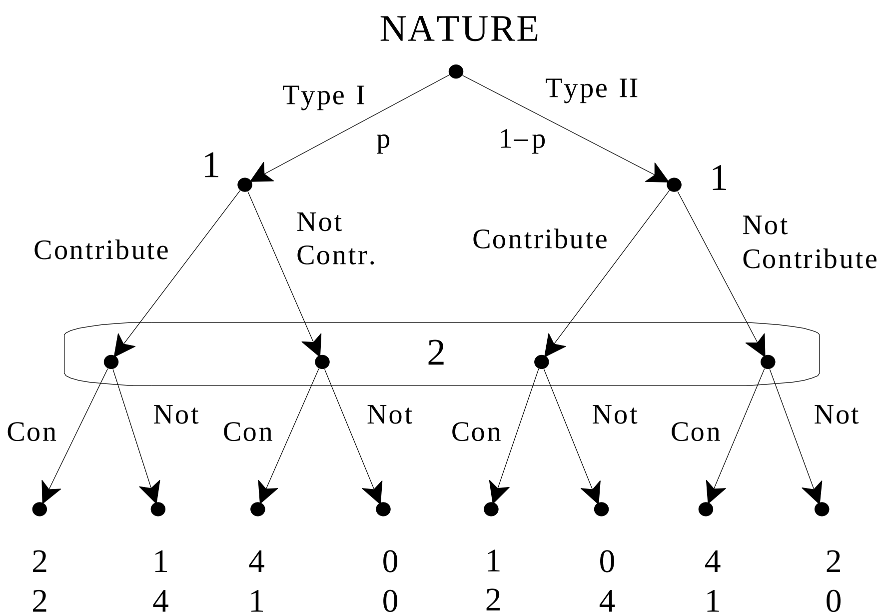
PRACTICE FINAL: ANSWERS

1.

		Player 2 Contribute	Not contribute
Player 1	Contribute	2 , 2	1 , 4
	Not contribute	4 , 1	0 , 0

Player 1 does **not** have a dominant strategy.

(b)



(c) Player 1 has four strategies. For example: (1) (Contribute if of Type I, Not Contribute if of Type II), (2) Contribute no matter what the type.
Player 2 has two strategies: Contribute and Not contribute.

(d) The normal form is as follows:

		Player 2 Contribute	Not contribute
Player 1	Contr. always	1 + p , 2	p , 4
	Never contr.	4 , 1	2 −2p , 0
	If Type I contr. if Type II not	4 −2p , 1 + p	2 −p, 4p
	If Type I not if Type II contr.	1 + 3p , 2 −p	0 , 4 −4p

(e) The following is a pure-strategy equilibrium where Player 1 makes the same choice at his two nodes:
Player 1: If Type I, then choose not contribute, If Type II, then choose not contribute
Player 2: choose contribute

(f) If $p \geq \frac{1}{3}$ (that is, if $4p \geq 1 + p$), then there is also the following equilibrium:
Player 1: If Type I, then choose contribute, If Type II, then choose not contribute
Player 2: choose not contribute

On the other hand, if $p < \frac{1}{3}$ then the equilibrium of part (e) is the only pure-strategy equilibrium.

2. (a)

Player 3 chooses E

Player 1 \ Player 2	GC	GD	HC	HD
A	2, 2, 2	2, 2, 2	2, 0, 1	2, 0, 1
B	1, 4, 3	4, 3, 2	1, 4, 3	4, 3, 2

Player 3 chooses F

Player 1 \ Player 2	GC	GD	HC	HD
A	4, 5, 3	4, 5, 3	0, 6, 0	0, 6, 0
B	1, 4, 3	4, 3, 2	1, 4, 3	4, 3, 2

(b) Player 1 does not have dominated strategies. For Player 2 GD is weakly dominated by GC. and HD is weakly dominated by HC. Player 3 does not have dominated strategies.

(c) The normal form of the subgame that starts at player 3's node is:

2 \ 3	E	F
G	2 , 2	5 , 3
H	0 , 1	6 , 0

In this subgame there are no pure-strategy Nash equilibria. To find the mixed strategy one, let p be the probability of G and q the probability of E. Then we need:

$$2q + 5(1-q) = 6(1-q) \quad \text{and} \quad 2p+(1-p) = 3p$$

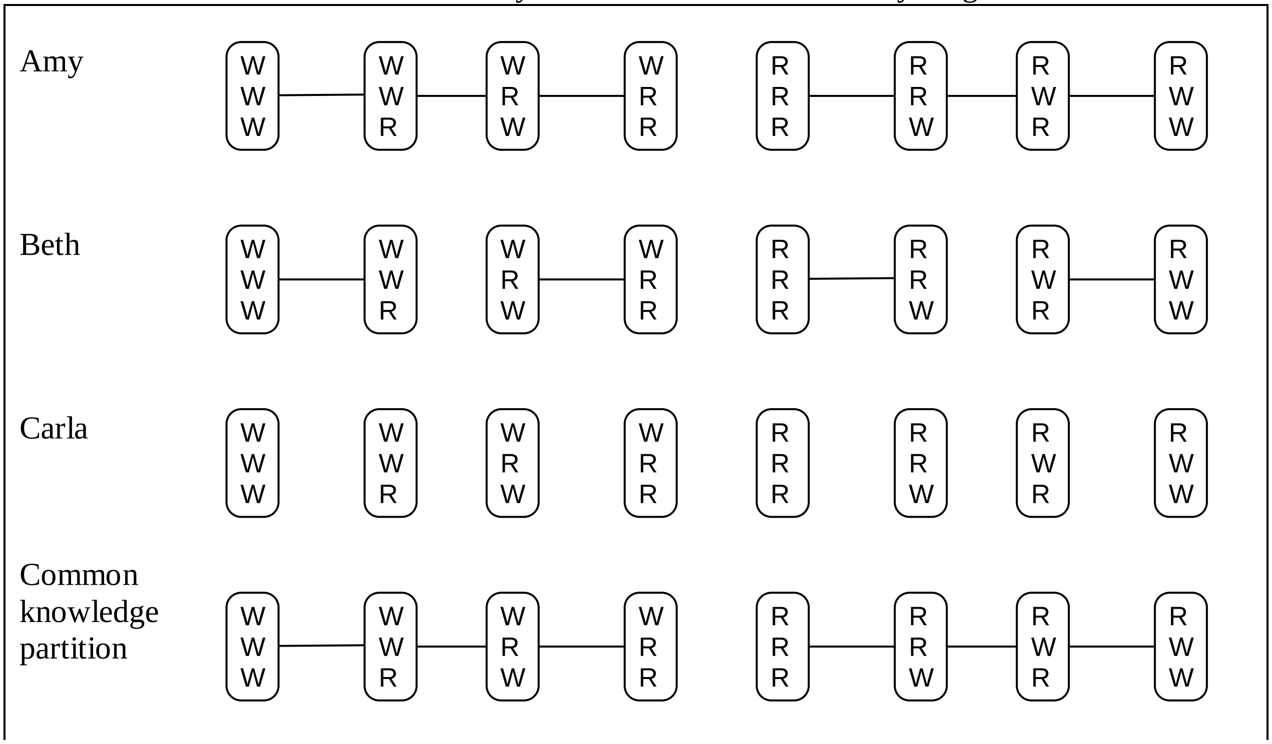
The solution is: $p = \frac{1}{2}$ and $q = \frac{1}{3}$. Thus Player 1's expected payoff at the Nash equilibrium of the subgame is: (1/6)2 + (1/6)2 + (2/6)4 + (2/6)0 = 2.

(d) Since player 1's expected payoff in the subgame is 2, he prefers choosing A to choosing B. Thus the unique subgame-perfect equilibrium is: $\left(\begin{array}{cc|cccc|cc} A & B & C & D & G & H & E & F \\ 1 & 0 & 1 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{3} & \frac{2}{3} \end{array}\right)$.

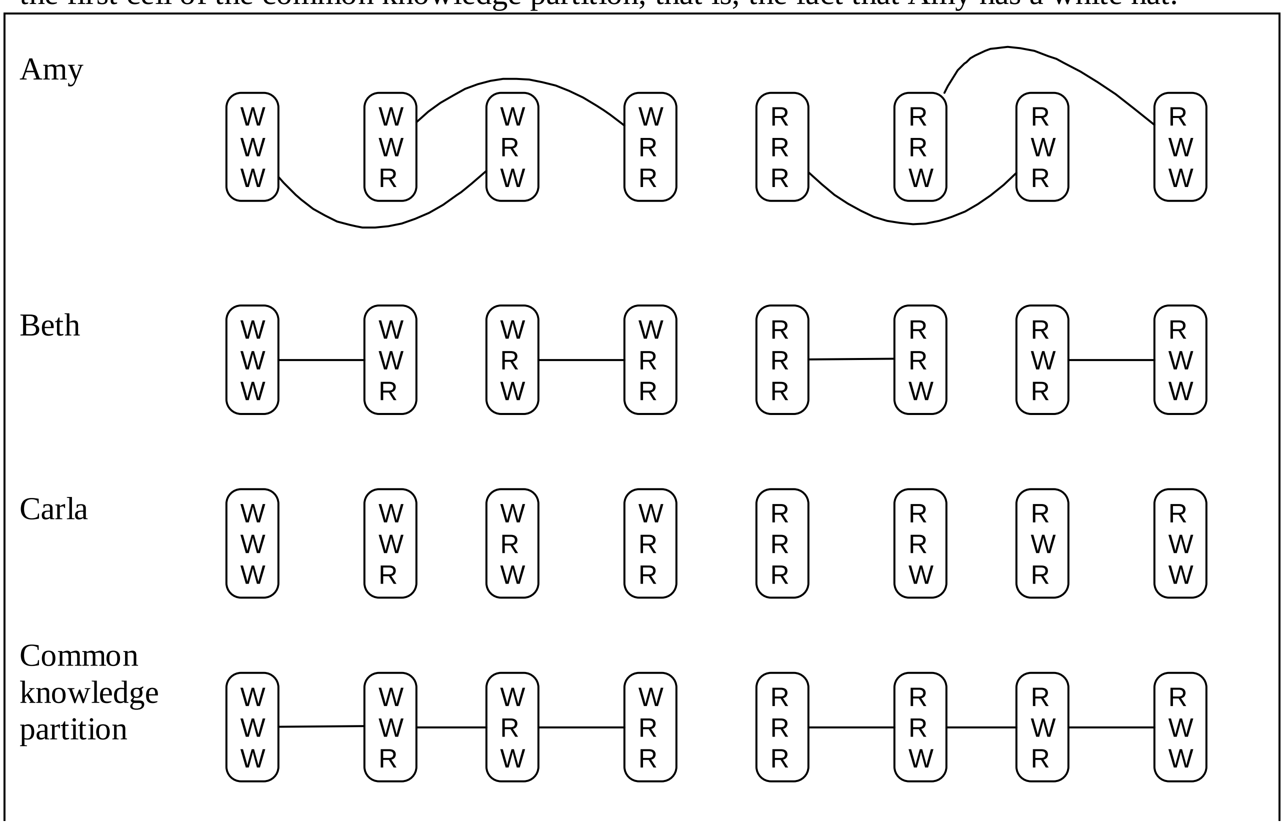
(e) A pure-strategy Nash equilibrium which is not subgame-perfect is (B, CH, F).

3. The Nash equilibria are: (1,1), (2,2), (1,2), (2,1), (2,3) and (3,2).

4. Represent the state as a triple, where the first coordinate denotes Amy's hat, the second Beth'a and the third Carla's. Beth can see Amy's hat and Carla can see everything. Thus



Since the true state is (W, R, W), the smallest event that is common knowledge among them is the first cell of the common knowledge partition, that is, the fact that Amy has a white hat.



The common knowledge partition is the same, hence the smallest event that is common knowledge among all three of them is that Amy has a white hat.

5. **(a)** (2, 4, 2): yes it is in the core.

(b) (2, 3, 3): yes it is in the core.

(c) (1, 2, 5): not in the core: it is blocked by coalition {1,2} [e.g. with imputation (1.5,2.5,0)]

(d) (2, 2, 4) : yes it is in the core.

6. The Shapley value is (2, 3.5, 2.5)

7. **(a)** N = {1,2,3,4}, v({1}) = v({3}) = 1, v({2}) = 2, v({4}) = 6

v({1,2}) = 4, v({1,3}) = 3, v({2,3}) = 5, v({1,4}) = 7, v({2,4}) = 8, v({3,4}) = 7

v({1,2,3}) = 8, v({1,2,4}) = 10, v({1,3,4}) = 9, v({2,3,4}) = 11

v(N) = 14.

(b) The Shapley value is (2, 3.5, 2.5, 6). Thus Players 1, 2 and 3 are not affected by the addition of dummy Player 4.