

**HOMEWORK # 1 (for due date see web page)**

1. Consider the following three-player game-frame (where  $o_1, o_2, \dots, o_{18}$  are the possible outcomes):

		<b>Player 2</b>		
		D	E	F
<b>Player 1</b>	<b>A</b>	o1	o2	o3
	<b>B</b>	o4	o5	o6
	<b>C</b>	o7	o8	o9
<b>Player 3: G</b>				
		<b>Player 2</b>		
		D	E	F
<b>Player 1</b>	<b>A</b>	o10	o11	o12
	<b>B</b>	o13	o14	o15
	<b>C</b>	o16	o17	o18
<b>Player 3: H</b>				

The players rank the outcomes as follows:

Player 1:	<table style="border-collapse: collapse;"> <tr><td style="padding: 2px 10px;"><math>o_7, o_{16}</math></td><td style="padding: 2px 10px;">best</td></tr> <tr><td style="padding: 2px 10px;"><math>o_4, o_{14}</math></td><td></td></tr> <tr><td style="padding: 2px 10px;"><math>o_{15}</math></td><td></td></tr> <tr><td style="padding: 2px 10px;"><math>o_1, o_{12}</math></td><td></td></tr> <tr><td style="padding: 2px 10px;"><math>o_9</math></td><td></td></tr> <tr><td style="padding: 2px 10px;"><math>o_5, o_{13}</math></td><td></td></tr> <tr><td style="padding: 2px 10px;"><math>o_2</math></td><td></td></tr> <tr><td style="padding: 2px 10px;"><math>o_{11}, o_{18}</math></td><td></td></tr> <tr><td style="padding: 2px 10px;"><math>o_6</math></td><td></td></tr> <tr><td style="padding: 2px 10px;"><math>o_3, o_8, o_{10}, o_{17}</math></td><td>worst</td></tr> </table>	$o_7, o_{16}$	best	$o_4, o_{14}$		$o_{15}$		$o_1, o_{12}$		$o_9$		$o_5, o_{13}$		$o_2$		$o_{11}, o_{18}$		$o_6$		$o_3, o_8, o_{10}, o_{17}$	worst	Player 2:	<table style="border-collapse: collapse;"> <tr><td style="padding: 2px 10px;"><math>o_{13}</math></td><td style="padding: 2px 10px;">best</td></tr> <tr><td style="padding: 2px 10px;"><math>o_5, o_{14}</math></td><td></td></tr> <tr><td style="padding: 2px 10px;"><math>o_1, o_7, o_{11}</math></td><td></td></tr> <tr><td style="padding: 2px 10px;"><math>o_6, o_{12}</math></td><td></td></tr> <tr><td style="padding: 2px 10px;"><math>o_2, o_8, o_{17}</math></td><td></td></tr> <tr><td style="padding: 2px 10px;"><math>o_3, o_4, o_9, o_{10}, o_{15}, o_{16}, o_{18}</math></td><td>worst</td></tr> </table>	$o_{13}$	best	$o_5, o_{14}$		$o_1, o_7, o_{11}$		$o_6, o_{12}$		$o_2, o_8, o_{17}$		$o_3, o_4, o_9, o_{10}, o_{15}, o_{16}, o_{18}$	worst
$o_7, o_{16}$	best																																		
$o_4, o_{14}$																																			
$o_{15}$																																			
$o_1, o_{12}$																																			
$o_9$																																			
$o_5, o_{13}$																																			
$o_2$																																			
$o_{11}, o_{18}$																																			
$o_6$																																			
$o_3, o_8, o_{10}, o_{17}$	worst																																		
$o_{13}$	best																																		
$o_5, o_{14}$																																			
$o_1, o_7, o_{11}$																																			
$o_6, o_{12}$																																			
$o_2, o_8, o_{17}$																																			
$o_3, o_4, o_9, o_{10}, o_{15}, o_{16}, o_{18}$	worst																																		
Player 3:	<table style="border-collapse: collapse;"> <tr><td style="padding: 2px 10px;"><math>o_{12}</math></td><td style="padding: 2px 10px;">best</td></tr> <tr><td style="padding: 2px 10px;"><math>o_{15}</math></td><td></td></tr> <tr><td style="padding: 2px 10px;"><math>o_3, o_6, o_{16}</math></td><td></td></tr> <tr><td style="padding: 2px 10px;"><math>o_{11}</math></td><td></td></tr> <tr><td style="padding: 2px 10px;"><math>o_2, o_7, o_{14}, o_{17}</math></td><td></td></tr> <tr><td style="padding: 2px 10px;"><math>o_8, o_{10}, o_{13}, o_{18}</math></td><td></td></tr> <tr><td style="padding: 2px 10px;"><math>o_1, o_4, o_5, o_9</math></td><td>worst</td></tr> </table>	$o_{12}$	best	$o_{15}$		$o_3, o_6, o_{16}$		$o_{11}$		$o_2, o_7, o_{14}, o_{17}$		$o_8, o_{10}, o_{13}, o_{18}$		$o_1, o_4, o_5, o_9$	worst																				
$o_{12}$	best																																		
$o_{15}$																																			
$o_3, o_6, o_{16}$																																			
$o_{11}$																																			
$o_2, o_7, o_{14}, o_{17}$																																			
$o_8, o_{10}, o_{13}, o_{18}$																																			
$o_1, o_4, o_5, o_9$	worst																																		

- (a) For each player write a utility function that represents her ranking, using consecutive integers with 0 being the lowest. Use these utility functions to obtain a game based on the above game-frame.
- (b) For each player find all the strategies that are strictly dominated.
- (c) What do you get by applying the iterated deletion of strictly dominated strategies?
- (d) Are there any Nash equilibria?

2. Consider the following game ( $x$  and  $y$  can be any non-negative real numbers):

		Player 2		
		F	G	H
Player 1	A	2 , 4	2 , 3	0 , 3
	B	2 , 1	3 , 4	1 , 0
	C	3 , 2	4 , 2	2 , $y$
	D	$x$ , 3	3 , 4	1 , 4
	E	1 , 2	3 , 2	0 , 1

- (a) For what values of  $x$  does Player 1 have a strictly dominant strategy? Name the strategy.
- (b) For what values of  $x$  does Player 1 have a weakly but not strictly dominant strategy? Name the strategy.
- (c) Are there values of  $y$  for which Player 2 has a weakly dominant strategy?
- (d) Find all the Nash equilibria when  $x = 1$  and  $y = 2$ .
- (e) Find all the Nash equilibria when  $x = 4$  and  $y = 2$ .
- (f) Let  $x = 4$  and  $y = 3$ . What do you get when you apply the procedure of iterative elimination of strictly dominated strategies? Write explicitly the various steps of the elimination procedure.