## ULTIMATUM GAME



Player 1 is given some money.
He makes an offer to Player 2.
If Player 2 says Yes, then the offer is implemented.

If Player 2 says No, then both players end up with nothing.

How much should Player 1 offer to Player 2?

Player 1 is given $\$ 40$.
He makes an offer to Player 2 ( $\$ 10$ or $\$ 20$ or $\$ 30$ ). If Player 2 says Yes, then the offer is implemented. If Player 2 says No, then both players end up with nothing.



Met hod of
backward induction
Suppose that Player 2 is fairness-minded and averse to greed:

Utility

$$
\begin{aligned}
\text { best }(\$ 20, \$ 20) & 4 \\
(\$ 10, \$ 30) & 3 \\
(\$ 0, \$ 0) & 2 \\
\text { worst }(\$ 30, \$ 10) & 1
\end{aligned}
$$

Centipede Game

- A referee past $\$ 20$ on the table.
- Player 1 can take it and end the game or Pass.
- If Player 1 passes, the referee adds $\$ 20$ to the pot and Player 2 can take it and end the game or Pass.
- If Player 2 passes then the referee adds another $\$ 20$ to the pot and Player 1 can take it and end the game or Pass. ... and so on.
- At the last move the active player can take the pot for herself or can Pass, in which case the pot is divided equally between the two players.


Suppose:
Suppose:
1 is selfish and greedy
2 is also selfish and greasy

$$
\begin{aligned}
\text { best } & (\$ 60, \$ 0) \\
& (\$ 30, \$ 30) \\
& (\$ 20, \$ 0) *
\end{aligned}
$$

$$
\text { best } \begin{aligned}
& (\$ 0, \$ 40) * \\
& (\$ 30, \$ 30)
\end{aligned}
$$

worst $\quad(\$ 0, \$ 40) *$

$$
\text { wort }(\$ 20, \$ 0),(\$ 60, \$ 0)^{*}
$$



Suppose that Player 1 is fairness-minded:

Utility
best $\left.\begin{array}{rl} & (\$ 30, \$ 30) \\ (\$ 60, \$ 0)\end{array}\right] *$
$(\$ 20, \$ 0) *$
worst $(\$ 0, \$ 40)$
Suppose that
$(\$ 30, \$ 30) \underset{2}{>}(\$ 0, \$ 40)$
Suppose that Player 2 is also fairness-minded:

Utility
best
worst

## A divorce

Mrs. Jones is seeking a divorce from Mr. Jones. Under the terms of her prenuptial agreement, her settlement will be $\mathbf{\$ 9 0 , 0 0 0}$ if she can prove that Mr. Jones had an affair, but $\mathbf{\$ 4 5 , 0 0 0}$ otherwise. Her lawyer, acting as her agent, can indeed prove the affair but only if he hires a private detective for $\mathbf{\$ 1 0 , 0 0 0}$, which he will have to pay out of his own pocket. The lawyer has offered Mrs. Jones a choice of two contracts. One contract involves a flat payment to the lawyer of $\mathbf{\$ 1 8 , 0 0 0}$, regardless of the outcome. The other contract involves a fee equal to one third of the settlement. What contract should Mrs. Jones choose?


Backward-induction solution

The race to 38. Players 1 and 2 take turns choosing a number from the set \{1,2,3,4,5\}. The first player to bring the total sum of the chosen numbers to 38 wins.


- Hen at each later stage if player 2 picks $n$ you pick $(6-n)$

$$
\begin{array}{llllllllll}
\text { Player 1: } & 2 & 2 & 5 & 3 & 4 & 1 & 5 & 38 & 1 \text { wins } \\
\text { Player 2: } & 4 & 1 & 3 & 2 & 5 & 1 &
\end{array}
$$

Player 1: 2 (1) mistake
Player 2: 41 from here on 2 can win by piecing ( $6-4$ ) where $n$ is No verwiter chosen by Player 1

A monopolist and a potential entrant

|  |  | Monopolist |  |
| :---: | :---: | :---: | :---: |
|  |  | fight | accommodate |
|  | Potential |  |  |
| entrant |  |  |  | In | 0,0 | 2,2 |
| :---: | :---: |
|  | Out |
|  | 1,5 |
|  |  |

Monopoly profit is 5
Alternative investment' yield 1 to $P E$


## A chain-store (monopolist) and many potential entrants

The chain store is a monopoly in $n$ towns. There are $n$ potential entrants, one in each town. They make decisions sequentially with perfect knowledge of what happened in the past.


