## PRACTICE THIRD MIDTERM: ANSWERS

1. (a)
\(\left.\begin{array}{|c|c|c|}\hline Total number of students \& 10,000 \& Percentage of total <br>
\hline Number of students who got an A \& 2,000 <br>

Number of students who got a B\end{array} \quad $$
\begin{array}{c}20,000\end{array}
$$\right]\)| 1500 |
| :---: |
| Number of students who got <br> P in midterm and A in the course <br> Number of students who got <br> F in midterm and A in the course <br> Number of students who got <br> P in midterm and B in the course <br> Number of students who got <br> F in midterm and B in the course |
| 5000 <br> $(25 \%$ of 2,000$)$ <br> $(50 \%$ of 8,000$)$ <br> 4,000 <br> $(50 \%$ of 8,000$)$ |

(b) 0.2 (or $20 \%$ ).
(c) $\frac{1,500}{1,500+4,000}=0.27($ or $27 \%)$.
(d) $\frac{4,000}{500+4,000}=0.89(0$ r $89 \%)$.
(e) 0.15 (or $15 \%$ ).
2. (a) For Ann $U_{0}(\$ 100$ in 4 years $)=(0.9)^{4}(\sqrt{100})=6.561$ and $U_{0}(\$ 400$ in 6 years $)=(0.9)^{6}(\sqrt{400})=10.629$. Thus she chooses to get $\$ 400$ in 6 years.
(b) For Christina, $U_{0}(\$ 100$ in 4 years $)=(0.7)(0.8)^{4}(\sqrt{100})=2.867$ and $U_{0}(\$ 400$ in 6 years $)=(0.7)(0.8)^{6}(\sqrt{400})=3.67$. Thus she too chooses to get $\$ 400$ in 6 years.
(c) For Ann $U_{4}(\$ 100$ now $)=\sqrt{100}=10$ and $U_{4}(\$ 400$ in 2 years $)=(0.9)^{2}(\sqrt{400})=16.2$. Thus she chooses $\$ 400$ in two years.
(d) For Christina $U_{4}(\$ 100$ now $)=\sqrt{100}=10$ and $U_{4}(\$ 400$ in 2 years $)=(0.7)(0.8)^{2}(\sqrt{400})=8.96$. Thus she changes her mind and chooses to get $\$ 100$ right away.
(e) Yes, because after 4 years she confirms her earlier choice.
(f) No, because after 4 years she changes her initial plan.
3. (a) $E U(a)=\frac{1}{3} 5+\frac{1}{12} 5+\frac{1}{6} 4+\frac{1}{4} 2+\frac{1}{6} 2=3.583, \quad E U(b)=\frac{1}{3} 6+\frac{1}{12} 1+\frac{1}{6} 6+\frac{1}{4} 1+\frac{1}{6} 1=3.5$ and $E U(c)=\frac{1}{3} 0+\frac{1}{12} 4+\frac{1}{6} 1+\frac{1}{4} 5+\frac{1}{6} 0=1.75$. Thus in the absence of information you would choose $a$ and have an expected utility of $\mathbf{3 . 5 8 3}$.
(b.1) If you receive information $\left\{s_{1}, s_{2}, s_{3}\right\}$, you will update you beliefs as follows:
$\begin{array}{llll}\text { probability } & \frac{4}{7} & \frac{1}{7} & \frac{2}{7}\end{array}$ state $\rightarrow \quad s_{1} \quad s_{2} \quad s_{3}$ act $\downarrow$

| $a$ | 5 | 5 | 4 |
| :--- | :--- | :--- | :--- |
| $b$ | 6 | 1 | 6 |
| $c$ |  | 0 | 4 |

$$
E U(a)=\frac{4}{7} 5+\frac{1}{7} 5+\frac{2}{7} 4=4.714
$$

c $\quad 0 \quad 4 \quad 1$

$$
\begin{aligned}
\text { and compute } & E U(b)=\frac{4}{7} 6+\frac{1}{7} 1+\frac{2}{7} 6=5.286 \\
& E U(c)=\frac{4}{7} 0+\frac{1}{7} 4+\frac{2}{7} 1=0.857
\end{aligned}
$$

Thus if informed that $\left\{s_{1}, s_{2}, s_{3}\right\}$ you would choose $\boldsymbol{b}$ and have an expected utility of 5.286.
(b.2) If you received information $\left\{s_{4}, s_{5}\right\}$ you will update you beliefs as follows:

| probability | $\frac{3}{5}$ | $\frac{2}{5}$ |  | $E U(a)=\frac{3}{5} 2+\frac{2}{5} 2=2$, |
| :---: | :---: | :---: | :--- | :--- |
| state $\rightarrow$ | $s_{4}$ | $s_{5}$ |  |  |
| act $\downarrow$ |  |  | and compute | $E U(b)=\frac{3}{5} 1+\frac{2}{5} 1=1$ |
| $a$ | 2 | 2 |  | $E U(c)=\frac{3}{5} 5+\frac{2}{5} 0=3$ |
| $b$ | 1 | 1 |  |  |

Thus if informed that $\left\{s_{4}, s_{5}\right\}$ you would choose $\boldsymbol{c}$ and have an expected utility of 3 .
(b.3) The probability of being informed that $\left\{s_{1}, s_{2}, s_{3}\right\}$ is $\frac{1}{3}+\frac{1}{12}+\frac{1}{6}=\frac{7}{12}$ and the probability of being informed that $\left\{s_{4}, s_{5}\right\}$ is $\frac{1}{4}+\frac{1}{6}=\frac{5}{12}$. Hence the expected utility of acquiring information is $\frac{7}{12} 5.286+\frac{5}{12} 3=4.33$. Hence acquiring information increases your expected utility from 3.583 to 4.33.

