

1. (a) Since  $(a, (d, f))$  and  $(b, (d, f))$  are both backward-induction solutions, it must be that Player 1 is indifferent between  $z_2$  and  $z_4$ . Since  $(b, (d, e))$  is a backward-induction solution but  $(a, (d, e))$  is not, it must be that Player 1 prefers  $z_3$  to  $z_2$ . Thus there are only two inferences that we can make about Player 1's preferences: (1)  $z_2 \sim_1 z_4$  and (2)  $z_3 \succ_1 z_2$  (and thus, by transitivity, also  $z_3 \succ_1 z_4$ ).

(a) Since both  $e$  and  $f$  are part of a backward-induction solution, it must be that Player 2 is indifferent between  $z_3$  and  $z_4$ . Since  $d$  is part of a backward-induction solution but  $c$  is not, it must be that Player 2 prefers  $z_2$  to  $z_1$ . Thus there are only two inferences that we can make about Player 2's preferences: (1)  $z_3 \sim_2 z_4$  and (2)  $z_2 \succ_2 z_1$ .

(b) There are five possibilities, depending on where  $z_1$  appears in the ranking:  $\left( \begin{array}{cc} \text{best} & z_1 \\ & z_3 \\ \text{worst} & z_2, z_4 \end{array} \right),$

$$\left( \begin{array}{cc} \text{best} & z_1, z_3 \\ \text{worst} & z_2, z_4 \end{array} \right), \left( \begin{array}{cc} \text{best} & z_3 \\ & z_1 \\ \text{worst} & z_2, z_4 \end{array} \right), \left( \begin{array}{cc} \text{best} & z_3 \\ \text{worst} & z_1, z_2, z_4 \end{array} \right), \left( \begin{array}{cc} \text{best} & z_3 \\ & z_2, z_4 \\ \text{worst} & z_1 \end{array} \right).$$

2. (a) (and (b)) The strategic form is as follows. The Nash equilibria are highlighted.

		Player 2			
		a c	a d	b c	b d
Player 1	L e	2 5	2 5	3 3	3 3
	L f	2 5	2 5	2 7	2 7
	R e	3 2	1 1	3 2	1 1
	R f	3 2	1 1	3 2	1 1

(c) There is a unique backward induction solution given by (Re, ac).

3. (a) For no values (if  $b \leq 2$   $H$  weakly dominates  $G$ , however,  $H$  does not dominate  $F$ ).

(b) (b.1) For  $a < 3$ .    (b.2) The strictly dominant strategy is B.

(c) (c.1) For  $a = 3$ .    (c.2) The weakly dominant strategy is B.

- (d) • for any  $a$ , if  $b > 2$  then (B,G) is the only pure-strategy Nash equilibrium.  
 • if  $a > 3$  and  $b < 2$  then (B,H) is the only pure-strategy Nash equilibrium.  
 • if  $a \leq 3$  and  $b = 2$ , then there are 3 pure-strategy Nash equilibria: (B,F), (B,G) and (B,H).

- (e) • if  $a \leq 3$  and  $b < 2$ , then there are 2 pure-strategy Nash equilibria: (B,F) and (B,H)  
 • if  $a \leq 3$  and  $b = 2$ , then there are 2 pure-strategy Nash equilibria: (B,G) and (B,H).

To answer (f) and (g) first note that, for every pair of values of  $a$  and  $b$ , in the first round of elimination strategies A, D and E of Player 1 are among those that get eliminated. If  $b > 2$  then G cannot be deleted in the first round, while if  $b \leq 2$  G can be deleted in the first

round.

If  $a > 3$  then C cannot be deleted in the first round, while if  $a \leq 3$  C can be deleted in the first round. Thus:

- If  $a > 3$  and  $b > 2$  then in the first round delete A, D and E, in the second round delete F and H and in the third round delete C. Left with (B,G).
- If  $a \leq 3$  and  $b > 2$  then in the first round delete A, C, D and E and in the second round delete F and H. Left with (B,G).
- If  $a \leq 3$  and  $b \leq 2$  then in the first round delete A, C, D, E and G and no more deletions are possible. Left with (B,F) and (B,H).
- If  $a > 3$  and  $b \leq 2$  then in the first round delete A, D, E and G, in the second round delete F and in the third round delete C. Left with (B,H).

Hence:

(f) For  $a \leq 3$  and  $b \leq 2$ .

(g) For  $a > 3$  and any value of  $b$ , or for  $a \leq 3$  and  $b > 2$ .

**4.** (c,k), (c,m), (d,k), (d,m)