First question to ask yourself: what is my ranking of the basic outcomes?

| state \rightarrow | S_1 | S_2 | <i>S</i> ₃ |
|---------------------|-----------------|-------|-----------------------|
| act↓ | | | |
| a_{1} | Z_1 | Z_2 | Z_3 |
| a_2 | Z_4 | Z_5 | Z_6 |
| a_3 | \mathcal{Z}_7 | Z_8 | Z_9 |

| state \rightarrow act \downarrow | S ₁ | <i>s</i> ₂ | <i>S</i> ₃ | best z_8 z_3 |
|---|----------------------|-----------------------|-----------------------|---|
| a_1 | Z_1 | Z_2 | Z_3 | Z_1, Z_9 |
| a_2 | Z_A | - Z5 | Z ₆ | <i>z</i> ₂ , <i>z</i> ₆ |
| a_2 | т Z. ₇ | Z.o | <i>Z</i> .o | z_4, z_5 |
| 3 | 5/ | -8 | ~9 | worst z_7 |

Note:

• a_1

Thus ...

| state → | C | C | C | Utility |
|---------|-----------------------|-----------------------|------------|-----------------|
| state 7 | s ₁ | s ₂ | 3 3 | λ_8 1 |
| act ↓ | | | | Z_3 |
| a_1 | Z_1 | Z_2 | Z_3 | z_1, z_9 |
| a_3 | Z_7 | Z_8 | Z_{0} | Z_2 |
| 5 | , | 0 | , | worst $z_7 = 0$ |

Three questions to ask yourself:

Note that p is the probability of the worst outcome, not the best

(1) What p is such that
$$\begin{pmatrix} z_3 \\ 1 \end{pmatrix} \sim \begin{pmatrix} z_7 & z_8 \\ p & 1-p \end{pmatrix}$$
? Suppose the answer is

(2) What p is such that
$$\begin{pmatrix} z_1 \\ 1 \end{pmatrix} \sim \begin{pmatrix} z_7 & z_8 \\ p & 1-p \end{pmatrix}$$
? Suppose the answer is

(3) What p is such that
$$\begin{pmatrix} z_2 \\ 1 \end{pmatrix} \sim \begin{pmatrix} z_7 & z_8 \\ p & 1-p \end{pmatrix}$$
? Suppose the answer is

$$\begin{array}{ccc} & & Utility\\ \text{best} & z_8 & 1\\ & z_3 & \frac{3}{4}\\ & z_1, z_9 & \frac{2}{3}\\ & z_2 & \frac{2}{5}\\ & \text{worst} & z_7 & 0 \end{array}$$

In order not to deal with fractions, rescale the utility function by multiplying each number by 60:

Utility

| best | Z_8 | 60 |
|-------|-----------------|----|
| | Z_3 | 45 |
| | z_1, z_9 | 40 |
| | Z_2 | 24 |
| worst | \mathcal{Z}_7 | 0 |
| | | |

state \rightarrow s_1 s_2 s_3 act \downarrow a_1 40 24 45 a_3 0 60 40

Next step: try to assign probabilities to the states (from objective data or some subjective assessment). Suppose you assess the following:

| state: | <i>S</i> ₁ | <i>S</i> ₂ | <i>S</i> ₃ |
|--------------|-----------------------|-----------------------|-----------------------|
| probability: | $\frac{1}{5}$ | $\frac{3}{5}$ | $\frac{1}{5}$ |

Then: $\mathbb{E}[U(a_1)] =$

 $\mathbb{E}[U(a_3)] =$

Hence you should take action



First question to ask yourself: how do I rank z_1 and z_2 ? Suppose that the answer is $z_2 \succ z_1$.



Second question to ask yourself: how do I rank z_4 and z_5 ? Suppose that the answer is $z_4 \succ z_5$.





Next question: how do I rank the remaining four outcomes? Suppose:

| | | Utility |
|-------|-------|---------|
| best | Z_2 | 1 |
| | Z_6 | |
| | Z_4 | |
| worst | Z_3 | 0 |

This is sufficient to eliminate the random event on the left:



Two more questions and then you are done!

(4) What p is such that
$$\begin{pmatrix} z_6 \\ 1 \end{pmatrix} \sim \begin{pmatrix} z_2 & z_3 \\ p & 1-p \end{pmatrix}$$
? Suppose the answer is $p = \frac{1}{2}$.

(5) What p is such that
$$\begin{pmatrix} z_4 \\ 1 \end{pmatrix} \sim \begin{pmatrix} z_2 & z_3 \\ p & 1-p \end{pmatrix}$$
? Suppose the answer is $p = \frac{1}{10}$.

Then the lottery corresponding to the random event on the right has an expected utility of



Hence the optimal decision is to first take action a and then, if a second choice is required between c and d, choose d:

