Set of alternatives among which society has to choose:

$$X = \left\{ x_1, x_2, \dots, x_m \right\}$$

Set of individuals (members of society or voters:

$$S = \{1, 2, \dots, n\}$$

Each voter *i* has a complete and transitive ranking \succeq_i of X

Social preference function:

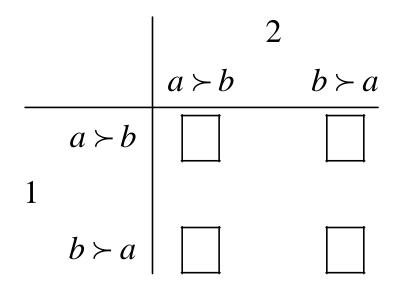
$$\underbrace{(\succeq_1, \succeq_2, ..., \succeq_n)}_{input} \mapsto \underbrace{\succeq}_{output}$$

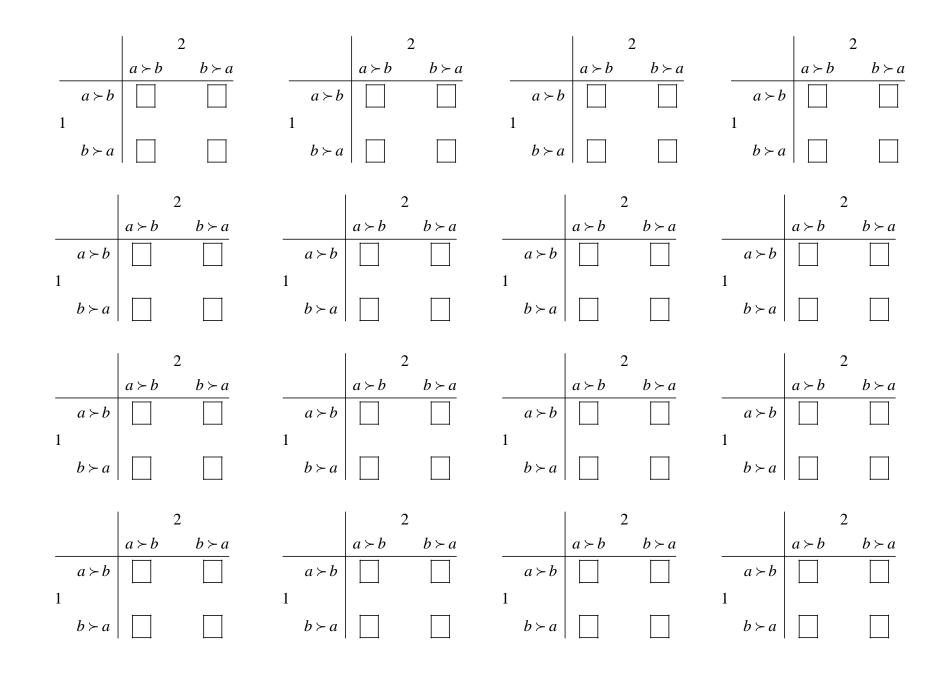
Social choice function:

$$\underbrace{\left(\succeq_{1}, \succeq_{2}, \dots, \succeq_{n}\right)}_{input} \mapsto \underbrace{x \in X}_{output}$$

Social Choice Function

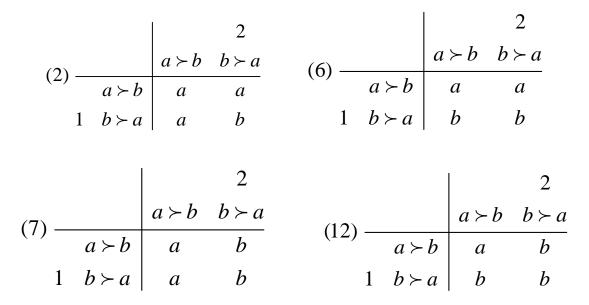
Two voters, two alternatives:





First requirement: UNANIMITY. A good SCF should be such that if both voters put the same alternative at the top of their reported ranking then that alternative should be chosen.

By imposing unanimity we are left with:

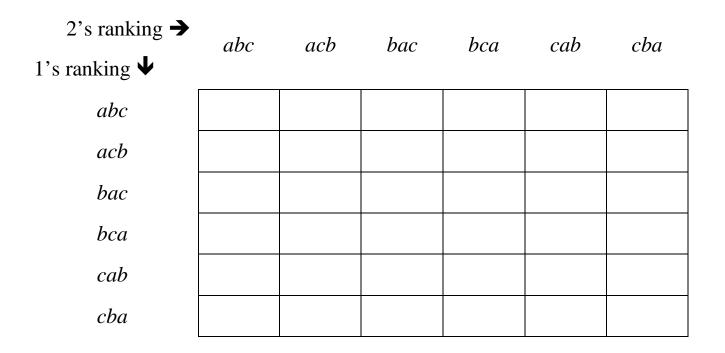


Second requirement: NON-DICTATORSHIP. A good SCF should be such that there is no individual whose top alternative is always chosen, that is, if he reports $a \succ b$ then *a* is chosen and if he reports $b \succ a$ then *b* is chosen.

By imposing Unanimity and Non-Dictatorship we are left with

Third requirement: NON-MANIPULABILITY. A good SCF should be such that there is no situation where an individual can gain by reporting a false ranking (that is, a ranking which is not her true ranking). Both of the remaining two rankings satisfy this requirement.

Now two voters but three alternatives: *a*, *b*, *c*.



2's ranking →	abc	acb	bac	bca	cab	cba
1's ranking $oldsymbol{\Psi}$						
abc	а	а	a	b	С	а

0.00	Ci	Ci	Cr	U	Ũ	
acb	а	а	b	а	а	С
bac	b	а	b	b	b	С
bca	а	b	b	b	С	b
cab	а	С	С	b	С	С
cba	С	а	b	С	С	С

Does it satisfy **Unanimity**?

2's ranking 🗲	abc	acb	bac	bca	cab	cba
1's ranking ↓						

abc	а	а	а	b	С	а
acb	а	а	b	а	а	С
bac	b	а	b	b	b	С
bca	а	b	b	b	С	b
cab	а	С	С	b	С	С
cba	С	а	b	С	С	С

Does it satisfy Non-Dictatorship?

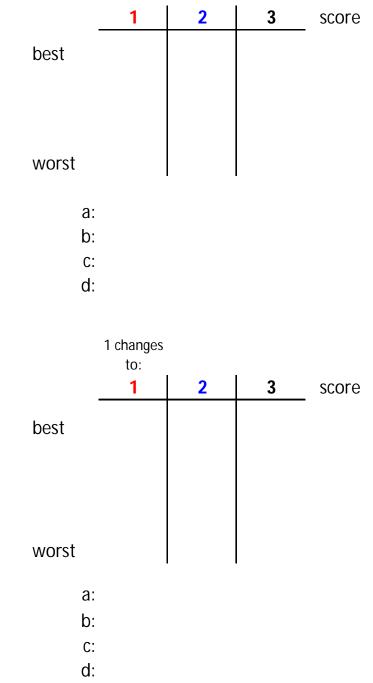
Satisfies Unanimity and Non-Dictatorship, but fails Non-Manipulability:

2's ranking →	abc	acb	bac	bca	cab	cba
1's ranking ↓						
abc	а	а	а	b	С	a
acb	а	а	b	а	а	С
bac	b	а	b	b	b	С
bca	а	b	b	b	С	b
cab	а	С	С	b	С	С
cba	С	а	b	С	С	С

Gibbard-Satterthwaite theorem:

MANIPULABILITY of the BORDA count

Four alternatives: a, b, c and d Three voters



MANIPULABILITY of the KEMENY-YOUNG method

The Kemeny-Young procedure is a social **preference** function. However, just like the Borda rule, it can be converted to a social **choice** function by picking the top-ranked alternative in the selected ranking.

Consider the following tie-breaking rule: if two or more rankings are selected by the Kemeny-Young procedure, then pick the one whose top alternative comes first in alphabetical order.

	voter 1	voter 2	voter 3
best	A	С	В
	В	A	С
worst	С	В	A
Ranking		Kemeny-Y	oung score
$\overline{A \succ B \succ C}$	$\#(A \succ$	$(B) + \#(A \succ$	$(C) + #(B \succ C) =$
$\overline{A \succ C \succ B}$	$\#(A \succ$	$-C) + \#(A \succ$	$(B) + \#(C \succ B) =$
$\overline{B \succ A \succ C}$	$\#(B \succ$	$(A) + \#(B \succ$	$(C) + \#(A \succ C) =$
$\overline{B \succ C \succ A}$	$\#(B \succ$	$-C) + \#(B \succ$	$(A) + \#(C \succ A) =$
$\overline{C \succ A \succ B}$	#(<i>C</i> ≻	$(C \succ A) + \#(C \succ C)$	$(B) + \#(A \succ B) =$
$C \succ B \succ A$	#(<i>C</i> >	$(-B) + \#(C \succ$	$(A) + \#(B \succ A) =$

If Voter 3 (for whom A is the worst alternative) lies and reports $C \succ B \succ A$ instead of the true $B \succ C \succ A$

	voter 1	voter 2	voter 3
best	A	С	С
	В	A	В
worst	С	В	A

Ranking	Kemeny-Young score
$\overline{A \succ B \succ C}$	$#(A \succ B) + #(A \succ C) + #(B \succ C) =$
$\overline{A \succ C \succ B}$	$#(A \succ C) + #(A \succ B) + #(C \succ B) =$
$\overline{B \succ A \succ C}$	$#(B \succ A) + #(B \succ C) + #(A \succ C) =$
$\overline{B \succ C \succ A}$	$#(B \succ C) + #(B \succ A) + #(C \succ A) =$
$\overline{C \succ A \succ B}$	$#(C \succ A) + #(C \succ B) + #(A \succ B) =$
$\overline{C \succ B \succ A}$	$#(C \succ B) + #(C \succ A) + #(B \succ A) =$