University of California, Davis -- Department of Economics

ECON 122 : GAME THEORY Professor Giacomo Bonanno

PRACTICE FOR FIRST MIDTERM EXAM

Answer all questions. Explain your answers.

1. Two players can make a total profit of $100 if they are not too greedy. The players are in different rooms and cannot communicate. Each player has to write a positive multiple of 10 not greater than 100 (thus either 10 or 20 or 30 … or 100) on a piece of paper and hand it to the referee. The referee opens the two bids and does the following:

- if the two amounts written down add up to 100 or less, she gives each player the amount he/she wrote (for example, if Player 1 writes 20 and Player 2 writes 60, then Player 1 gets $20 and Player 2 gets $60),
- if the two amounts written down add up to **more** than $100, the referee gives nothing to the players.

Each player only cares about how much money he/she gets: he/she does not care about how much money the other player gets (e.g. whether the other player gets more or less than him/her).

(a) Consider Player 1. Is the strategy of writing 10 strictly dominated by another strategy?

(b) What are Player 1's strictly dominated strategies?

(c) Consider Player 1. Is the strategy of writing 100 weakly dominated by another strategy?

(d) What are Player 1's weakly dominated strategies?

(e) What are the Nash equilibria of this game?

2. Consider the following game, where in each cell the first number is the payoff of Player 1 and the second number is the payoff of Player 2.

Player 2

		L	C	R
	T	10, 18	7, 20	1, 18
Player 1	M	12, 15	8, 16	1, 12
	B	10, 9	4, 8	0, 0

(a) Does player 1 have a weakly dominant strategy? If Yes, state which strategy.

(b) Does player 2 have a weakly dominant strategy? If Yes, state which strategy.

(c) Is there a dominant-strategy equilibrium?

(d) What is the output of the iterated deletion of weakly dominated strategies (IDWDS)? For each step in the procedure state which strategies are deleted.

3. Consider a simultaneous two-player second-price auction concerning a single indivisible good. The game-frame is as follows: $S_1 = S_2 = B$ where $B = \{p_1, p_2, ..., p_m\}$ is a *finite* set of positive numbers with $p_1 < p_2 < ... < p_m$, the set of outcomes is the set of pairs (i, p) where $i \in \{1, 2\}$ is the winner of the auction and $p \in B$ is the price that the winner has to pay and the outcome function is as follows (b_i denotes the bid of Player *i*): $f(b_1, b_2) = \begin{cases} (1, b_2) & \text{if } b_1 \geq b_2 \\ (2, b_1) & \text{otherwise} \end{cases}$. Let v_i be the value of the object to Player *i* (that is, Player *i* views getting the object as equivalent to getting $\$v_i$). We shall consider various kinds of preferences. We state them in terms of Player 1, but the same definitions apply to Player 2. The following apply to all three preferences (this is the "**selfish**" part):

for every $p < v_1$ and for every p', $(1, p) \succ_1 (2, p')$;

for every p and p', $(1, p) \succ_1 (1, p')$ if and only if $p < p'$.

- Player 1 is *selfish and **uncaring*** if, in addition, her preferences are as follows:
  for every p and p', $(2, p) \sim_1 (2, p')$; for every p, $(2, p) \sim_1 (1, v_1)$;
  and everything that follows from the above by transitivity.
- Player 1 is *selfish and **benevolent*** if, in addition, her preferences are as follows:
  for every p and p', $(2, p) \succ_1 (2, p')$ if and only if $p < p'$; $(2, p_m) \sim_1 (1, v_1)$;
  and everything that follows from the above by transitivity.
- Player 1 is *selfish and **spiteful*** if her preferences are as follows:
  for every p and p', $(2, p) \succ_1 (2, p')$ if and only if $p > p'$; $(2, p_1) \sim_1 (1, v_1)$;
  and everything that follows from the above by transitivity.

(a) Suppose that Player 1 is **selfish and uncaring** and $v_1 = 80$. How does she rank the following three outcomes: $(1,72), (2,60), (2,35)$?

(b) Suppose that Player 1 is **selfish and benevolent** and $v_1 = 64$. How does she rank the following three outcomes: $(1,73), (2,57), (2,46)$?

(c) Suppose that Player 1 is **selfish and spiteful** and $v_1 = 25$. How does she rank the following three outcomes: $(1,18), (2,50), (2,39)$?

In parts (d)-(f) assume that $m > 3$, $v_1, v_2 \in B$, $p_1 < v_1 < p_m$ and $p_1 < v_2 < p_m$.

(d) Suppose that Player 1 is selfish and ***uncaring***. Does she have a weakly or strictly dominant strategy? If your answer is Yes, say what that strategy is and state whether it is weak or strict dominance; if your answer is No explain why not.

(e) Suppose that Player 1 is selfish and ***benevolent***. Is bidding v_1 a dominant strategy? Explain your answer.

(f) Suppose that it is common knowledge that both players are selfish and ***uncaring,*** $B = \{1,2,3,4,5\}, v_1 = 3$ and $v_2 = 5$. Find all the pure-strategy Nash equilibria.

(g) Suppose that it is common knowledge that both players are selfish and ***benevolent,*** $B = \{1,2,3,4,5\}, v_1 = 3$ and $v_2 = 5$. **(g.1)** Is (3,5) a Nash equilibrium? Explain your answer. **(g.2)** Find all the pure-strategy Nash equilibria.

(h) Suppose that it is common knowledge that both players are selfish and ***spiteful,*** $B = \{1,2,3,4,5\}, v_1 = 3$ and $v_2 = 5$. **(h.1)** Is (3,5) a Nash equilibrium? Explain your answer. **(h.2)** Find all the pure-strategy Nash equilibria.

4. Consider the following situation. There are three voters, 1, 2, and 3 and four candidates for a job, a, b, c and d. The voting procedure is as follows: Voter 1 moves first and can either veto a or veto b (vetoing a candidate means that that candidate is out: he cannot be chosen for the job). If she vetoes a, voter 2 can either choose b (in which case b is elected) or veto b. If voter 1 vetoes b, voter 2 can either choose a (who is therefore elected) or veto a. Finally, if a candidate has not been chosen yet, voter 3 chooses one of the remaining two (i.e. one of the two who have not been vetoed). At every stage the vote of each player is public, that is, observed by the other voters.

(a) Represent this situation as an extensive-form game-frame (with perfect information).

(b) What are Player 1's strategies?

(c) What are Player 2's strategies?

(d) What are Player 3's strategies?

(e) Suppose that the voters rank the candidates as follows

	Voter 1	Voter 2	Voter 3
Best	*c*	*a*	*c*
	a	*c*	*d*
	b	*b*	*b*
Worst	*d*	*d*	*a*

What is the backward induction solution of this game?

(f) Describe a Nash equilibrium which is different from the backward-induction solution. Explain what is intuitively wrong with this equilibrium.