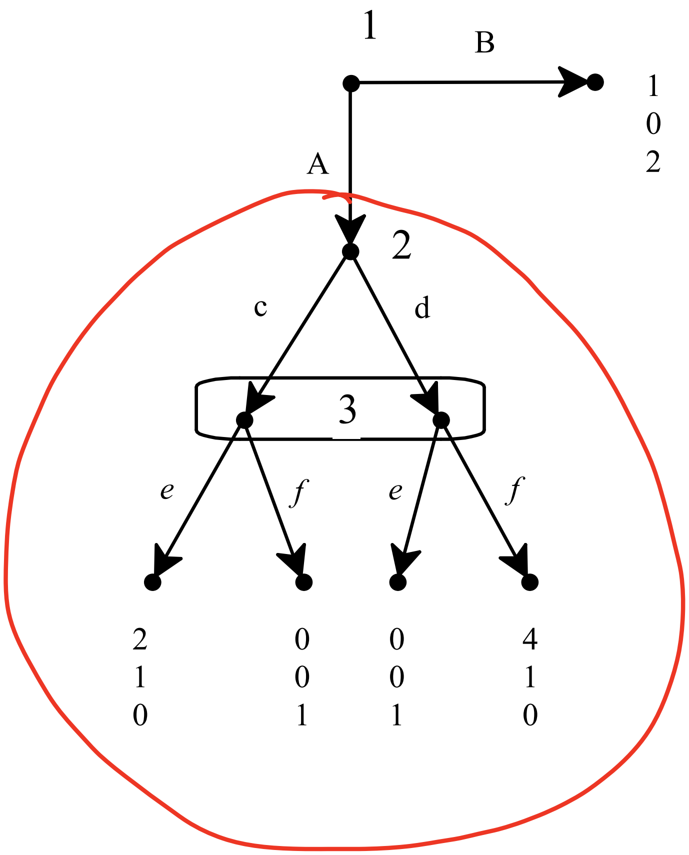
No

Is (E, h, L) a N.E?

No:

1 and 2 happy
3 unhappy

There may be no subgame-perfect equilibria



3

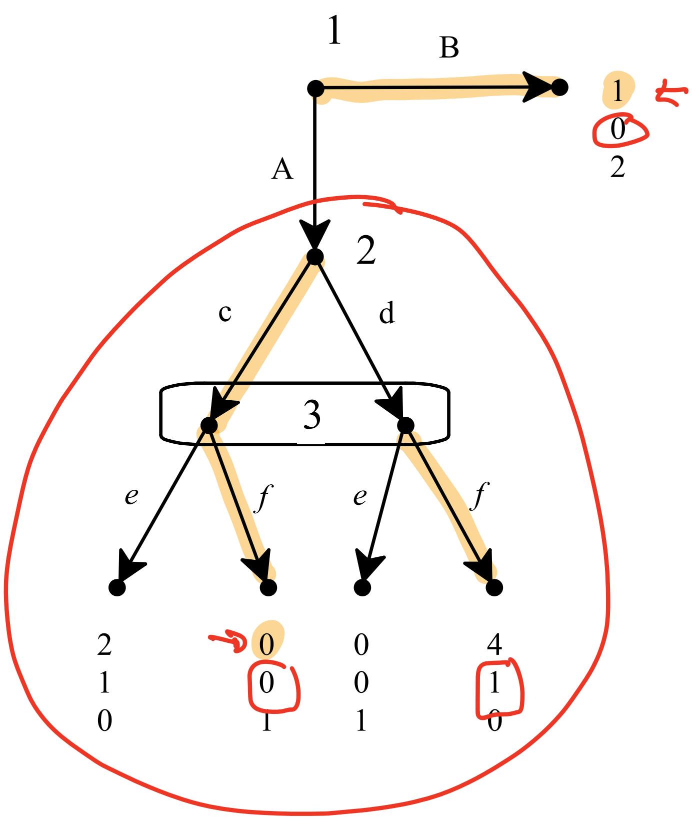
2 \ 3	e	f
c	1 0	0 1
d	0 1	1 0

No N. E.

Hence the entire game does not have any subgame-perfect equilibria

SPE: a N.E. of the entire game that remains a N.E. in every subgame

There may be no subgame-perfect equilibria



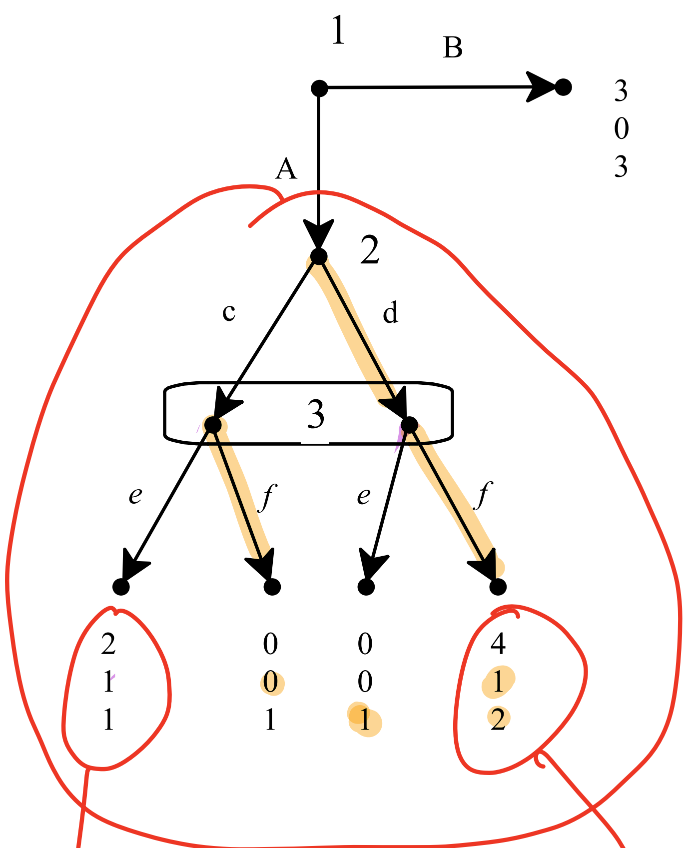
(B, c, f) is a N.E.

not subgame perfect because (c, f) is not a N.E. of the subgame because c is not a best reply to f for Player 2

Another N.E. is (B, d, e) also not SPE because (d, e) not NE of subgame

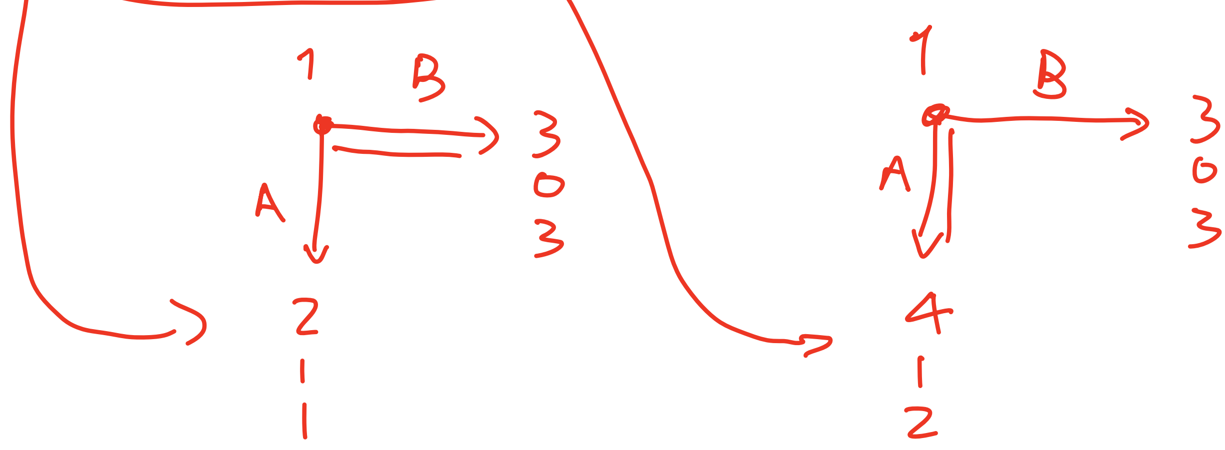
SPE: a N.E. of the entire game that remains a N.E. in every subgame

There may be several subgame-perfect equilibria



(c, e) a N.E. of subgame

(d, f) is also a NE of subgame



SPE: (B, c, e) (A, d, F)

Set of NE = set of SPE in the class of games that ...

have no proper subgames