## WINTER 2024 - SECOND MIDTERM EXAM: ANSWERS for VERSION 1

1. (a) The information is (1) $P(D)=0.02$ (from which we deduce that $P(\neg D)=0.98$ ),
(2) $P(-\mid D)=0.01$ (from which we deduce that $P(+\mid D)=0.99$ ) and (3) $P(+\mid \neg D)=0.05$.

Now, $P(+)=P(+\mid D) P(D)+P(+\mid \neg D) P(\neg D)=0.99(0.02)+0.05(0.98)=0.0688=6.88 \%$.
(b) $2 \%$ of 10,000 is 200 ; thus 200 have the disease and 9,800 don't. Of the 200 who have the disease, $1 \%$, that is, 2 give a negative result; thus the remaining 198 give a positive result. Of the 9,800 who don't have the disease, $5 \%$ (that is, 490) give a positive result. Thus the table is as follows:

|  | positive blood test | negative <br> blood test | Total |
| :---: | :---: | :---: | :---: |
| have the disease | 198 | 2 | 200 |
| don't have the disease | 490 | 9,310 | 9,800 |
| Total | 688 | 9,312 |  |

(c) $\frac{490}{688}=0.7122=71.22 \%$.
2. (a) Putting the money in the checking account corresponds to the lottery $\binom{\$ 1,200}{1}$, whose expected value is 1,200 , while putting it in the mutual fund corresponds to the lottery $\left(\begin{array}{ccc}\$ 1,800 & \$ 1,400 & \$ 700 \\ \frac{8}{100} & \frac{74}{100} & \frac{18}{100}\end{array}\right)$ whose expected value is 1,306 . Hence Trevor would be risk averse.
(b) The possible levels of wealth are: $\$ 1,800, \$ 1,400, \$ 1,200$ and $\$ 700$. Assign utility 1 to $\$ 1,800$ and zero to $\$ 700$. Since $\underbrace{\binom{\$ 1,400}{1}}_{C D} \sim \underbrace{\left(\begin{array}{cc}\$ 1,800 & \$ 700 \\ \frac{1}{2} & \frac{1}{2}\end{array}\right)}_{\text {startup }}$ the utility of $\$ 1,400$ is $\frac{1}{2} 1+\frac{1}{2} 0=\frac{1}{2}=0.5$. Finally, since $\underbrace{\binom{\$ 1,200}{1}}_{\text {Checking }} \sim \underbrace{\left(\begin{array}{cc}\$ 1,400 & \$ 700 \\ \frac{96}{100} & \frac{4}{100}\end{array}\right)}_{\text {foreign venture }}$, the utility of $\$ 1,200$ is $\frac{96}{100} 0.5+\frac{4}{100} 0=0.48$. Hence Trevor's utility function is $\begin{array}{cccc}\$ 1,800 & \$ 1,400 & \$ 1,200 & \$ 700 \\ 1 & 0.5 & 0.48 & 0\end{array}$
(c) $\mathbb{E}[U($ checking $)]=0.48, \quad \mathbb{E}[U($ mutual $)]=\frac{8}{100} 1+\frac{74}{100} 0.5+\frac{18}{100} 0=0.45 \quad \mathbb{E}[U(C D)]=0.5$, $\mathbb{E}[U($ startup $)]=0.5(1)+0.5(0)=0.5, \quad \mathbb{E}[U($ foreign $)]=\frac{96}{100} 0.5+\frac{4}{100} 0=0.48$.
(d) For a risk neutral person we compute expected values. $\mathbb{E}[$ checking $]=1,200$, $\mathbb{E}[$ mutual $]=\frac{8}{100} 1,800+\frac{74}{100} 1,400+\frac{18}{100} 700=1,306, \quad \mathbb{E}[C D]=1,400$,
$\mathbb{E}[$ startup $]=0.5(1,800)+0.5(700)=1,250, \quad \mathbb{E}[$ foreign $]=\frac{96}{100} 1,400+\frac{4}{100} 700=1,372$. Thus the ranking is $C D \succ$ foreign $\succ$ mutual $\succ$ startup $\succ$ checking .
3.

(a) First we need to convert outcomes into utilities: \begin{tabular}{c:cccc}
<br>
\hdashline$a$ \& $s_{1}$ \& $s_{2}$ \& $s_{3}$ \& $s_{4}$ <br>
$b$ \& 5 \& 13 \& 3 \& 8 <br>
$c$

. . Thus the regret table is: 

<br>
\hdashline \& $s_{1}$ \& $s_{2}$ \& $s_{3}$ \& $s_{4}$ <br>
$a$ \& 7 \& 6 \& 0 <br>
$b$ \& 6 \& 0 \& $\boxed{7}$ \& 1
\end{tabular} where the maximum regret for each act is highlighted.

(b) The MinMax Regret solution is: $\{a, b\}$.
(c) (c.1) $H\left(a, \frac{1}{4}\right)=4 \times \frac{1}{4}+9 \times \frac{3}{4}=\frac{31}{4}=7.75$. (c.2) $H\left(b, \frac{1}{4}\right)=3 \times \frac{1}{4}+13 \times \frac{3}{4}=\frac{42}{4}=10.5$. (c.1) $H\left(c, \frac{1}{4}\right)=0 \times \frac{1}{4}+11 \times \frac{3}{4}=\frac{33}{4}=8.25$.
(d) $\mathbb{E}[U(a)]=\frac{4+6+4+9}{4}=\frac{23}{4}, \quad \mathbb{E}[U(b)]=\frac{5+13+3+8}{4}=\frac{29}{4}, \quad \mathbb{E}[U(c)]=\frac{11+1+10+0}{4}=\frac{22}{4}$ Thus the act that maximizes expected utility is $b$.

