University of California, Davis -- Department of Economics

ECON 122 : GAME THEORY Professor Giacomo Bonanno

PRACTICE FOR FINAL EXAM

Answer all questions. Explain your answers.

1. Consider the following game. There are two players. Each has to decide whether or not to contribute to public television. If neither of them contributes, then there is no public television and each gets a payoff of zero. If a player decides to contribute, she gives her credit card number to the potential provider of public television. The provider then proceeds as follows: if both players supplied their credit card numbers, each credit card is charged \$2 and the broadcasting takes place; if only one player provided her credit card number, then the card is charged \$3 and the broadcasting takes place. Public television is a public good, that is, once it is provided everybody can enjoy it, whether or not she contributed to it. Each player derives a benefit from being able to watch public television equivalent to getting \$4. The players make their decisions independently of each other (i.e. with no information about what the other player chose).

(a) Write the strategic form of this game (there are only two players, namely the potential contributors; the provider of public television is not considered as a player). Give as payoff the **net benefit** to each player. Does Player 1 have a dominant strategy?

Now consider the more complex situation where there is incomplete information. Player 1 knows Player 2's payoffs, while Player 2 is uncertain as to whether Player 1 has the payoffs explained above (refer to this case as "Player 1 is of Type I") or the following **net** payoffs (refer to this case as "Player 1 is of Type II") :

		Player 2: Contribute	Player 2: Not contribute
Player 1	Contribute	1	0
	Not contribute	4	2

Player 2 believes that Player 1 is of Type I with probability p and of Type II with probability $(1 - p)$, where $0 < p < 1$. All this is common knowledge between the two players. Both players are **risk neutral**.

(b) Represent the above incomplete information game as a game with imperfect information.

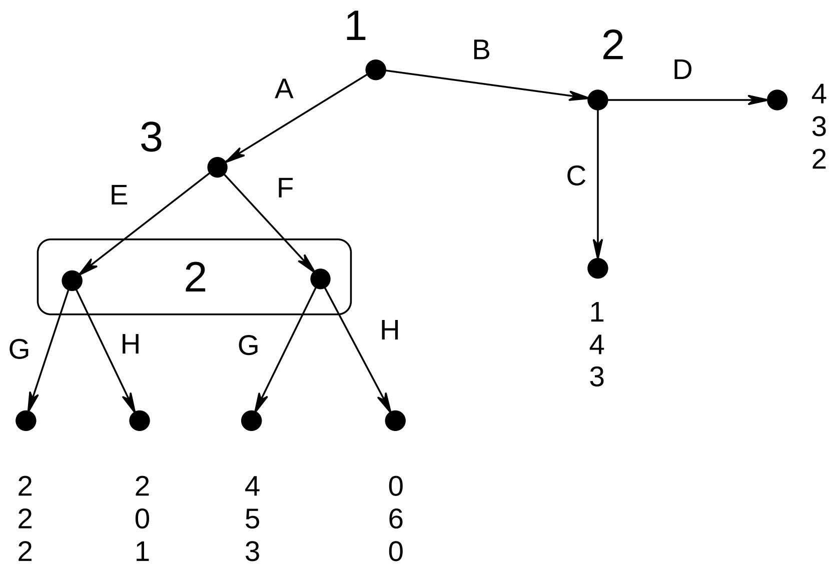
(c) How many strategies does Player 1 have? Write down at least two of them. How many strategies does Player 2 have?

(d) Write the strategic form corresponding to the extensive form of part (b). [Hint: payoffs are **expected** payoffs and thus are expressions involving p).

(e) Find a pure-strategy Nash equilibrium where Player 1 makes the same choice no matter what his type is.

(f) Are there any other pure-strategy Nash equilibria? [Hint: your answer must be conditional on the value of p.]

2. Consider the following extensive game



(a) Write the corresponding strategic-form game

(b) Does player 1 have any (weakly or strictly) dominated strategies? And player 2? and player 3?

(c) Find the mixed-strategy Nash equilibrium of the subgame that starts at player 3's node

(d) Find the subgame-perfect equilibrium of the entire game.

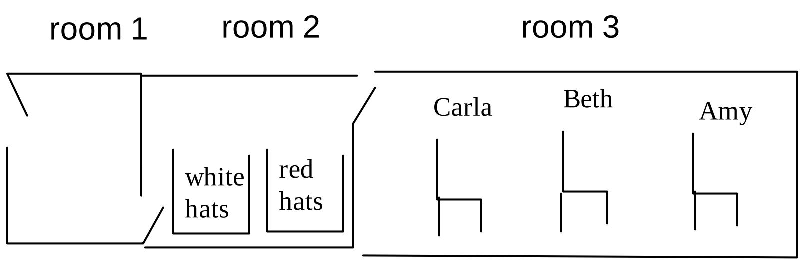
(e) Find a pure-strategy Nash equilibrium which is **not** subgame perfect.

3. Consider the following two-player game. Each player has to write down a positive integer not exceeding 900 (thus either 1 or 2 or … or 900). Payoffs are as follows (u_1 is the payoff of player 1, and u_2 the payoff of Player 2), where *x* is the number written by Player 1 and *y* the number written by Player 2:

$$u_1(x,y) = \begin{cases} x-1 & \text{if } x<y \\ 0 & \text{if } x \geq y \end{cases}, \qquad u_2(x,y) = \begin{cases} y-1 & \text{if } x>y \\ 0 & \text{if } x \leq y \end{cases}$$

Find **all** the pure strategy Nash equilibria of this game.

4. Amy, Beth and Carla are now in room 1. They are asked to proceed, one at a time, to room 3 through room 2. In room 2 there are two large containers, one with red hats only and the other with white hats only. They have to choose one hat (obviously each of them knows the color of the hat she chooses), put it on their head and then go and sit in the chair in room 3 that has their name on it.



Amy goes first, then Beth then Carla. The chairs are with the back to the door. Thus a person entering room 3 can see whoever is already seated there, but cannot be seen by them. Suppose that Amy chooses a white hat, Beth a red hat and Carla a white one.

(a) Represent the interactive knowledge situation that arises when they are all seated in room 3 (that is, use an information partition model to represent what each individual knows).

(b) Find the smallest event that is common knowledge among them. Give also a verbal interpretation of this event.

(c) Repeat (a) and (b) assuming now that there is a mirror in room 3 that allows Amy to see the hat of Carla (but not that of Beth).

5. Consider the following cooperative game: $N = \{1,2,3\}$, $v(\{1\}) = v(\{3\}) = 1$, $v(\{2\}) = 2$,

$v(\{1,2\}) = 4$, $v(\{1,3\}) = 3$, $v(\{2,3\}) = 5$, $v(N) = 8$. For each of the following imputations state whether or not it belong to the core (if not, explain why not).

(a) (2, 4, 2) **(b)** (2, 3, 3) **(c)** (1, 2, 5) **(d)** (2, 2, 4)

6. Find the Shapley value of the game of Question 5.

7. Consider again the game of Question 5. Suppose we add a fourth player who is a dummy and is such that $v(\{4\}) = 6$.

(a) Write the full game (i.e. write the characteristic function).

(b) Compute the Shapley value and compare it to what you obtained in Question 6: are Players 1, 2 and 3 affected by the addition of Player 4?