

- Axiom 1: Unrestricted Domain or Freedom of Expression
 - At the individual level, any complete and transitive ranking should be allowed.

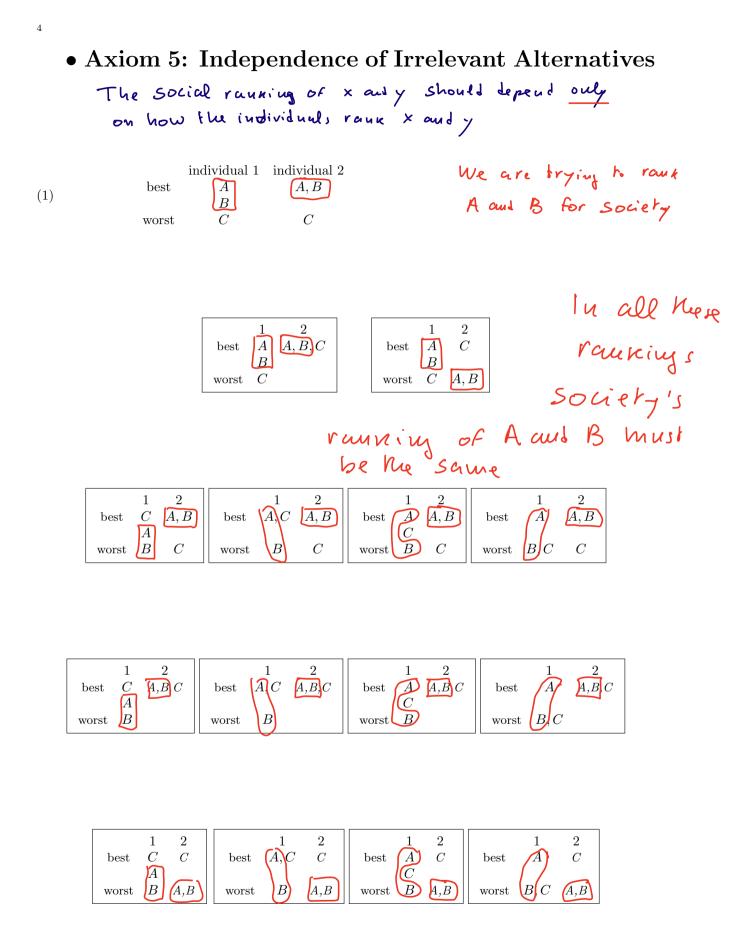
• Axiom 2: Rationality

Also the social ranking should be complete and transitive

• Axiom 3: Unanimity or Pareto Principle If, for every ie {1,2,..., n3, x>; y they x>y

E∡a w م best worst		2's ranking C A B	3's ranking B C A Vuqu	A versus B no nucuimous ranking A versus C " " B versus C " " imity principle imposes no restrictions
Example Z best worst	1's ranking A B C	2's ranking C A B	3's ranking A, B C Vчач	Av.B: no unanimous ranking Av.C: " BvC: " imity principle imposes no restrictions
E×ouple 3 best worst	1's ranking A B C	2's ranking C A B	3's ranking A C B	A>B, A>B, A>B, A>B Munnimous them Manimity requires A>B for A and C and for B and C unanimity imposes no restictions

Axiom 4: Non-dictatorship no dictator, that is, Mora is There is { no individual i such that, for every two alternatives x and y, if x >; y then x>y.
Equivalently: For every i ∈ {1, 2, --, n} there exists at least one pair (x,y) such that x >; y but y ≥ x



If there are only two alternatives the Independence of Irrelevant Alternatives axiom is trivially satisfied.

Remark 1. If there are only two alternatives (and any number of individuals) then the method of majority voting satisfies all of Arrow's axioms.

Arrow's Impossibility Theorem

If the number of alternatives is at least three, there is no social preference function that satisfies the five axioms.

equivalently: if you find a SPF Mar Satisfies four of the axioms, then it must violate the fifth axiom.

Arrow's axioms

Unrestricted Domain or Freedom of Expression	FE
Rationality	R < Completeness transitivity
Unanimity or Pareto	U
Non-Dictatorship	ND
Independence of Irrelevant Alternatives	IIA
Majority Rule with 2 alternatives 7 The	e two satisfy all
Plurality Rule with 2 alternatives	Arrow's axidus
Majority Rule with 2 alternatives These of the second	Completeness res Transitivity V yes fails
Plurality Rule with more than 2 alternatives	ND yes
	ITA yes
	FE yes

if n is even: number or individuals $\ge \frac{n}{2} + 1$ **if n is odd:** number or individuals $\ge \frac{n+1}{2}$

n voters

Majority rule: if a majority prefers x to y then society prefers x to y if a majority prefers y to x then society prefers y to x otherwise society is indifferent between x and y

Plurality rule: if the number of individuals who prefer x to y is grater than the number of individuals who prefer y to x then society prefers x to y

> if the number of individuals who prefer y to x is grater than the number of individuals who prefer x to y then society prefers y to x

otherwise society is indifferent between x and y