

# INSURANCE WITH MORAL HAZARD

Insurance market: customers with

- initial wealth  $W$
- facing potential loss  $L$
- with probability  $p$  fixed
- utility-of-money function  $U(m)$  with  $U'(m) > 0$  and  $U''(m) < 0$

# Moral hazard

The agent can either incur an expense or take an action that reduces the probability of loss.

We use  $e$  for either 'expense' or 'effort'

$P_n$  probability of loss if no effort

$P_e$  probability of loss if effort

$$0 < P_e < P_n < 1$$

Effort is costly:

either monetary cost:  $\$C$

or psychological cost:  $U(m, e)$   $\frac{\partial U}{\partial m} > 0$  utility is increasing in  $m$

$\frac{\partial U}{\partial e} < 0$  " decreasing in  $e$

## Monetary cost of effort: $\$C$

NI (No Insurance):

- if no “effort”:  $EU_n(NI) = p_n U(W-L) + (1-p_n) U(W)$

- if “effort”:  $EU_e(NI) = p_e U(W-L-C) + (1-p_e) U(W-C)$

## Psychological cost of effort

Suppose that there are two levels of effort: zero effort and some positive level of effort  $e > 0$

NI (No Insurance):

- if no effort:  $EU_n(NI) = p_n U(W-L, 0) + (1-p_n) U(W, 0)$

- if effort:  $EU_e(NI) = p_e U(W-L, e) + (1-p_e) U(W, e)$

Example:

$$W = 10,000 \quad L = 1,900 \quad p_n = \frac{4}{10} \quad p_e = \frac{1}{10}$$
$$U_n(m) \equiv \underline{U(m, 0)} = \sqrt{m}$$
$$U_e(m) \equiv \underline{U(m, e)} = \underline{\sqrt{m} - c}$$

$c$  = utility  
cost of  
effort

Then

$$\mathbb{E}[U_n(NI)] = \frac{4}{10} \sqrt{10,000 - 1,900} + \frac{6}{10} \sqrt{10,000} = 96$$

$$\mathbb{E}[U_e(NI)] = \frac{1}{10} \left( \sqrt{10,000 - 1,900} - c \right) + \frac{9}{10} \left( \sqrt{10,000} - c \right) = 99 - c$$

if  $c < 3$  then she chooses effort

$c > 3$  " no effort

$c = 3$  indifferent

If offered an insurance contract  $(h, d)$  the agent has four possible choices:

1. not insure and not exert effort
2. not insure and exert effort
3. purchase the contract and not exert effort
4. purchase the contract and exert effort

We are mainly interested in the “distortionary” effects of insurance and thus we will **assume that, under no insurance, the agent will choose effort.**

In the above example, suppose that  $c = 2$ :

$$W = 10,000 \quad L = 1,900 \quad p_n = \frac{4}{10} \quad p_e = \frac{1}{10} \quad U_n(m) \equiv U(m, 0) = \sqrt{m}$$

$$U_e(m) \equiv U(m, e) = \sqrt{m} - 2$$

$$\mathbb{E}[U_n(NI)] = 96$$

$$\mathbb{E}[U_e(NI)] = 99 - c = 99 - 2 = 97 \quad \text{if not insured she will choose effort}$$

$$W = 10,000 \quad L = 1,900 \quad p_n = \frac{4}{10} \quad p_e = \frac{1}{10}$$

Consider the full-insurance contract with premium  $h = \$190$ .

$$\begin{aligned}
 EU_n(NI) &= 96 & EU_e(NI) &= 97 & EU_e(h=190, d=0) &= \sqrt{10,000 - 190} - 2 \\
 & & & & &= 99.05 \\
 EU_n(h=190, d=0) & & & & EU_n(h=190, d=0) &= \sqrt{10,000 - 190} = 101.05
 \end{aligned}$$

What will the profit from this contract be? Will it be

$$190 - \frac{1}{10}(1,900) = 0?$$

$$\text{real profit} \quad 190 - \frac{4}{10}(1,900) = -570$$

In the fixed-probability case a monopolist would offer a full-insurance contract at the intersection of the reservation indifference curve (that is, the indifference curve that goes through the NI point) and the 45° line. Also in the case of moral hazard the monopolist would want to leave the consumer with no surplus, that is, **keep her at her reservation utility level**, but what is the reservation indifference curve in this case?

Through any point  $(W_1, W_2)$  in the wealth diagram there are now **two** indifference curves:

Slope of ind. curve at point A

Fixed  $p$

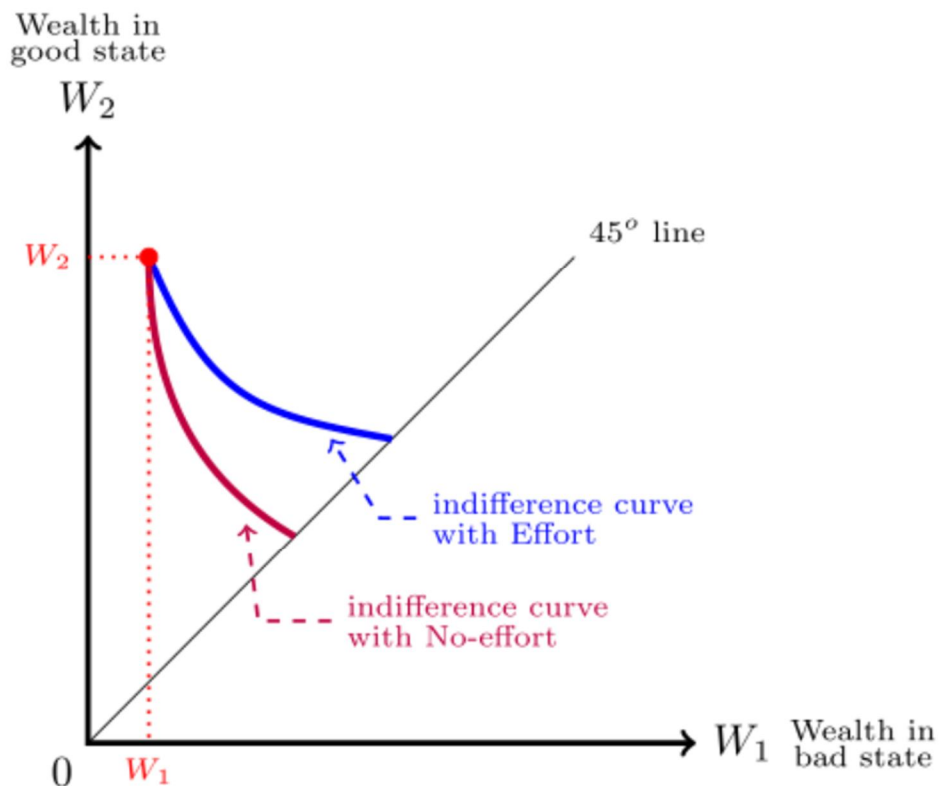
$$- \frac{p}{1-p} \frac{U'(W_1, A)}{U'(W_2, A)}$$

$$- \frac{p_n}{1-p_n} \frac{U'(W_1, A)}{U'(W_2, A)} = \frac{\frac{\partial U}{\partial m}(W_1^A, D)}{\frac{\partial U}{\partial m}(W_2^A, D)}$$

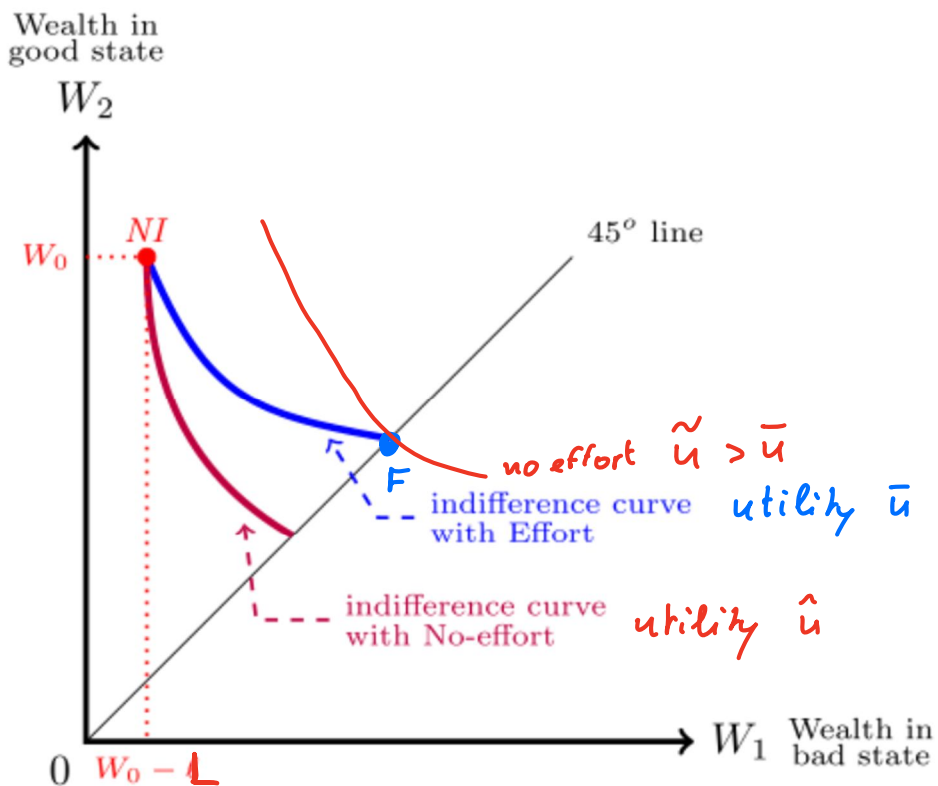
$$- \frac{p_e}{1-p_e} \frac{U'(W_1, A)}{U'(W_2, A)} = \frac{\frac{\partial U}{\partial m}(W_1^A, e)}{\frac{\partial U}{\partial m}(W_2^A, e)}$$

Since  $p_e < p_n$

$$\frac{p_e}{1-p_e} < \frac{p_n}{1-p_n}$$



This is true at every point, in particular also at the NI point:



In the previous example:  
 $\hat{u} = 96, \bar{u} = 97$

Assume that  
 $\bar{u} > \hat{u}$

reservation level of utility:  $\bar{u}$

What is the reservation locus?  
 utility  
 not an indifference curve



