## DECISION TREES

## Decision to buy a house

- NEW (built 2015), costs \$350,000
- OLD (built 1980), costs $\$ 300,000$

You worry about the total cost over the next 5 years.

- New houses have a $25 \%$ probability of requiring a repair within 5 years and, on average, the repair would cost $\$ 20,000$.
- Old houses have a $60 \%$ probability of requiring a repair within 5 years and, on average, the repair would cost $\$ 100,000$.

Your options are: (1) buy house $\mathbf{N}$, (2) buy house $\mathbf{O}$ or (3) pay $\$ 1,000$ to an inspector to inspect both houses. The inspector will be able to tell you if each house is good or bad.

- A good new house has probability $20 \%$ of requiring a repair (that costs $\$ 20,000$ ) and probability $80 \%$ of requiring no repair.
- A bad new house has probability $30 \%$ of requiring a repair (that costs $\$ 20,000$ ) and probability $70 \%$ of requiring no repair.
- A good old house has probability $50 \%$ of requiring a repair (that costs $\$ 100,000$ ) and probability $50 \%$ of requiring no repair.
- A bad old house has probability $70 \%$ of requiring a repair (that costs $\$ 100,000$ ) and probability $30 \%$ of requiring no repair.

Based on past data, the probabilities that the inspector will come up with the various verdicts are:

- Both good: 20\%
- Both bad: 30\%
- Old house good, new house bad: $20 \%$
- Old house bad, new house good: $30 \%$.


## THIS IS A LOT OF INFORMATION!

- NEW costs $\$ 350,000$. New houses have a $25 \%$ probability of requiring a repair within 5 years and, on average, the repair would cost \$20,000.
- OLD costs $\$ 300,000$. Old houses have a $60 \%$ probability of requiring a repair within 5 years and, on average, the repair would cost $\$ 100,000$.
- You can also hire an inspector and pay her $\$ 1,000$

Assuming risk neutrality

The "hire inspector" module is as follows:


The expected values of the lotteries are:

- For (1):
- For (2):
- For (3):
- For (4):

Thus we can reduce this part of the tree to:

## OBJECTIVE: pay the LOWEST 5-year cost



Thus we can reduce the option of hiring the inspector to the following lottery:

Whose expected value is

## The optimal decision is:

1. hire the inspector and then
2. (a) if both good, buy
(b) if $\mathbf{N} \operatorname{good}$ and $\mathbf{O}$ bad, buy
(c) if $\mathbf{N}$ bad and $\mathbf{O}$ good, buy
(d) if both bad, buy
