**IDSDS.** *The Iterated Deletion of Strictly Dominated Strategies*

A is not a dominant strategy for Player 1

B 〃 〃 〃

C 〃 〃 〃

| | | Player 2 | | |
|---|---|---|---|---|
| | | *D* | *E* | *F* |
| Player 1 | *A* | 8, 6 | 0, 9 | 3, 8 |
| | *B* | 3, 2 | 2, 1 | 4, 3 |
| | *C* | 2, 8 | 1, 5 | 3, 1 |

For Player 1 C is strictly dominated by B

After deleting C, now D becomes strictly dominated by F

After deleting D, now A 〃 〃 〃 〃 B

$R_1$ = player 1 is rational $\qquad$ $B_1 x$ = player 1 believes $x$

$R_2$ = " 2 " $\qquad$ $B_2 x$ = " 2 "

**IDSDS.** *The Iterated Deletion of Strictly Dominated Strategies*

Player 2

| Player 1 | ~~D~~ | ~~E~~ | F |
|---|---|---|---|
| ~~A~~ | ~~8, 6~~ | ~~0, 9~~ | ~~3, 8~~ |
| B | ~~3, 2~~ | ~~2, 1~~ | $\boxed{4, 3}$ |
| ~~C~~ | ~~2, 8~~ | ~~1, 5~~ | ~~3, 1~~ |

(3) $B_1 B_2 R_1$, $B_1 R_2$ (row A)

(1) $R_1$ (row C)

(2) $B_2 R_1$, $R_2$ (column D)

(4) $B_2 B_1 B_2 R_1$, $B_2 B_1 R_2$ (column E)

$R_1$

$R_2$, $B_2 R_1$

$B_1 B_2 R_1$

$B_1 R_2$

$B_2 B_1 B_2 R_1$

$B_2 B_1 R_2$

At any stage delete strategies that are either strictly or weakly dominated

**IDWDS.** *The Iterated Deletion of Weakly Dominated Strategies*

Player 2

Player 1

| | L | | ~~R~~ | |
|---|---|---|---|---|
| A | 4 | 0 | ~~0~~ | ~~0~~ |
| ~~T~~ | ~~3~~ | ~~2~~ | ~~2~~ | ~~2~~ |
| ~~M~~ | ~~1~~ | ~~1~~ | ~~0~~ | ~~0~~ |
| ~~B~~ | ~~0~~ | ~~0~~ | ~~1~~ | ~~1~~ |

3 bec. T and M str. dom. by A

1 because str. dom. by T

2 because R weakly dom. by L

**IDWDS.** *The Iterated Deletion of Weakly Dominated Strategies*

| | | Player 2 | |
|---|---|---|---|
| | | L | R |
| Player 1 | A | 4 , 0 | 0 , 0 |
| | T | 3 , 2 | 2 , 2 |
| | M | 1 , 1 | 0 , 0 |
| | B | 0 , 0 | 1 , 1 |

3

1 bec. str. dom. by T

2 bec. L weakly dom. by R

For every player

Definition: at every stage identify all the strategies that are weakly or strictly dominated. Then delete all of them at the same time.

Repeat

**IDWDS.** *The Iterated Deletion of Weakly Dominated Strategies*

| | | Player 2 | |
|---|---|---|---|
| | | L | R |
| Player 1 | A | 4 0 | 0 0 |
| | T | 3 2 | 2 2 |
| | * ~~M~~ | ~~1 1~~ | ~~0 0~~ |
| | * ~~B~~ | ~~0 0~~ | ~~1 1~~ |

1

| | | 2 | |
|---|---|---|---|
| | | L | R |
| 1 | T | 1, 1 | 0, 2 |
| | B | 0, – | 0, 0 |

For 1 B is weakly dom. by T

IDSDS leaves the game unchanged

# Nash equilibrium

$x \in S_1$ (= set of strategies of Player 1)

$y \in S_2$ ( " " 2)

| | | Player 2 | | |
|---|---|---|---|---|
| | | *D* | *E* | *F* |
| Player 1 | *A* | 1 0 | 2 3 | 3 1 |
| | *B* | 3 3 | 1 5 | 4 4 |
| | *C* | 3 2 | 0 1 | 3 1 |

Two Nash equilibria:

(A, E)

(C, D)

(x, y) is a Nash equilibrium if

$$\pi_1(x, y) \geq \pi_1(z, y) \quad \text{for every } z \in S_1$$

$$\pi_2(x, y) \geq \pi_2(x, w) \quad \text{for every } w \in S_2$$

| | | **Player 2** | |
|---|---|---|---|
| | | C | D |
| **Player 1** | A | 2 2 2 | 4 3 6 |
| | B | 5 3 2 | 3 4 2 |

**Player 3** chooses F

| | | **Player 2** | |
|---|---|---|---|
| | | C | D |
| **Player 1** | A | 0 0 0 | 1 2 5 |
| | B | 3 6 1 | 0 0 1 |

**Player 3** chooses G

(A, D, F) is a Nash equilibrium

# Nash equilibrium

Player 2

| Player 1 | | D | E | F |
|---|---|---|---|---|
| | A | 1 0 | 2 3 | 3 1 |
| | B | 3 3 | 1 5 | 4 4 |
| | C | 3 2 | 0 1 | 3 0 |

**Player 2**

| **Player 1** | | C | D |
|---|---|---|---|
| | A | 2 2 2 | 4 3 6 |
| | B | 5 3 2 | 3 4 2 |

**Player 3** chooses F

**Player 2**

| | C | D |
|---|---|---|
| T | 0 0 0 | 1 2 5 |
| B | 3 6 1 | 0 0 1 |

**Player 3** chooses G

# Large game.

150 students in a class, they simultaneously ask for a grade (A, B or C); if 20% or less ( i.e. ≤ 30) ask for an A then all requests are granted, otherwise they all get a C.

Selfish players

first set : exactly 30 choose A, 120 choose B

Second set : at least 32 choose A

**Example with uncertain outcomes.** A simple auction. There are two players, Charlie and Doreen. There is an object (e.g. a painting) which Charlie values at $120 and Doreen values at $180. Each player has to submit a bid of either $50 or $80. The highest bidder gets the object and pays his/her bid (the loser does not pay anything). If the bids are equal, a fair coin is tossed.

Outcomes:

*a* Charlie wins and pays $50

*b* Charlie wins and pays $80

*c* Doreen wins and pays $50

*d* Doreen wins and pays $80

Player's utility = value – price paid (if wins, otherwise 0)

| | | Doreen (value: $180) bid $50 | bid $80 |
|---|---|---|---|
| **Charlie** | bid $50 | | |
| (value: $120) | bid $80 | | |

Outcomes: *a* Charlie wins and pays \$50

*b* Charlie wins and pays \$80

*c* Doreen wins and pays \$50

*d* Doreen wins and pays \$80

Player's utility = value – price paid (if wins, otherwise 0)

**Doreen** (value: \$180)

| | | bid \$50 | bid \$80 |
|---|---|---|---|
| **Charlie** | bid \$50 | $\begin{pmatrix} b & d \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ | $d$ |
| (value: \$120) | bid \$80 | $b$ | $\begin{pmatrix} b & d \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ |

| | | Doreen | |
|---|---|---|---|
| | | bid $50 | bid $80 |
| **Charlie** | bid $50 | 35 , 65 | 0 , 100 |
| | bid $80 | 40 , 0 | 20 , 50 |