2. A continuum of contracts

Again $W = 900,\ L = 700,\quad p = \frac{1}{50}\ ,\quad U(m) = \sqrt{m}\ ,\quad B = \left(h_B = 60,\ d_B = 100\right)$.

The profit from contract B is

$\pi(B) =$

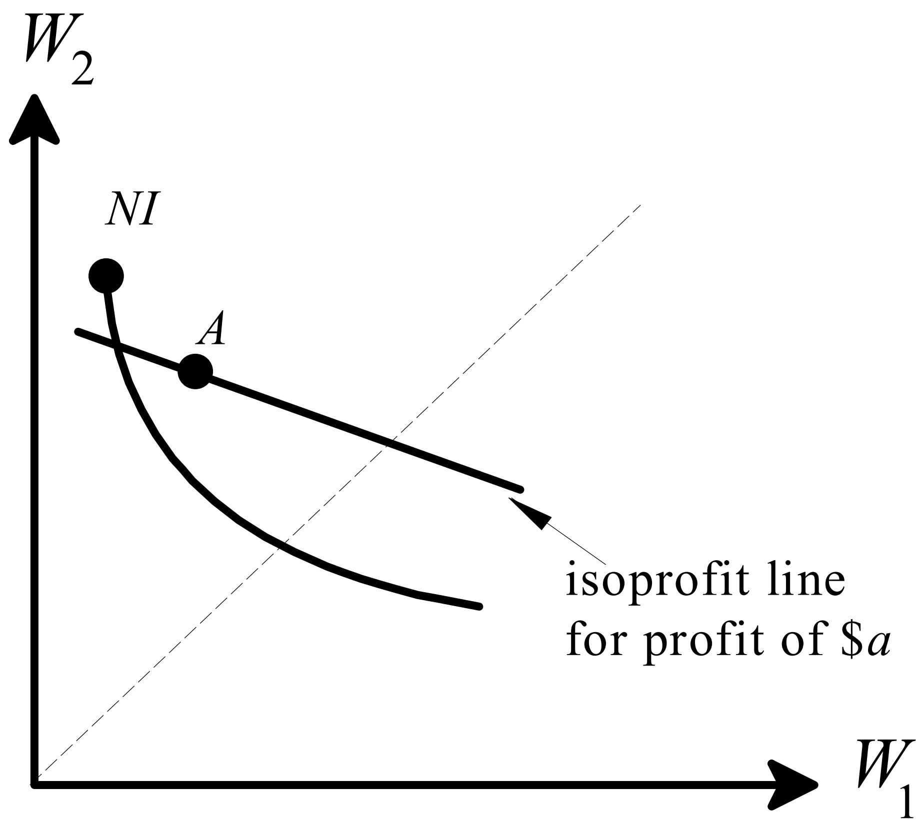
Suppose that the insurance company tells the consumer that she can choose any other contract that guarantees a profit of $48 to the insurer,

$h(d) =$

Examples: $h(50) = 61$, $h(100) = 60$ (this is contract B), $h(150) = 59$, $h(200) = 58$

Will the consumer still choose contract $B = \left(h_B = 60,\ d_B = 100\right)$?

By the familiar slope argument…



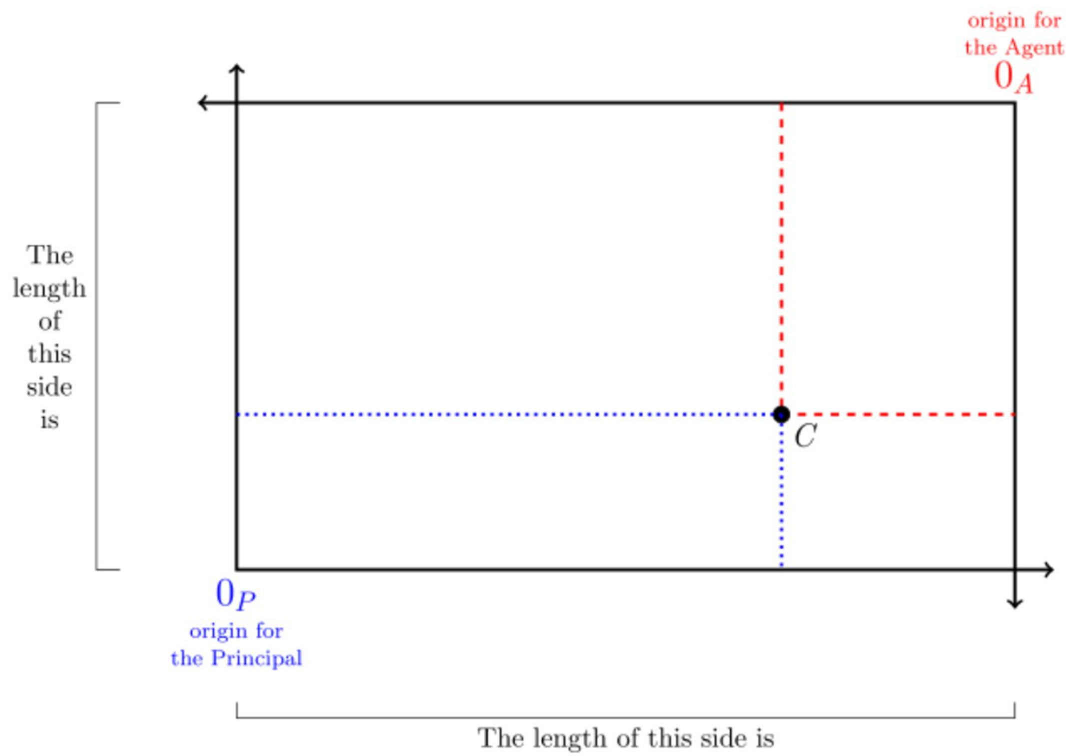
Contractual relationships between two individuals: Principal and Agent. Examples:

Principal	**Agent**	**Contract**
Owner of firm	Manager	Division of profits
Client	Lawyer	Lawyer's fee
Land-owner	Farmer	Division of crop
Patient	Doctor	Doctor's fee

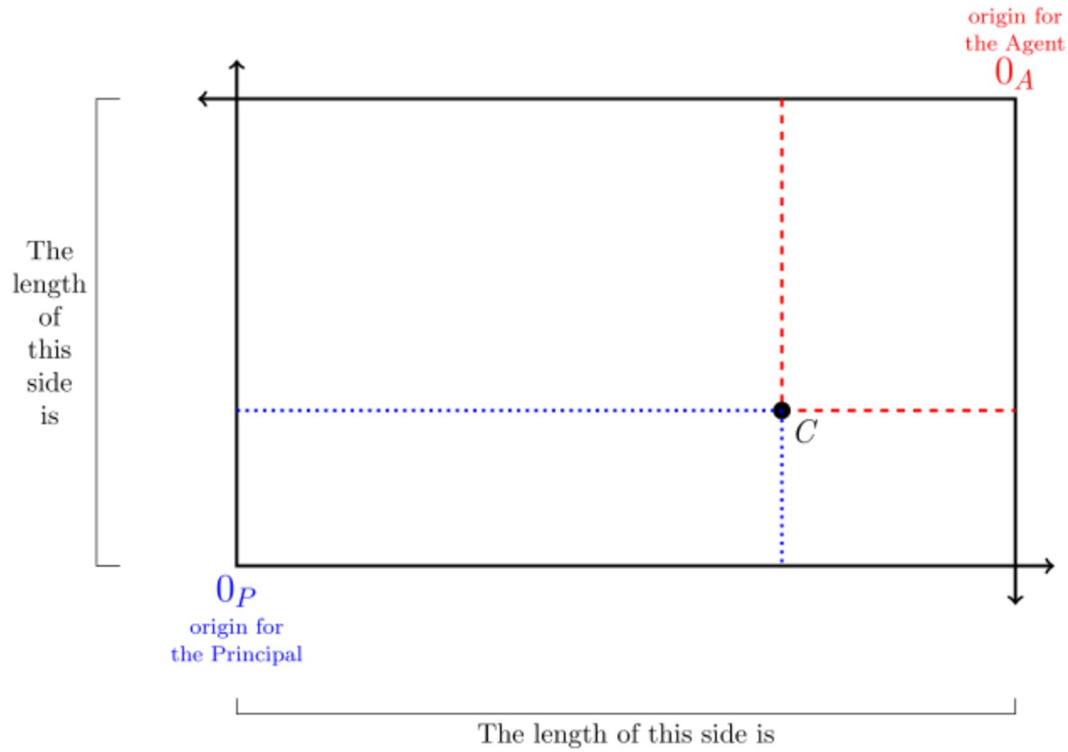
Assume that neither individual has any additional wealth to draw from.
The outcome of the relationship is uncertain:

A contract is specified as a pair (w^G, w^B)

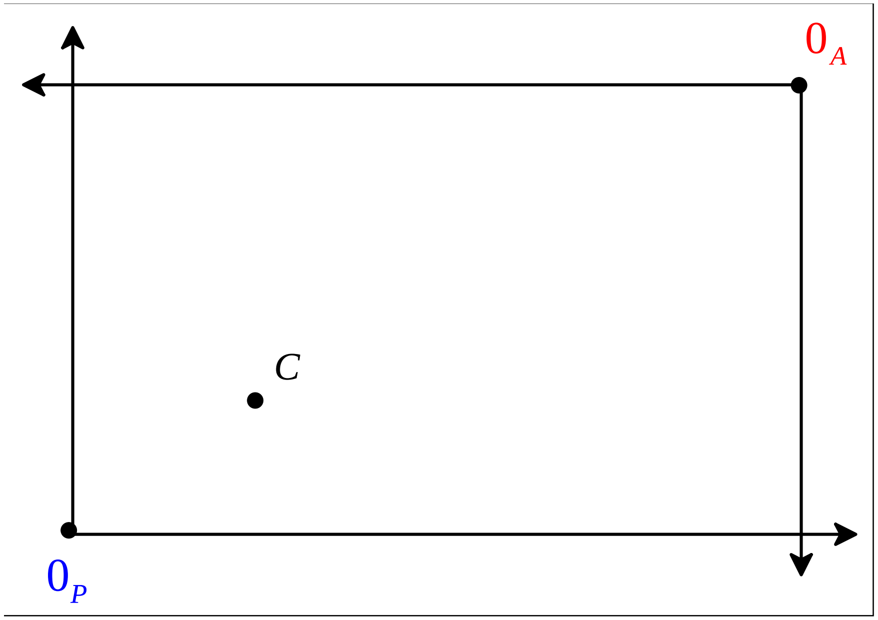
The set of possible contracts can be represented graphically by means of an Edgeworth box

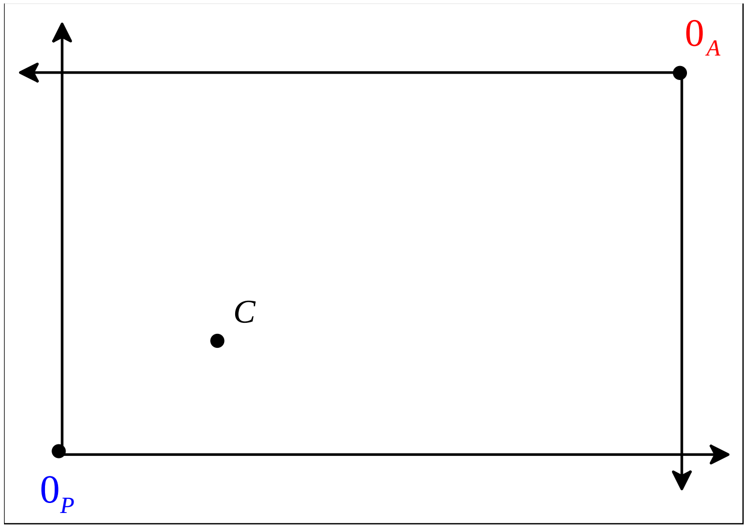


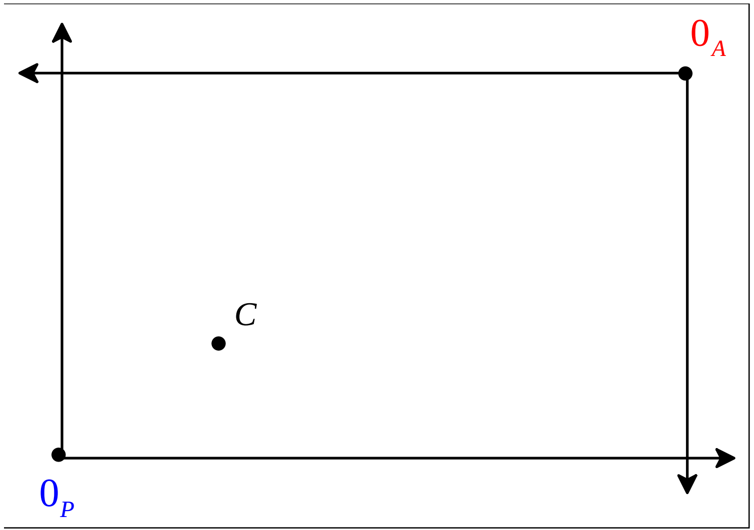
Example: $X^G = \$800,\ \ X^B = \$500,\ \ C = (w^G = 300, w^B = 400)$



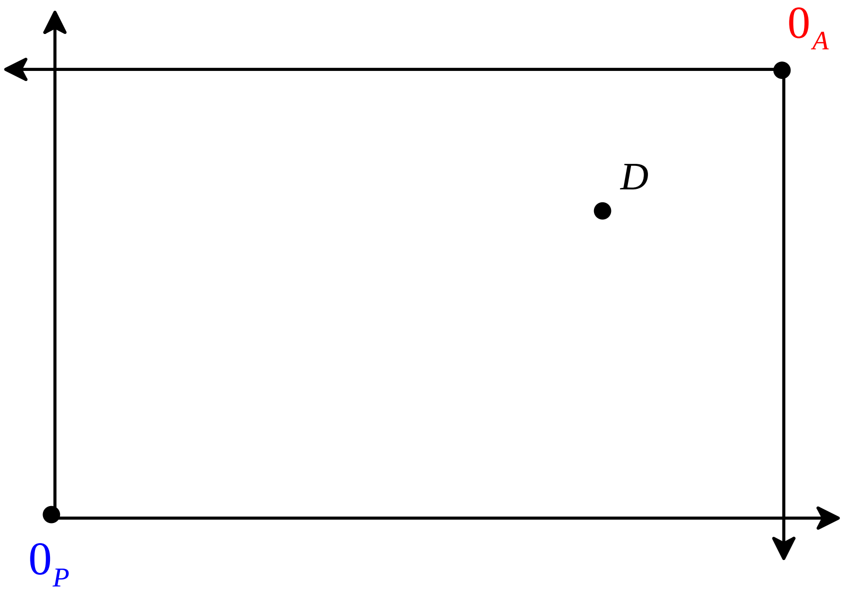
INDIFFERENCE CURVES Start with the Principal

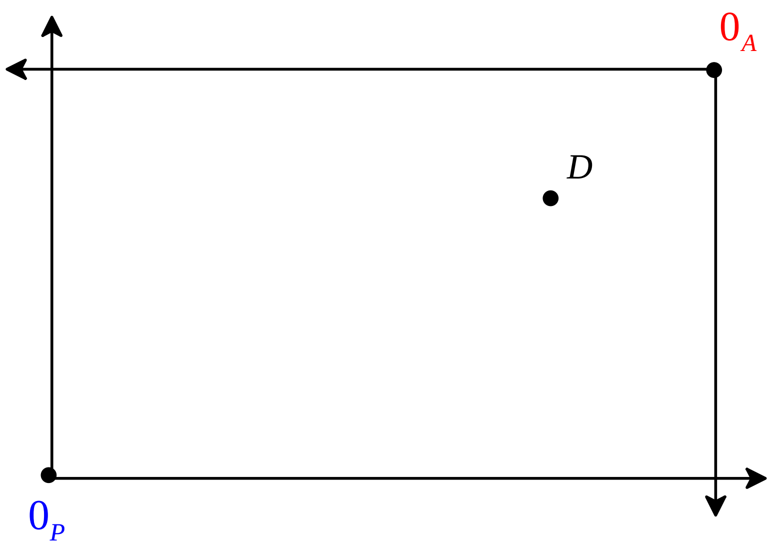


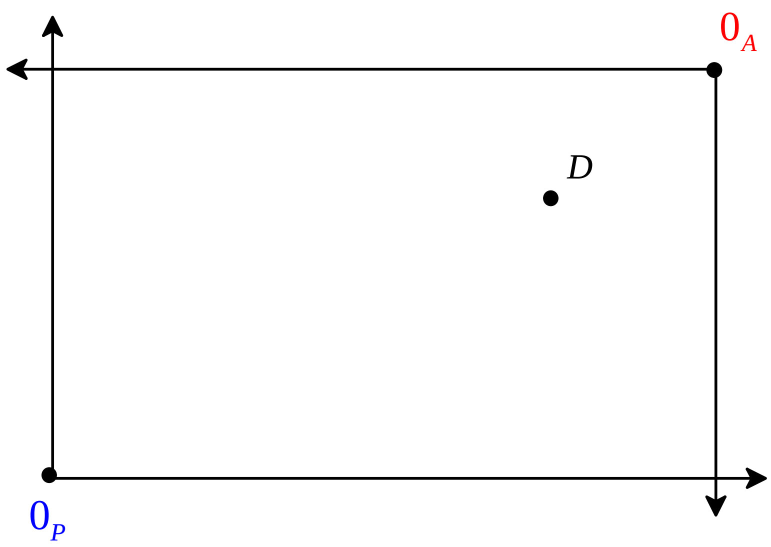




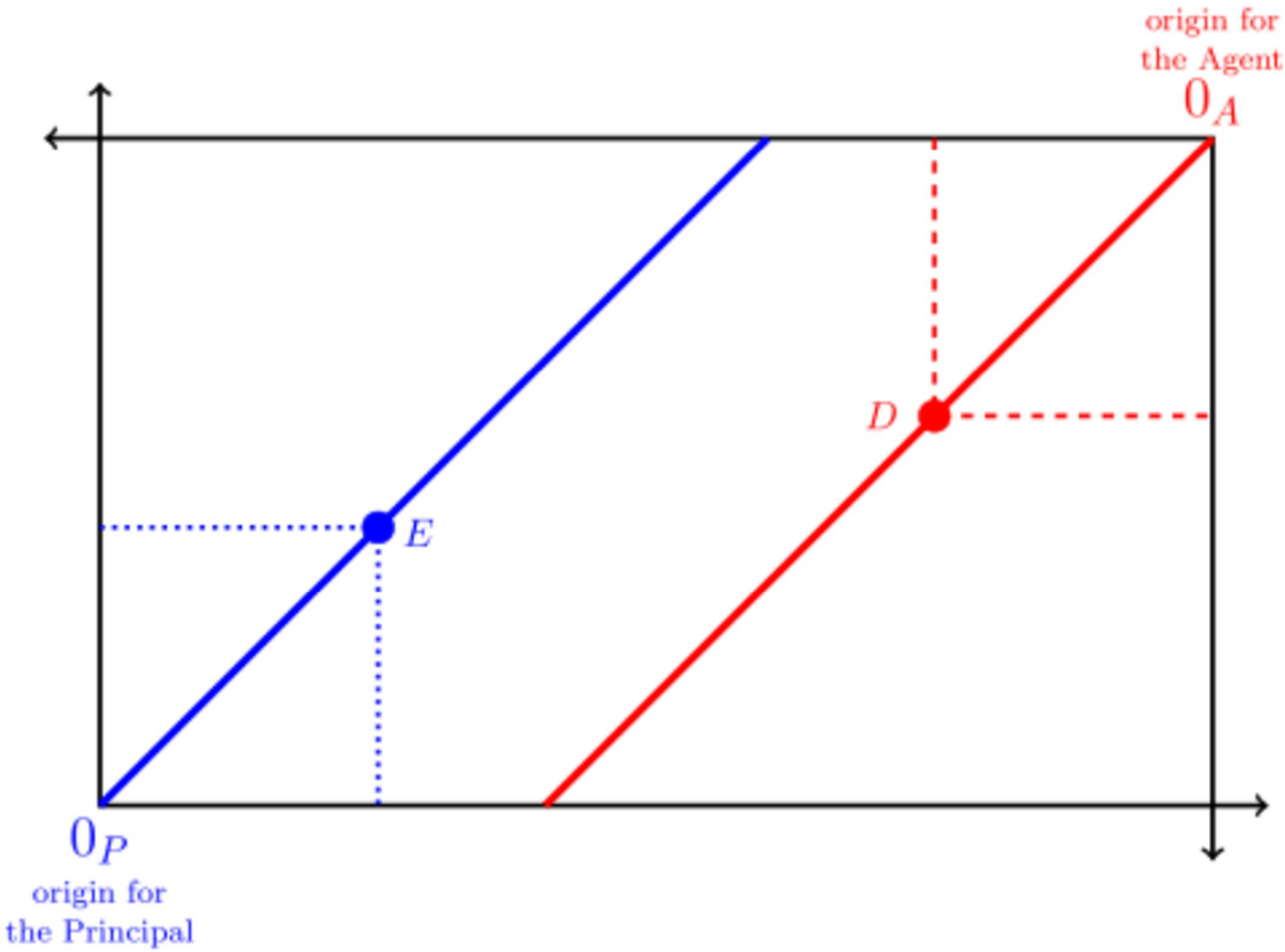
INDIFFERENCE CURVES now the Agent



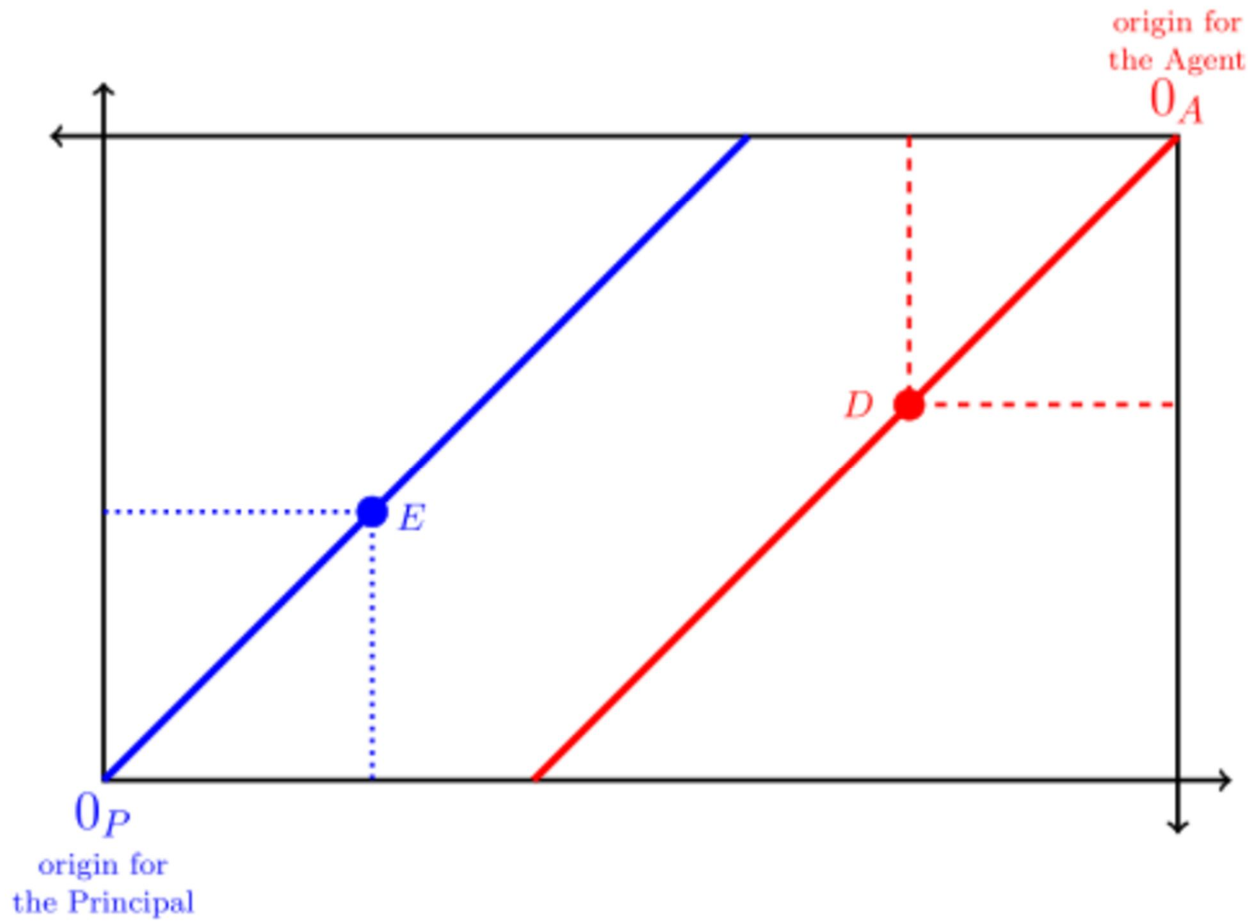




The 45° lines



Example: $X^G = \$800$, $X^B = \$500$, $D = (w^G = 150, w^B = 150)$, $E = (w^G = 600, w^B = 300)$



Recall: a contract is a pair $\left(w^G, w^B\right)$ where w^G is the payment to the Agent if the outcome is X^G and w^B is the payment to the Agent if the outcome is X^B.

$U_P(m)$ Principal's utility function

$U_A(m)$ Agent's utility function.

Given a contract C $=\left(w^G, w^B\right)$, the Principal's expected utility is:

$$\mathbb{E}\left[U_P(C)\right]=$$

while the Agent's expected utility is:

$$\mathbb{E}\left[U_A(C)\right]=$$

We want to characterize the set of **Pareto efficient contracts**.

Definition. Contract ***C*** **is *Pareto dominated* by contract *B*** if:

either

that is,

or

that is,

Definition. A contract that is not Pareto dominated is called ***Pareto efficient*** **(or Pareto optimal)**. Thus contract C is Pareto efficient if for every other contract D, either

or

or both.

Example. $X^G = 1{,}000$, $X^B = 600$, $p = \frac{1}{3}$ $U_P(\mathrm{m}) = \sqrt{\mathrm{m}}$ and $U_A(m) = m$.

C = (400, 400) is Pareto dominated by contract B = (676, 276):

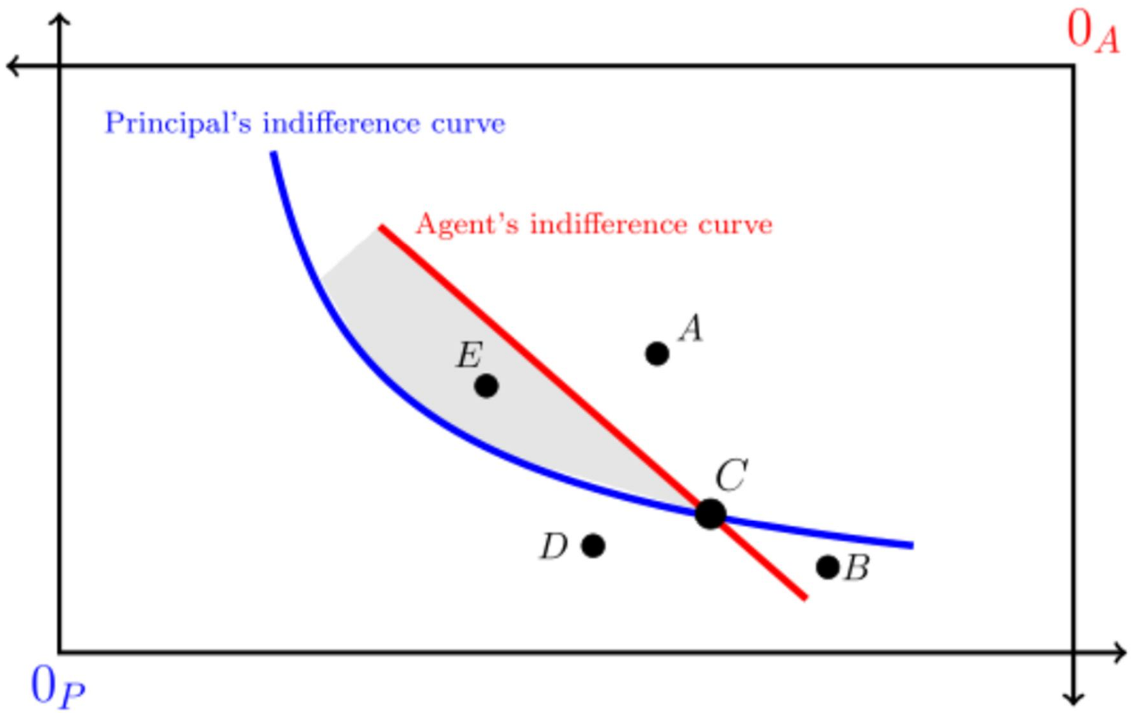
$$\mathbb{E}\left[U_P(B)\right] =$$

$$\mathbb{E}\left[U_P(C)\right] =$$

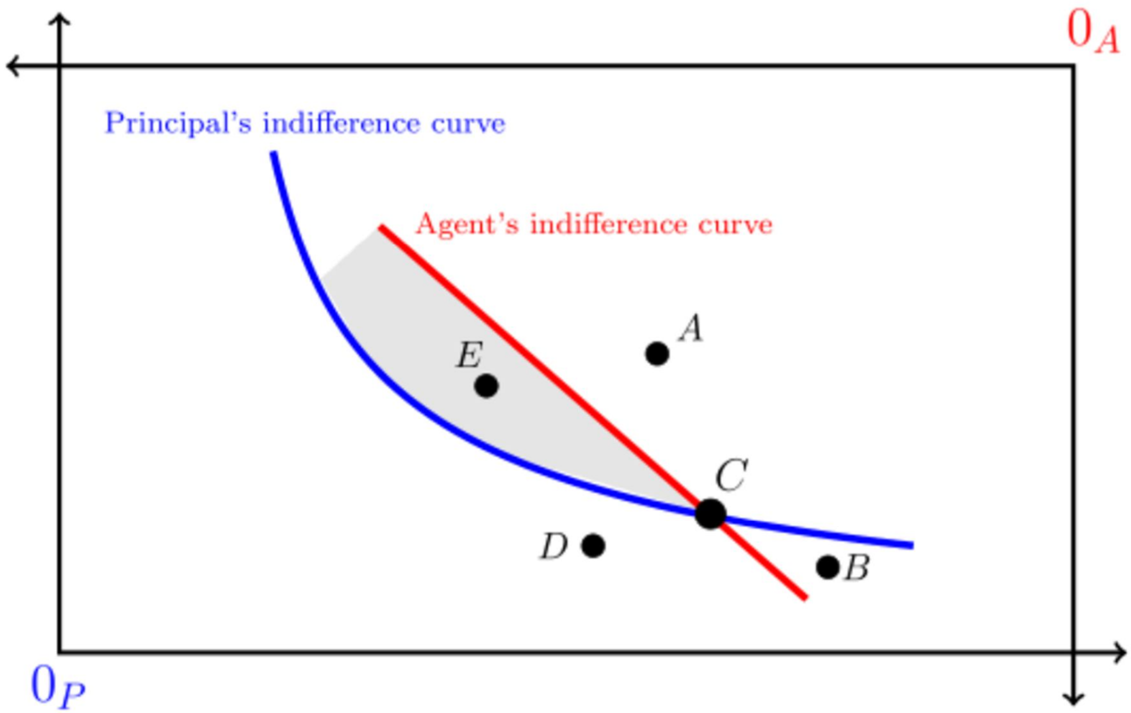
and

$$\mathbb{E}\left[U_A(B)\right] =$$

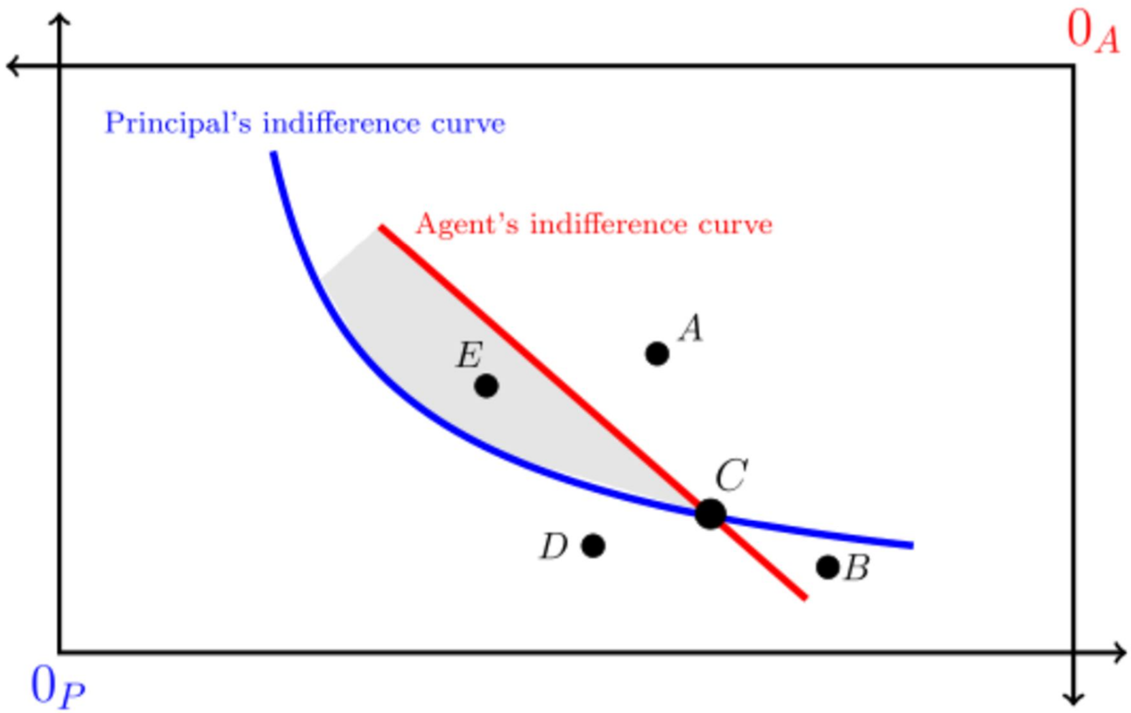
$$\mathbb{E}\left[U_A(C)\right] =$$



For the Principal:



For the Agent:



For the Principal: $C \succ_P D$ $\quad C \succ_P B$ $\quad E \succ_P C$ $\quad A \succ_P C$ $\quad A \succ_P E$

For the Agent: $C \succ_A A$ $\quad B \succ_A A$ $\quad C \succ_A B$ $\quad E \succ_A C$ $\quad D \succ_A C$ $\quad D \succ_A E$

Thus C is Pareto dominated by E (or E Pareto dominates C). So C is not Pareto efficient.

Any contract C at which the indifference curves cross cannot be Pareto efficient, because any contract in the area between the two curves is Pareto superior to (or Pareto dominates) C.

Thus a contract C **in the interior of the box** is Pareto efficient if and only if the two indifference curves (of Principal and Agent) are **tangent** at C.

Example: