

HOMEWORK # 6 (for due date see web page)

1. . You look awful: dark circles under your eyes, hanging chin, absent look, ... You must have one of the following:

- Flu (**F**): symptoms are tiredness and a headache.
- Hangover (**H**): symptoms are headache and dizziness.
- Unprepared for Final Syndrome (**UF**): symptoms are dizziness, trembling.
- Prepared but no Sleep Syndrome (**PNS**): symptoms are trembling, tiredness.

These are the only possibilities and it is well known to everybody what symptoms go with what form of sickness. It is also known that (1) you always tell your mother whether or not you have a headache, (2) you always tell your roommate whether or not you are tired and (3) you have been examined by your doctor who can always determine with absolute accuracy what your sickness is.

- (a) Represent what each individual (mother, roommate and doctor) knows by means of information partitions.
- (b) Let E be event that you either have the either flu or a hangover. Find the events “Your mother knows E ”, “Your roommate knows E ” and “Your doctor knows E ”.
- (c) Suppose that you have the flu. Does your mother know that you have the flu? Does your roommate know? Does your doctor know?
- (d) Let S be the event that you have either F or H or UF . Find the events $K_{mother}S$ (your mother knows S), $K_{doc}K_{mother}S$ (the doctor knows that your mother knows S) and $K_{roommate}K_{mother}S$ (your roommate know that your mother knows S).

2. Ann, Bob and Carla are taken to a room where there is a tray with two white balls and two red balls. Thus it becomes common knowledge among them that there are four balls, two of which are white and two are red. Then they leave the room. A referee then leads Ann to the room and asks her to pick one ball and hide it in her pocket. Then it becomes Bob’s turn to enter the room and pick one of the remaining balls and put it in his pocket. Finally, Carla enters the room, picks one of the remaining balls and puts it in her pocket. It is commonly known among them that this is what happened, that is, that first Ann, then Bob, then Carla went to the room and picked a ball. Nobody gets to see the balls picked by the other two. However, they do see what balls are left on the tray when they make their own choice.

- (a) Using a set of states and information partitions, represent the state of knowledge of Ann, Bob and Carla (concerning who has what balls).
- (b) Draw the partition representing what is common knowledge **between Ann and Bob**.
- (c) Draw the partition representing what is common knowledge **between Bob and Carla**.
- (d) What strategy should Bob use in picking his ball if he wants to prevent Carla from knowing what color ball Ann has? And what if he wants to prevent Carla from knowing what color ball he himself has?