MANIPULABILITY of the BORDA count Satisfies unanimity Four alternatives: a. b. c and d and non-dictatorship

Four alternatives: a, b, c and d Three voters





d:

MANIPULABILITY of the KEMENY-YOUNG method

The Kemeny-Young procedure is a social **preference** function. However, just like the Borda rule, it can be converted to a social **choice** function by picking the top-ranked alternative in the selected ranking.

Consider the following tie-breaking rule: if two or more rankings are selected by the Kemeny-Young procedure, then pick the one whose top alternative comes first in alphabetical order.

| | voter 1 | voter 2 | voter 3 |
|--------------------------------|---------------|------------------------------|-------------------------|
| best | A | С | В |
| | В | A | С |
| worst | С | В | A |
| Ranking | | Kemeny-Y | oung score |
| $\overline{A \succ B \succ C}$ | $\#(A \succ$ | $(B) + \#(A \succ$ | $C) + #(B \succ C) =$ |
| $\overline{A \succ C \succ B}$ | $\#(A \succ$ | $-C) + \#(A \succ$ | $(B) + \#(C \succ B) =$ |
| $\overline{B \succ A \succ C}$ | $\#(B \succ$ | $(A) + \#(B \succ$ | $C) + #(A \succ C) =$ |
| $\overline{B \succ C \succ A}$ | $\#(B \succ$ | $-C) + #(B \succ$ | $(A) + \#(C \succ A) =$ |
| $\overline{C \succ A \succ B}$ | #(<i>C</i> > | $(C \succ A) + #(C \succ A)$ | $(B) + \#(A \succ B) =$ |
| $C \succ B \succ A$ | #(<i>C</i> > | -B) + #($C >$ | $(A) + \#(B \succ A) =$ |
| | | | |

If Voter 3 (for whom A is the worst alternative) lies and reports $C \succ B \succ A$ instead of the true $B \succ C \succ A$

| | voter 1 | voter 2 | voter 3 |
|-------|---------|---------|---------|
| best | A | C | С |
| | В | A | В |
| worst | C | В | A |

| Ranking | Kemeny-Young score |
|--------------------------------|--|
| $\overline{A \succ B \succ C}$ | $#(A \succ B) + #(A \succ C) + #(B \succ C) =$ |
| $A \succ C \succ B$ | $#(A \succ C) + #(A \succ B) + #(C \succ B) =$ |
| $\overline{B \succ A \succ C}$ | $#(B \succ A) + #(B \succ C) + #(A \succ C) =$ |
| $\overline{B \succ C \succ A}$ | $#(B \succ C) + #(B \succ A) + #(C \succ A) =$ |
| $\overline{C \succ A \succ B}$ | $#(C \succ A) + #(C \succ B) + #(A \succ B) =$ |
| $\overline{C \succ B \succ A}$ | $#(C \succ B) + #(C \succ A) + #(B \succ A) =$ |

The Psychology of Decision Making

1. Manipulation of Choices Through Decoys





Dan Arieli, Predictably Irrational: The Hidden Forces That Shape Our Decisions, 2010





2. Framing Effects: Gains versus Losses

I will give you \$200:



and then you will have to choose one of:

OPTION 1 : I give you an additional \$100:



HEADS: I give you an additional \$200



OPTION 2 : I toss a coin

TAILS: I give you no additional money





and then you will have to choose one of:

OPTION 1 : You give me back \$100:





In both cases:

Option 1 = you end up with \$300

Option 2 = you face the uncertain prospect (lottery)

 $\begin{pmatrix} You end up with \$400 & You end up with \$200 \\ Probability \frac{1}{2} & Probability \frac{1}{2} \end{pmatrix}$

Non-monetary example of effect of FRAMING in terms of GAINS vs LOSSES

You have been diagnosed with cancer. Two treatments are available:

- Surgery, which incurs some risk of dying on the operating table. Out of every 100 patients who chose surgery 90 survived the operation, 68 were alive after 1 year and 34 were alive after 5 years.
- Radiation. Out of every 100 patients who chose radiation 100 survived the treatment, 77 were alive after 1 year and 22 were alive after 5 years.

About 80% of experimental subjects chose surgery

You have been diagnosed with cancer. Two treatments are available:

- Surgery, which incurs some risk of dying on the operating table. Out of every 100 patients who chose surgery 10 died during the operation, 32 died after 1 year and 66 died within 5 years.
- Radiation. Out of every 100 patients who chose radiation none died during the treatment, 23 after 1 year and 78 died within 5 years.

About 50% of experimental subjects chose surgery.

Loss Aversion:

We are happy when we gain something, but

Twice unhappy when we lose it

The Pain of Paying

fMRI studies show that the pain centers of the brain light up when one has to part with one's cash.



The Pain of Paying

FMRI studies show that the pain centers of the brain light up when one has to part with one's cash.

Less pain with credit.



Functional magnetic resonance imaging (fMRI) is a procedure that measures brain activity by detecting associated changes in blood flow.

People tend to be risk-averse towards gains, but risk-loving towards losses.

Can such an attitude be compatible with expected utility?

Choice between
$$A:\begin{pmatrix} +\$50\\1 \end{pmatrix}$$
 and $B:\begin{pmatrix} +\$100 & +\$0\\\frac{1}{2} & \frac{1}{2} \end{pmatrix}$

Suppose that **she prefers** the sure gain: she prefers **A**. Then she displays **risk-aversion towards gains** (the expected value of these two options is the same).

Choice between
$$C: \begin{pmatrix} -\$50 \\ 1 \end{pmatrix}$$
 and $D: \begin{pmatrix} -\$100 & -\$0 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$.

Suppose that **she prefers** the risky prospect: she prefers **D**. **Then she is risk-loving towards losses** (the expected value of these two options is the same).

Is there a von Neumann-Morgenstern utility function that is consistent with these choices?

 $U \qquad A = \begin{pmatrix} +\$50 \\ 1 \end{pmatrix} \succ B = \begin{pmatrix} +\$100 & +\$0 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ outcome \$200 \$150 \$100 (\$50 \$0

Suppose that her initial wealth is \$100.

Hence it is possible for an expected-utility maximizing individual to display risk aversion towards a gain and risk love towards a symmetric loss.

However, this cannot happen at every wealth level.

Beginning wealth: \$200. Choice between
$$A: \begin{pmatrix} +\$50 \\ 1 \end{pmatrix}$$
 and $B: \begin{pmatrix} +\$100 & +\$0 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$.

Beginning wealth: \$200. Choice between
$$C: \begin{pmatrix} -\$50\\ 1 \end{pmatrix}$$
 and $D: \begin{pmatrix} -\$100 & -\$0\\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$.

Can she prefer A to B and also D to C? Let's see.

| | | Since she prefers D to C, she prefers |
|---------|---|---------------------------------------|
| outcome | U | |
| \$200 | 1 | |
| \$150 | a | |
| \$100 | b | |
| \$50 | С | |
| \$0 | 0 | |
| | | |
| | | |
| | | |
| | | |

Thus people who are consistently (that is, at every initial level of wealth) riskaverse towards gains and risk-loving towards losses cannot satisfy the axioms of expected utility. If those axioms capture the notion of rationality, then those people are irrational.