First question to ask yourself: what is my ranking of the basic outcomes?

$$
\begin{array}{cccc}
\text { state } & \rightarrow & S_{1} & S_{2}
\end{array} S_{3}
$$

| state $\rightarrow$ | $S_{1}$ | $S_{2}$ | $s_{3}$ | best | $z_{8}$ |
| :---: | :---: | :---: | :---: | :--- | :---: |
| act $\downarrow$ |  |  |  |  | $z_{3}$ |
| $a_{1}$ |  | $z_{1}$ | $z_{2}$ | $z_{3}$ |  |
| $a_{2}$ | $z_{4}$ | $z_{5}$ | $z_{6}$ |  | $z_{1}, z_{9}$ |
| $a_{3}$ | $z_{7}$ | $z_{8}$ | $z_{9}$ |  | $z_{2}, z_{6}$ |
|  |  |  |  | $z_{4}, z_{5}$ |  |

Note:

- $a_{1}$

Thus ...


Three questions to ask yourself:
Note that $p$ is the probability of the worst outcome, not the best
(1) What $p$ is such that $\binom{z_{3}}{1} \sim\left(\begin{array}{cc}z_{7} & z_{8} \\ p & 1-p\end{array}\right)$ ? Suppose the answer is
(2) What $p$ is such that $\binom{z_{1}}{1} \sim\left(\begin{array}{cc}z_{7} & z_{8} \\ p & 1-p\end{array}\right)$ ? Suppose the answer is
(3) What $p$ is such that $\binom{z_{2}}{1} \sim\left(\begin{array}{cc}z_{7} & z_{8} \\ p & 1-p\end{array}\right)$ ? Suppose the answer is

## Utility

| best | $z_{8}$ | 1 |
| :---: | :---: | :---: |
|  | $z_{3}$ | $\frac{3}{4}$ |
|  | $z_{1}, z_{9}$ | $\frac{2}{3}$ |
|  | $z_{2}$ | $\frac{2}{5}$ |
| worst | $z_{7}$ | 0 |

In order not to deal with fractions, rescale the utility function by multiplying each number by 60 :

## Utility

|  |  |  | best | $z_{8}$ | 60 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | $z_{3}$ | 45 |
|  |  |  |  | $z_{1}, z_{9}$ | 40 |
|  |  |  |  | $z_{2}$ | 24 |
|  |  |  | worst | $z_{7}$ | 0 |
| $\begin{gathered} \text { state } \rightarrow \\ \text { act } \downarrow \\ a_{1} \end{gathered}$ | $s_{1}$ | $s_{2}$ | $s_{3}$ |  |  |
|  | 40 | 24 | 45 |  |  |
| $a_{3}$ |  | 60 | 40 |  |  |

Next step: try to assign probabilities to the states (from objective data or some subjective assessment). Suppose you assess the following:

$$
\begin{array}{cccc}
\text { state: } & S_{1} & S_{2} & S_{3} \\
\text { probability: } & \frac{1}{5} & \frac{3}{5} & \frac{1}{5}
\end{array}
$$

Then: $\mathbb{E}\left[U\left(a_{1}\right)\right]=$
$\mathbb{E}\left[U\left(a_{3}\right)\right]=$

Hence you should take action

Decision tree


First question to ask yourself: how do $\operatorname{Irank} z_{1}$ and $z_{2}$ ? Suppose that the answer is $z_{2} \succ z_{1}$.


Second question to ask yourself: how do I rank $z_{4}$ and $z_{5}$ ? Suppose that the answer is $z_{4} \succ z_{5}$.



Next question: how do I rank the remaining four outcomes? Suppose:

|  |  | Utility |
| :---: | :---: | :---: |
| best | $z_{2}$ | 1 |
|  | $z_{6}$ |  |
|  | $z_{4}$ |  |
| worst | $z_{3}$ | 0 |

This is sufficient to eliminate the random event on the left:


Two more questions and then you are done!
(4) What $p$ is such that $\binom{z_{6}}{1} \sim\left(\begin{array}{cc}z_{2} & z_{3} \\ p & 1-p\end{array}\right)$ ? Suppose the answer is $p=\frac{1}{2}$.
(5) What $p$ is such that $\binom{z_{4}}{1} \sim\left(\begin{array}{cc}z_{2} & z_{3} \\ p & 1-p\end{array}\right)$ ? Suppose the answer is $p=\frac{1}{10}$.

Then the lottery corresponding to the random event on the right has an expected utility of


Hence the optimal decision is to first take action $a$ and then, if a second choice is required between $c$ and $d$, choose $d$ :


