Definition. Contract ***C*** **is** ***Pareto dominated*** **by contract** ***B*** if: or B Pareto dominates C

either

$$\begin{cases} E[U_A(B)] > E[U_A(C)] \\ \text{and} \\ E[U_P(B)] \geq E[U_P(C)] \end{cases}$$

that is, $B \succ_A C$ and $B \succsim_P C$

or

$$\begin{cases} E[U_P(B)] > E[U_P(C)] \\ \text{and} \\ E[U_A(B)] \geq E[U_A(C)] \end{cases}$$

that is, $B \succ_P C$, $B \succsim_A C$

Definition. A contract that is not Pareto dominated by any other contract is called ***Pareto efficient*** **(or Pareto optimal)**. Thus contract C is Pareto efficient if for every other contract D, either

or

or both.

Example. $X^G = 1{,}000$, $X^B = 600$, $p = \frac{1}{3}$ $U_P(\text{m}) = \sqrt{\text{m}}$ and $U_A(m) = m$.

C = (400, 400) is Pareto dominated by contract B = (676, 276):

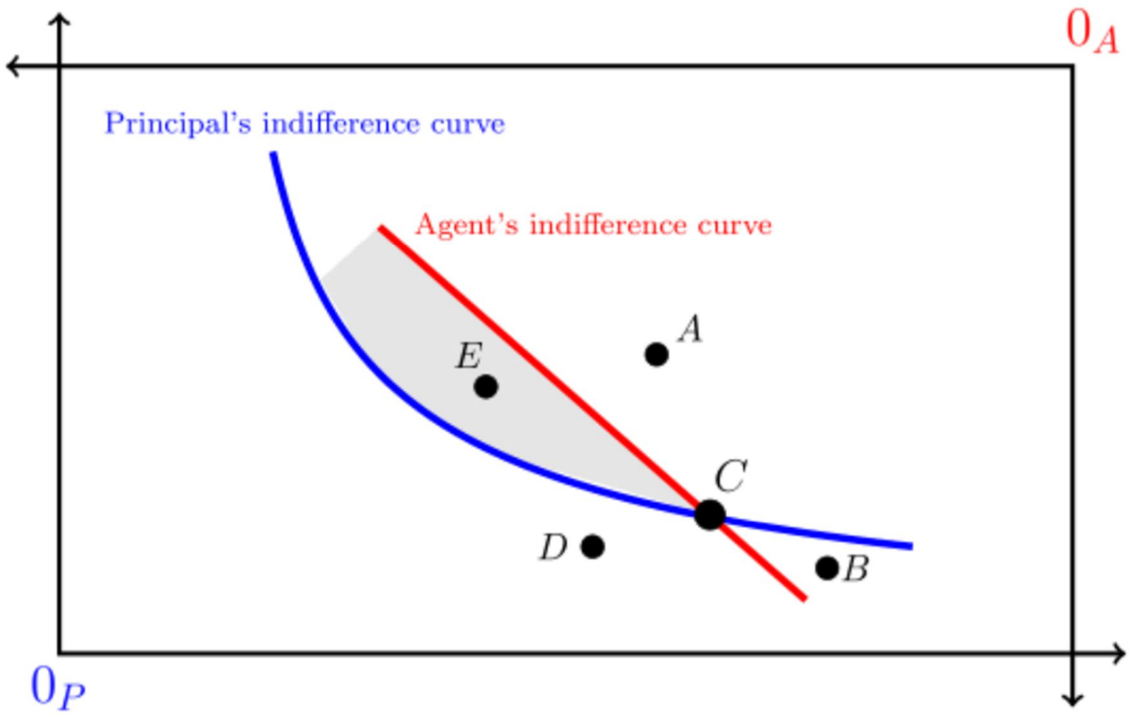
$$\mathbb{E}\left[U_P(B)\right] =$$

$$\mathbb{E}\left[U_P(C)\right] =$$

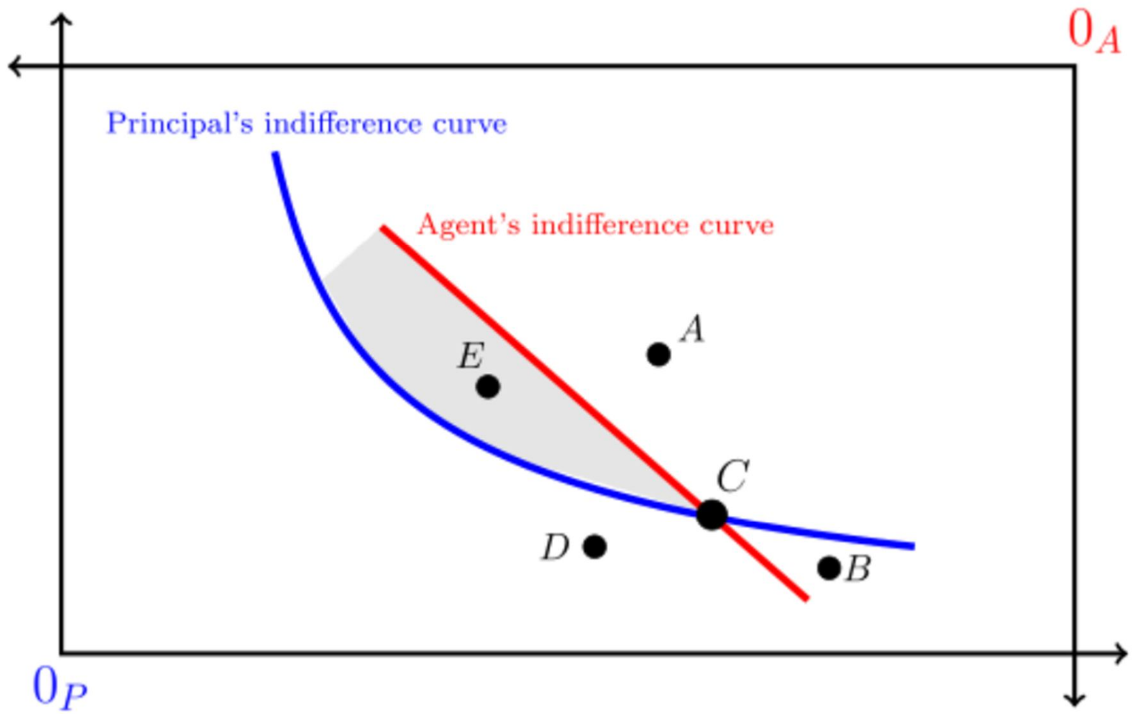
and

$$\mathbb{E}\left[U_A(B)\right] =$$

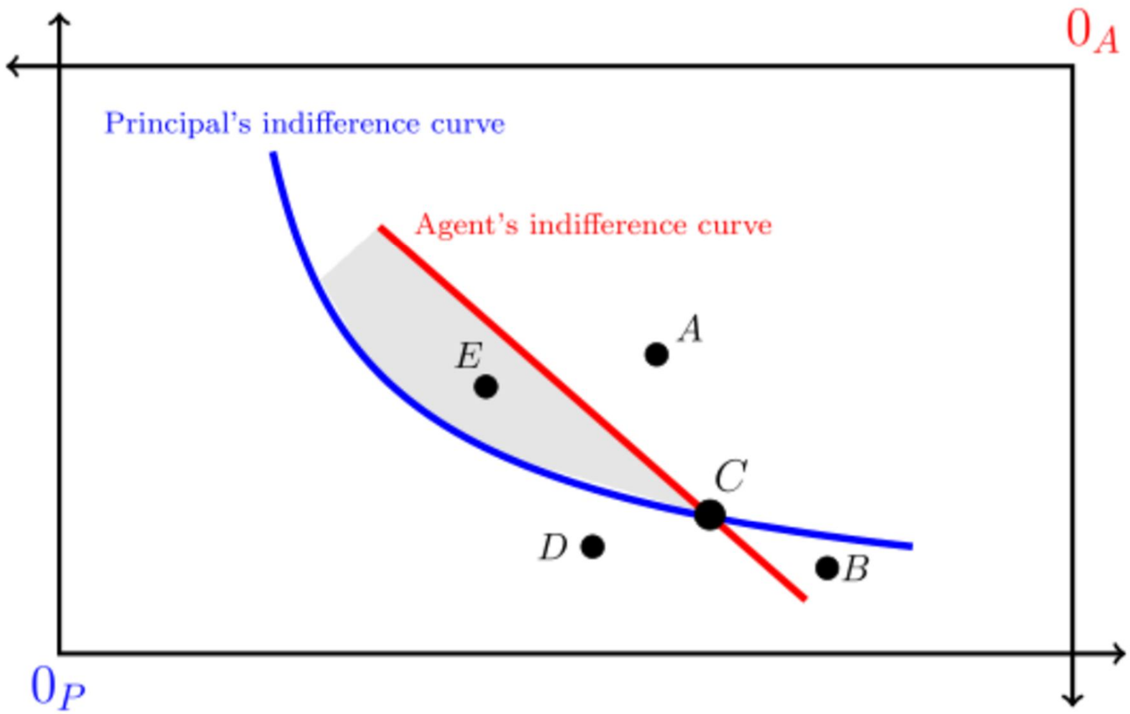
$$\mathbb{E}\left[U_A(C)\right] =$$



For the Principal:



For the Agent:



For the Principal: $C \succ_P D$ $C \succ_P B$ $E \succ_P C$ $A \succ_P C$ $A \succ_P E$

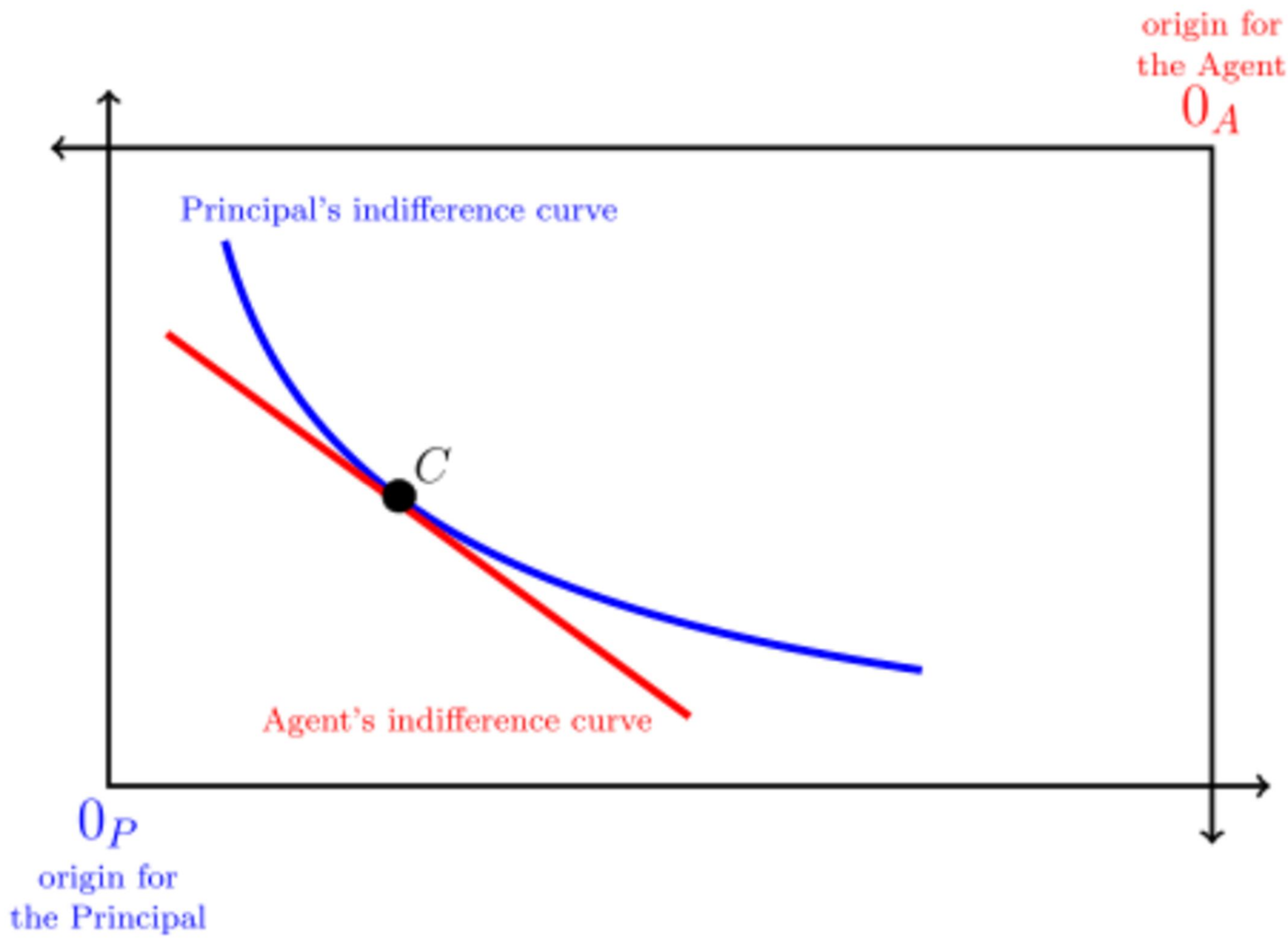
For the Agent: $C \succ_A A$ $B \succ_A A$ $C \succ_A B$ $E \succ_A C$ $D \succ_A C$ $D \succ_A E$

Thus *C* is Pareto dominated by *E* (or *E* Pareto dominates *C*). So *C* is not Pareto efficient.

Any contract *C* at which the indifference curves cross cannot be Pareto efficient, because any contract in the area between the two curves is Pareto superior to (or Pareto dominates) *C*.

Thus a contract C **in the interior of the box** is Pareto efficient if and only if the two indifference curves (of Principal and Agent) are **tangent** at C.

Example:



Pareto efficient risk sharing

We saw that a contract $C = \left(w_C^G, w_C^B\right)$ in the **interior** of the Edgeworth box ($0 < w^G < X^G$ and $0 < w^B < X^B$) is Pareto efficient if and only if the two indifference curves through C are tangent at that point.

- Slope of Principal's indifference curve at $C = \left(w_C^G, w_C^B\right)$:

- Slope of Agent's indifference curve at $C = \left(w_C^G, w_C^B\right)$:

Thus the two are equal if and only if

Case 1: Principal risk averse, Agent risk neutral

Agent's utility function can be taken to be

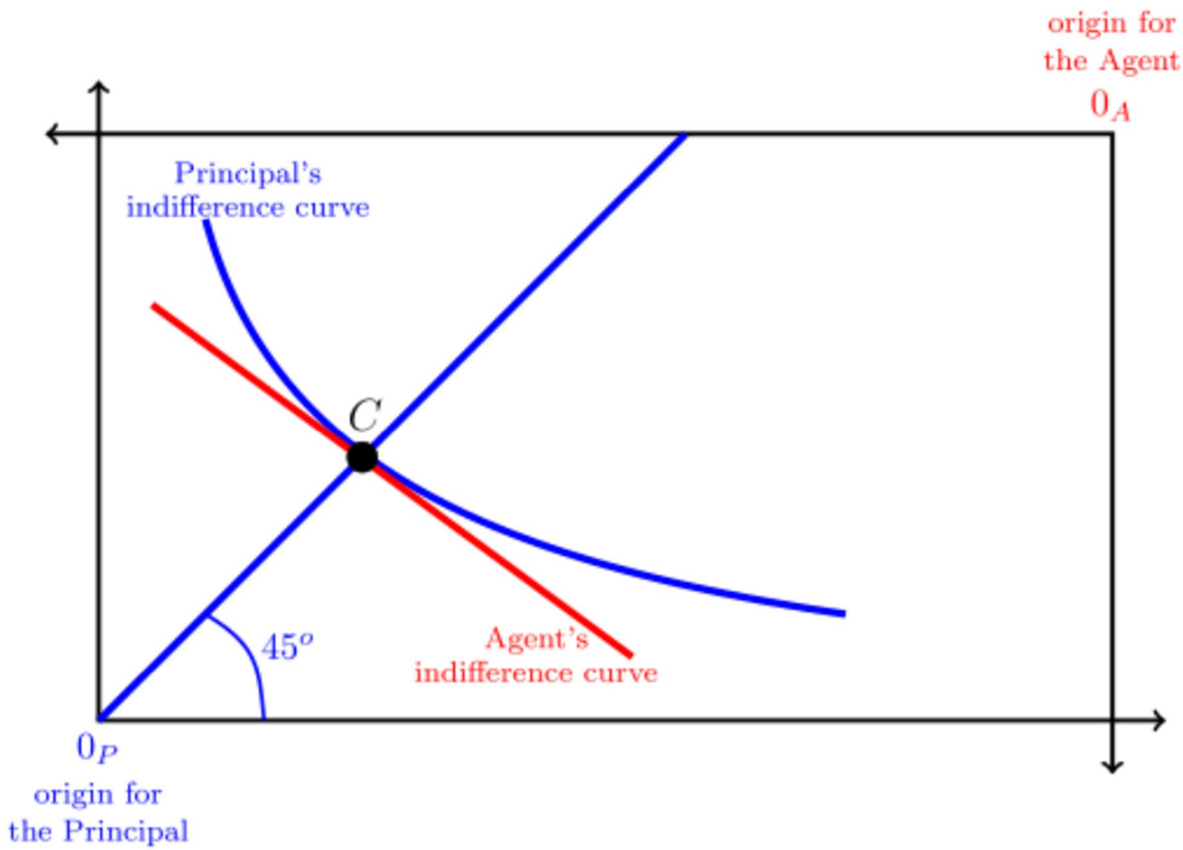
Hence the required equality $\frac{U_P'(X^G - w_C^G)}{U_P'(X^B - w_C^B)} = \frac{U_A'(w_C^G)}{U_A'(w_C^B)}$ reduces to

(*)

Since U_P is strictly concave,

Thus (*) is satisfied if and only if

That is, contract C must be on the 45° line out of the origin for the Principal:



Case 2: Principal risk neutral, Agent risk averse

Principal's utility function can be taken to be

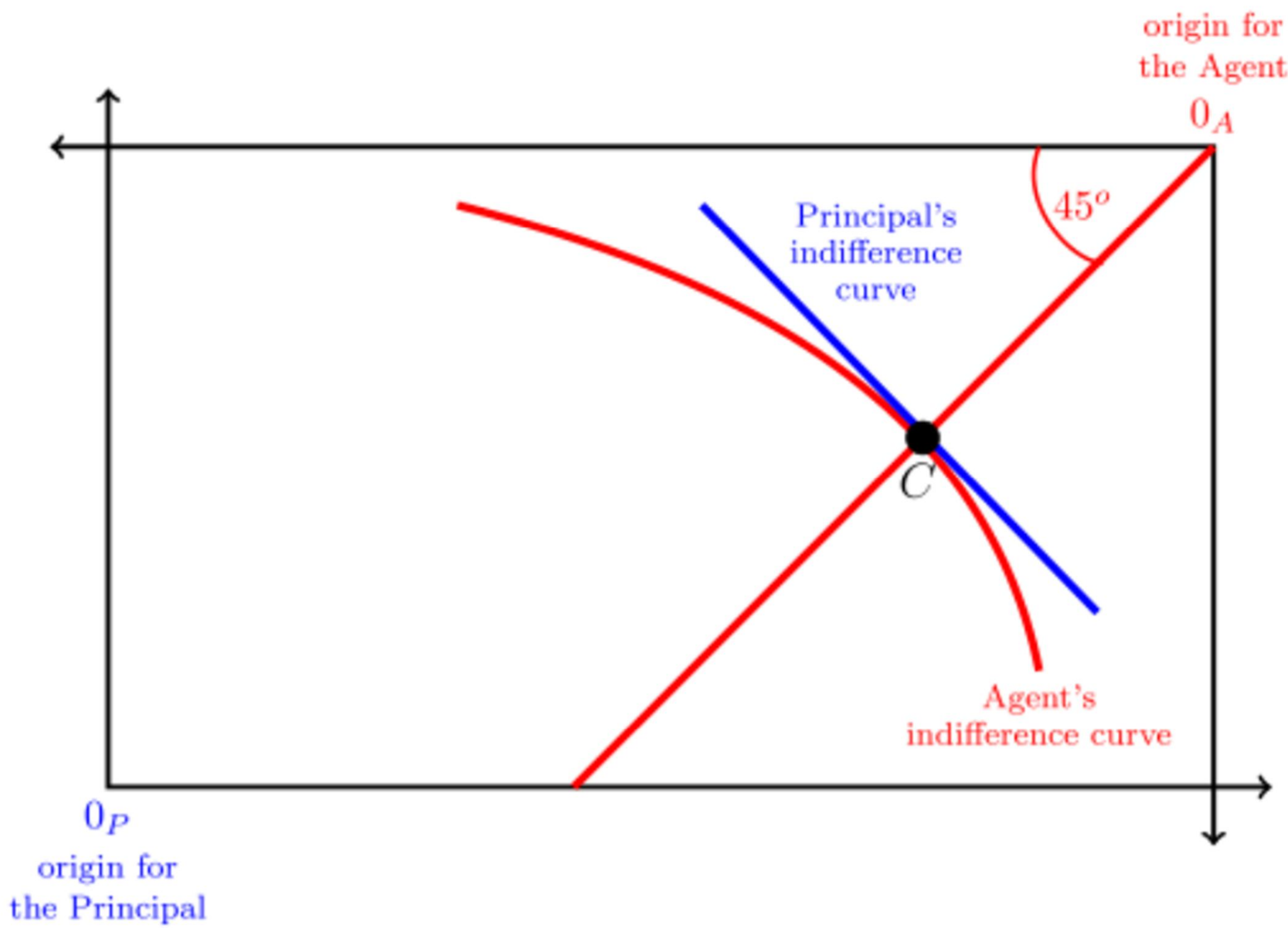
Hence the required equality $\frac{U_P'(X^G - w_C^G)}{U_P'(X^B - w_C^B)} = \frac{U_A'(w_C^G)}{U_A'(w_C^B)}$ reduces to

(*)

Since U_A is strictly concave,

Thus (*) is satisfied if and only if

Thus contract C must be on the 45^o line out of the origin for the Agent:



General principle: when one party is risk averse and the other is risk neutral, the risk-neutral party must bear all the risk (that is, the risk-averse party must be guaranteed a fixed level of wealth).

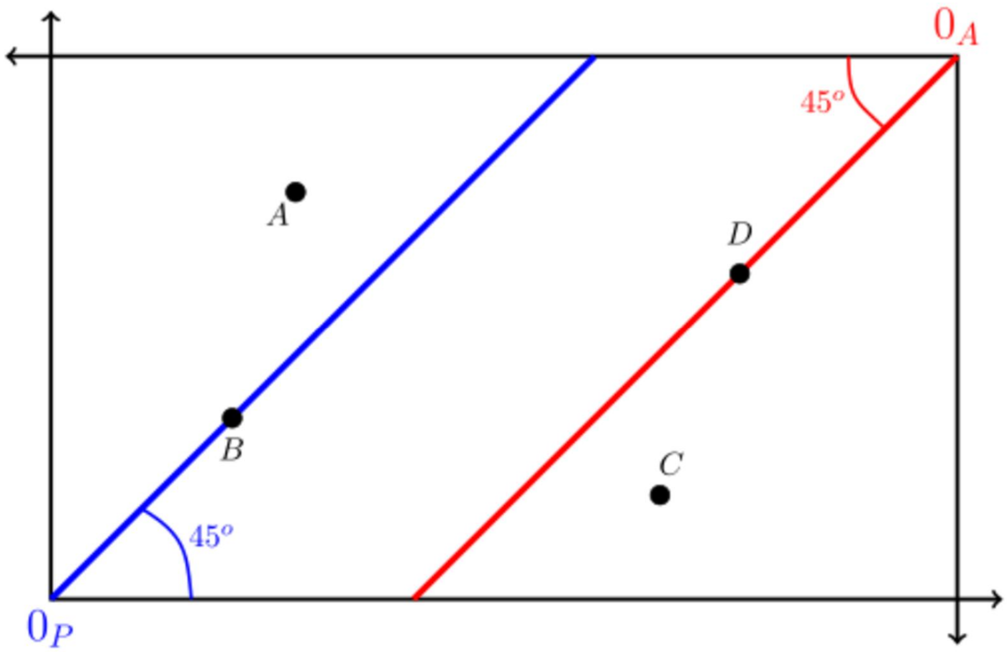
Case 3: both Principal and Agent risk averse

Recall from Week 4 (04A) that (IC = indifference curve)

- at a point **above** the 45^o line, **slope of IC is, in absolute value, greater than**

- at a point **on** the 45^o line, **slope of IC is, in absolute value, equal to**

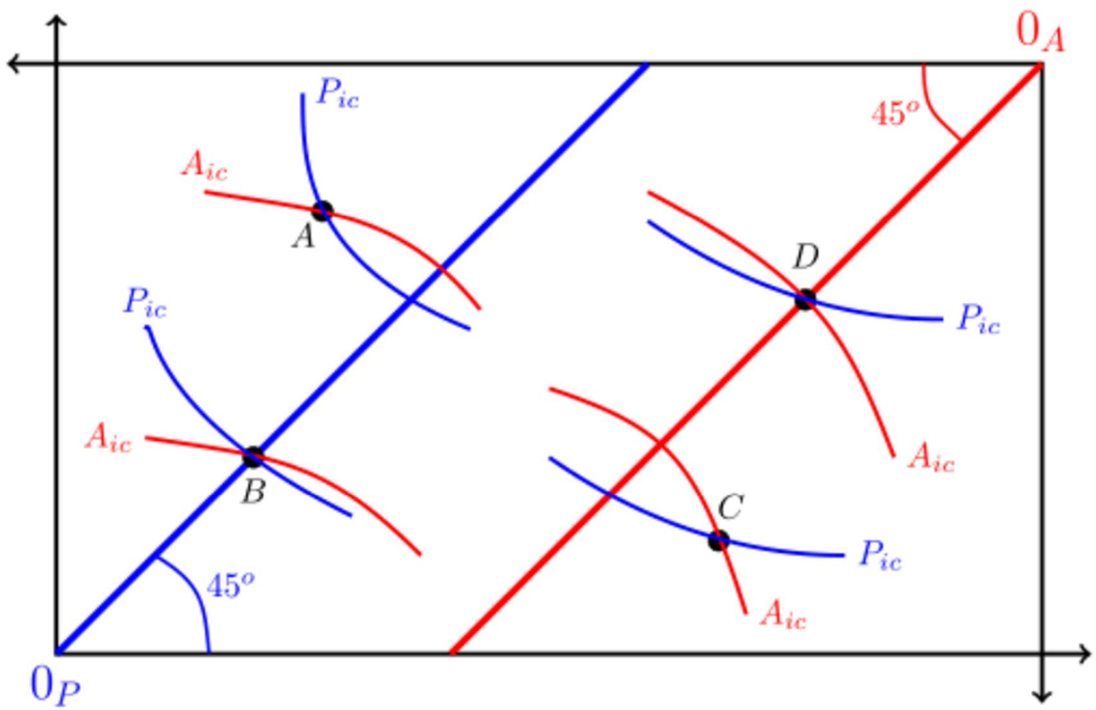
- at a point **below** the 45^o line, **slope of IC is, in absolute value, less than**



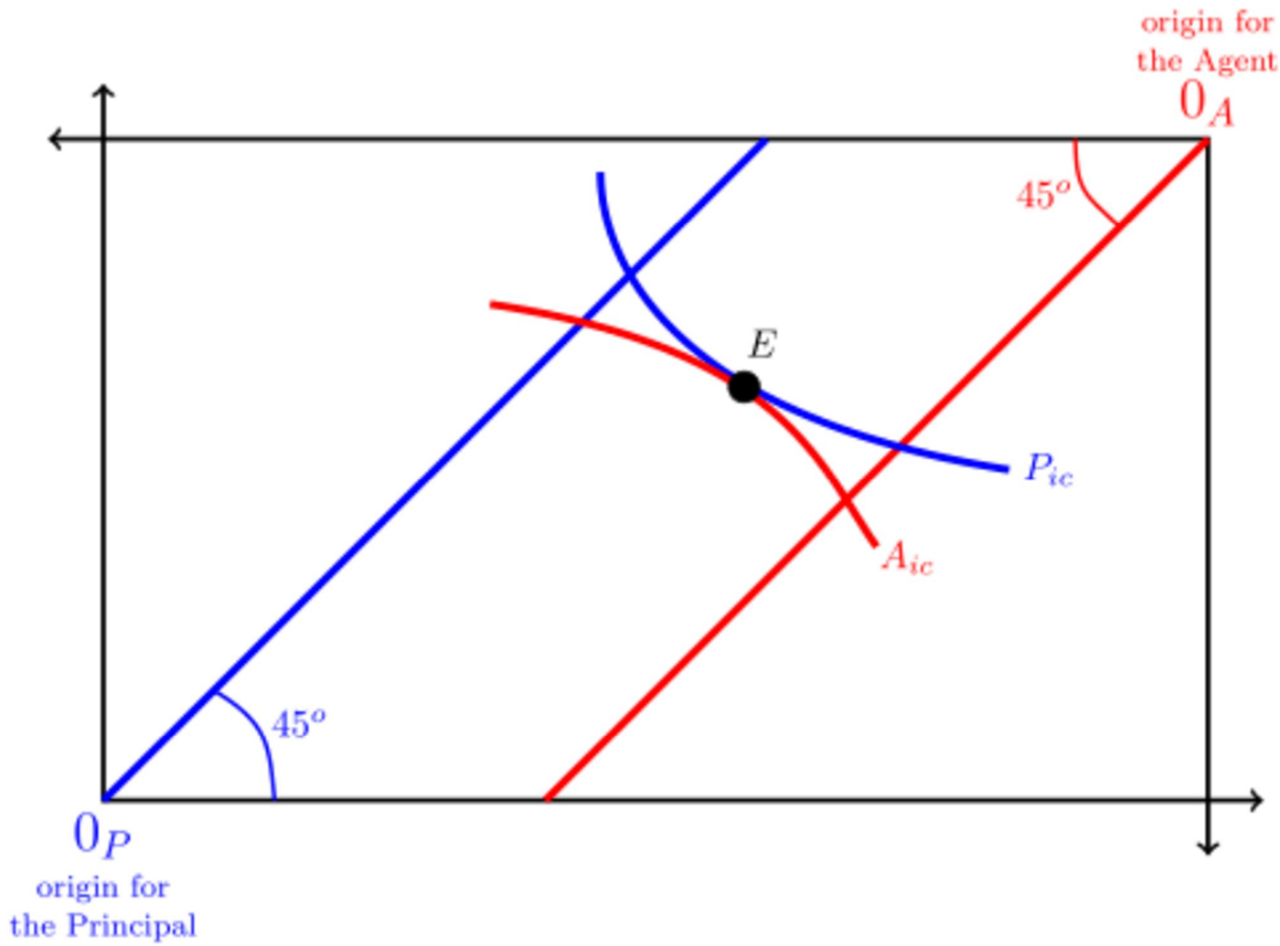
P-slope = absolute value of slope of Principal's indifference curve

A-slope = absolute value of slope of Agent's indifference curve

- At point *A*,
- At point *B*,
- At point *D*,
- At point *C*,



Thus the tangency can occur only at points **between the two 45º lines.** Hence **both individuals must bear some of the risk.**


origin for
the Agent
0_A
45^o
E
P_{ic}
A_{ic}
45^o
0_P
origin for
the Principal

Example. $U_P(m) = \sqrt{m}$ and $U_A(m) = 82 - \left(10 - \frac{m}{100}\right)^2 - 1$. Let $X^G = 800$ and $X^B = 200$. Consider the contract **($w^G = 400$, $w^B = 100$)**. Is it Pareto efficient? We have to check if equality of the slopes holds.

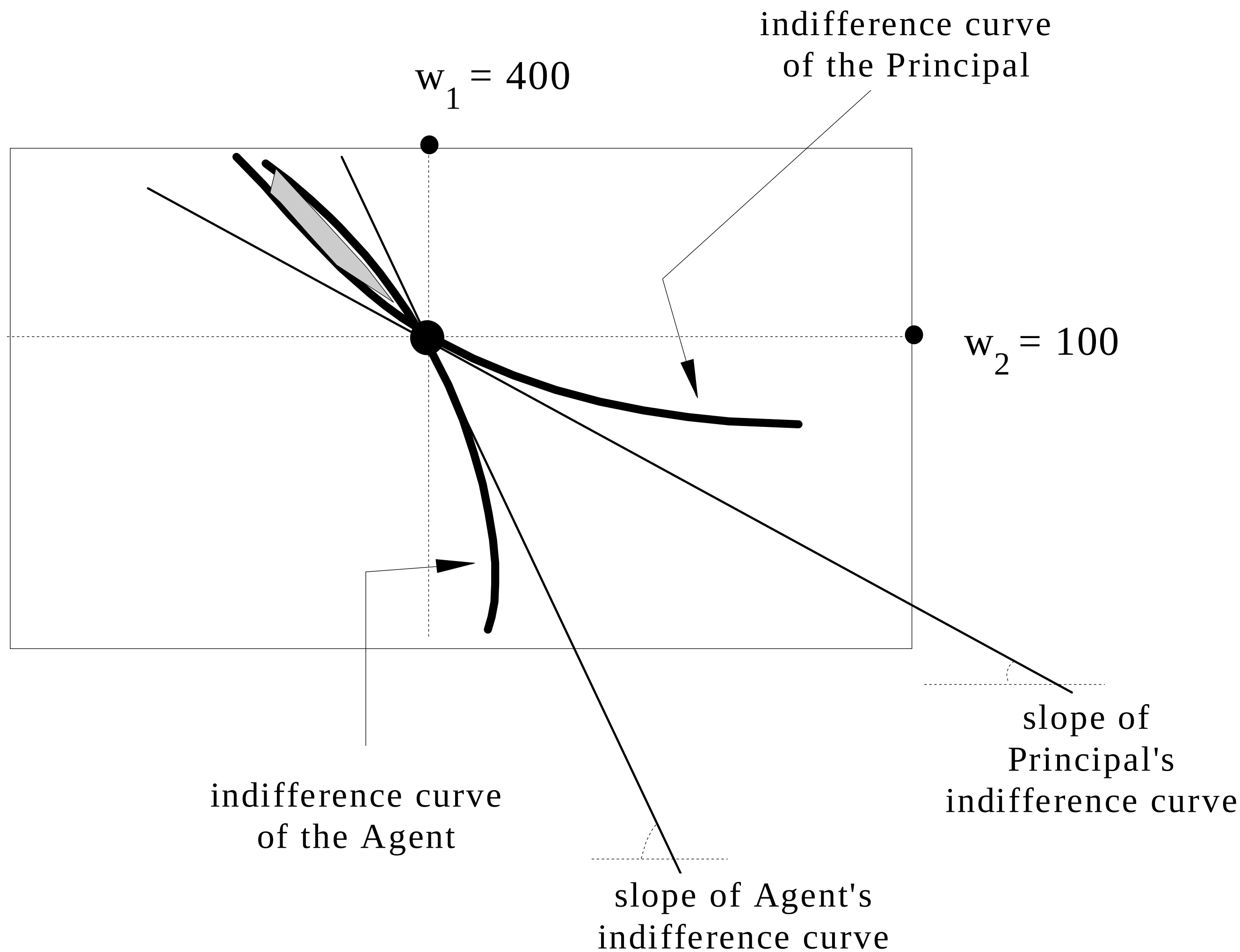
$U'_P(m) =$

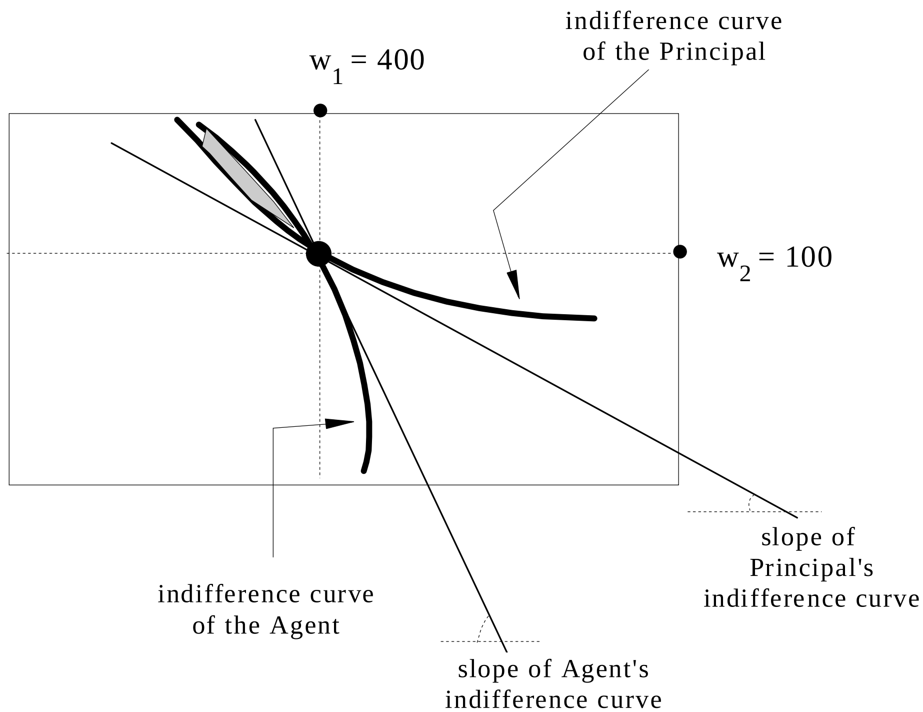
$$U'_A(m) =$$

Principal: $$\frac{U'_P(X^G - w^G)}{U'_P(X^B - w^B)}$$

Agent:

The indifference curve of the Principal is less steep than the indifference curve of the Agent at point (w^G= 400, w^B= 100): see the following figure





Let S = (w^G = 400, w^B = 100) be the contract under consideration and let T = (w^B = 401, w^B = 99.7). Then

$\mathbb{E}\left[U_P(S)\right] =$

$\mathbb{E}\left[U_P(T)\right] =$

$\mathbb{E}\left[U_A(S)\right] =$

$\mathbb{E}\left[U_A(T)\right] =$

Case 4: both Principal and Agent risk neutral