SPRING 2024 - FIRST MIDTERM EXAM: ANSWERS for VERSION 1

1. **(a)** **(a.1)** For $x < 3$. **(a.2)** The strictly dominant strategy is C.

(b) **(b.1)** For $x = 3$. **(b.2)** The weakly dominant strategy is C.

(c) For no values (if $y \leq 2$ G weakly dominates H; however, G does not dominate F).

(d) • for any x, if $y > 2$ then (C,H) is the only pure-strategy Nash equilibrium. Odd.
• if $x > 3$ and $y < 2$ then (C,G) is the only pure-strategy Nash equilibrium. Odd.
• if $x \leq 3$ and $y = 2$, then there are 3 pure-strategy Nash equilibria: (C,F), (C,G) and (C,H). Odd.

(e) • $x \leq 3$ and $y < 2$, then there are two Nash equilibria: (C,F) and (C,G). Even.
• $x > 3$ and $y = 2$, then there are two Nash equilibria: (C,G) and (C,H). Even.

To answer **(f)** and **(g)** first note that, for every pair of values of x and y, in the first round of elimination strategies A, B and E of Player 1 are among those that get eliminated.
If $y > 2$ then H cannot be deleted in the first round, while if $y \leq 2$ H can be deleted in the first round.
If $x > 3$ then D cannot be deleted in the first round, while if $x \leq 3$ D can be deleted in the first round. Thus:

- If $x > 3$ and $y > 2$ then in the first round delete A, B and E, in the second round delete F and G and in the third round delete D. Left with (C,H).
- If $x \leq 3$ and $y > 2$ then in the first round delete A, B, D and E and in the second round delete F and G. Left with (C,H).
- If $x > 3$ and $y \leq 2$ then in the first round delete A, B, E and H, in the second round delete F and in the third round delete D. Left with (C,G).
- If $x \leq 3$ and $y \leq 2$ then in the first round delete A, B, D, E and H and then no more deletions are possible. Left with (C,F) and (C,G).

Hence: **(f)** For $x \leq 3$ and $y \leq 2$. **(g)** For $x > 3$ and any value of y, or for $x \leq 3$ and $y > 2$.

2. **(a)** Since $(a,(c,e))$ and $(b,(c,e))$ are both backward-induction solutions, it must be that Player 1 is indifferent between z_1 and z_3. Since $(b,(c,f))$ is a backward-induction solution but $(a,(c,f))$ is not, it must be that Player 1 prefers z_4 to z_1. Thus there are only two inferences that we can make about Player 1's preferences: (1) $z_1 \sim_1 z_3$ and (2) $z_4 \succ_1 z_1$ (and thus, by transitivity, also $z_4 \succ_1 z_3$)

(b) Since both e and f are part of a backward-induction solution, it must be that Player 2 is indifferent between z_3 and z_4. Since c is part of a backward-induction solution but d is not, it must be that Player 2 prefers z_1 to z_2. Thus there are only two inferences that we can make about Player 2's preferences: (1) $z_3 \sim_2 z_4$ and (2) $z_1 \succ_2 z_2$.

(c) There are five possibilities, depending on where z_2 appears in the ranking:

$$\begin{pmatrix} best & z_2 \\ & z_4 \\ worst & z_1, z_3 \end{pmatrix}, \begin{pmatrix} best & z_2, z_4 \\ worst & z_1, z_3 \end{pmatrix}, \begin{pmatrix} best & z_4 \\ & z_2 \\ worst & z_1, z_3 \end{pmatrix}, \begin{pmatrix} best & z_4 \\ worst & z_1, z_2, z_3 \end{pmatrix}, \begin{pmatrix} best & z_4 \\ & z_1, z_3 \\ worst & z_2 \end{pmatrix}.$$

2. **(a) and (b)** The strategic form is as follows. The Nash equilibria are highlighted.

Player 2

Player 1		b1 c1		b1 c2		b2 c1		b2 c2	
	a1 d1	2	1	2	1	0	0	0	0
	a1 d2	2	1	2	1	0	0	0	0
	a2 d1	1	4	2	2	1	4	2	2
	a2 d2	1	4	1	6	1	4	1	6

(c) There is a unique backward induction solution given by $\left((a_1, d_1), (b_1, c_1)\right)$.

3. (A,f), (A,g), (B,f), (B,g)