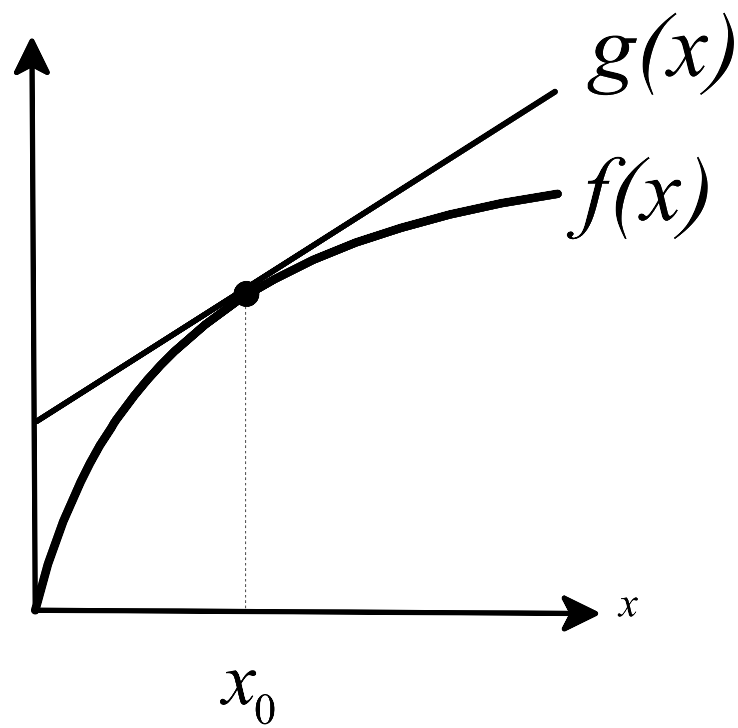
Slope of an indifference curve

Preliminaries on the meaning of the derivative.

$f(x) = \sqrt{x} + \frac{x^2}{3}$. Then $f'(x) =$ The derivative is used to

construct a linear function to approximate the function $f(x)$ at a point x_0:



$f(x) = \sqrt{x} + \frac{x^2}{3}$. Then $f'(x) = \frac{1}{2\sqrt{x}} + \frac{2x}{3}$. Let $x_0 = 9$.

$f(9) =$ and $f'(9) =$

so that $g(x) =$ Let's see how well g approximates f

Take $x = 9.1$. Then $f(9.1) =$ $g(9.1) =$

Take $x = 12$. Then $f(12) =$ $g(12) =$

END OF PRELIMINARIES

Slope of indifference curve

Let A and B be two points that lie on the same indifference curve: $\mathbb{E}[U(A)] = \mathbb{E}[U(B)]$,

(*)

- Since x_B is close to x_A, $U(x_B) \simeq$

- Since y_B is close to y_A, $U(y_B) \simeq$

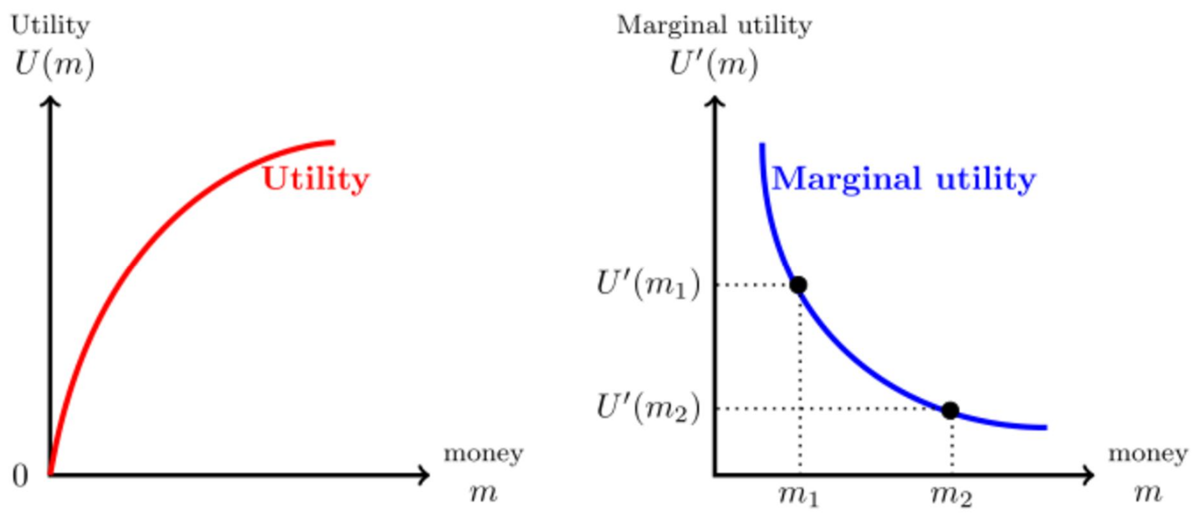
Thus the RHS of (*) can be written as

So (*) becomes

that is, which can be written as

Comparing the slope at a point with the ratio $\frac{p}{1-p}$

Look at the case of risk aversion but the other cases are similar.



- at a point **above** the 45° line, where $x < y$,

- at a point **on** the 45° line, where $x = y$,

- at a point **below** the 45° line, where $x > y$,

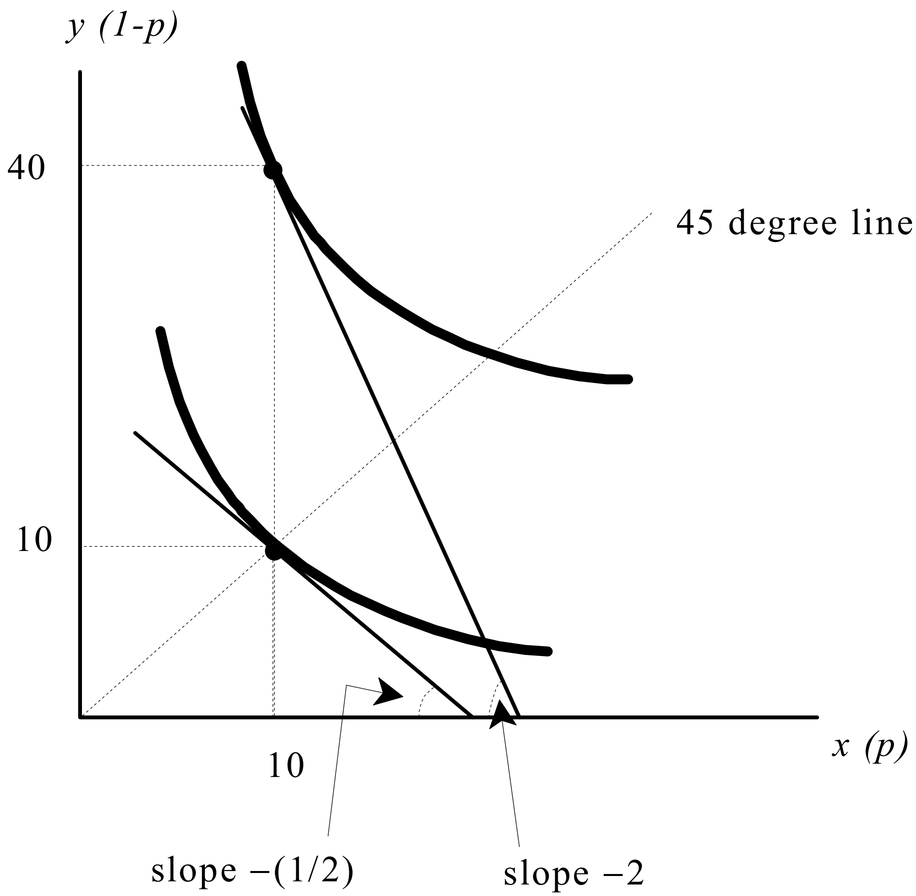
Example. $U(m) = \ln(m)$, $p = \frac{1}{3}$. What is the slope of the indifference curve at points $A = (10,40)$ and $B = (10,10)$?

The expected utility of lottery $A = \begin{pmatrix} 10 & 40 \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$ is

The slope of the indifference curve at point A is equal to .

The expected utility of lottery $B = \begin{pmatrix} 10 & 10 \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$ is

The slope of the indifference curve at point B is equal to .



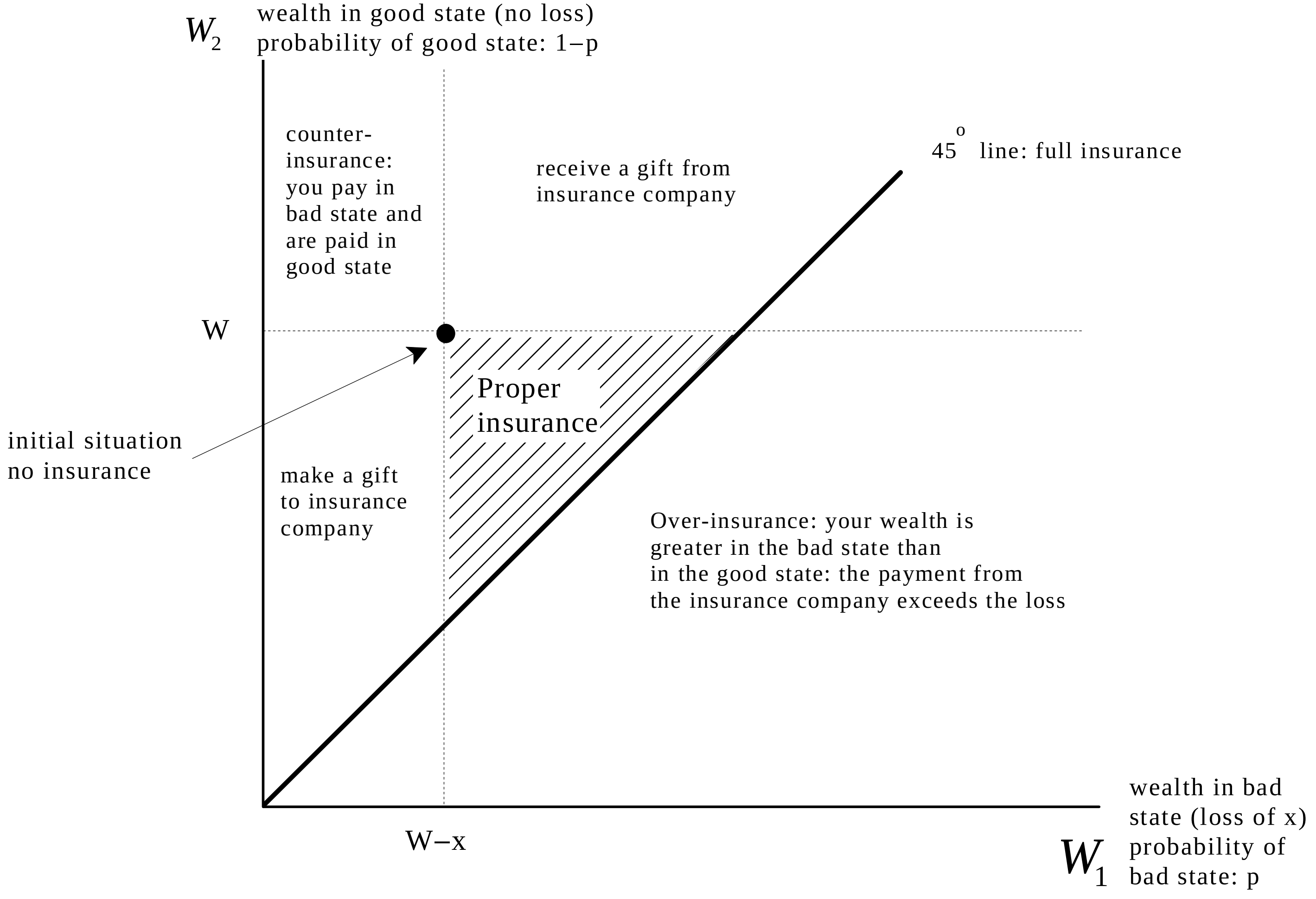
$A = (10, 40)$

$B = (10, 10)$

$p = \frac{1}{3}$

Thus $-\frac{p}{1-p} = -\frac{1}{2}$

DEMAND SIDE OF INSURANCE



$W = 40{,}000$, $L = 5{,}000$, probability of loss $p = \frac{1}{50}$, $U(m) = \ln(m)$

$NI =$ $\mathbb{E}[U(NI)] =$

Suppose the consumer is offered $A = (h_A = 200,\ d_A = 0)$ which would yield a profit of

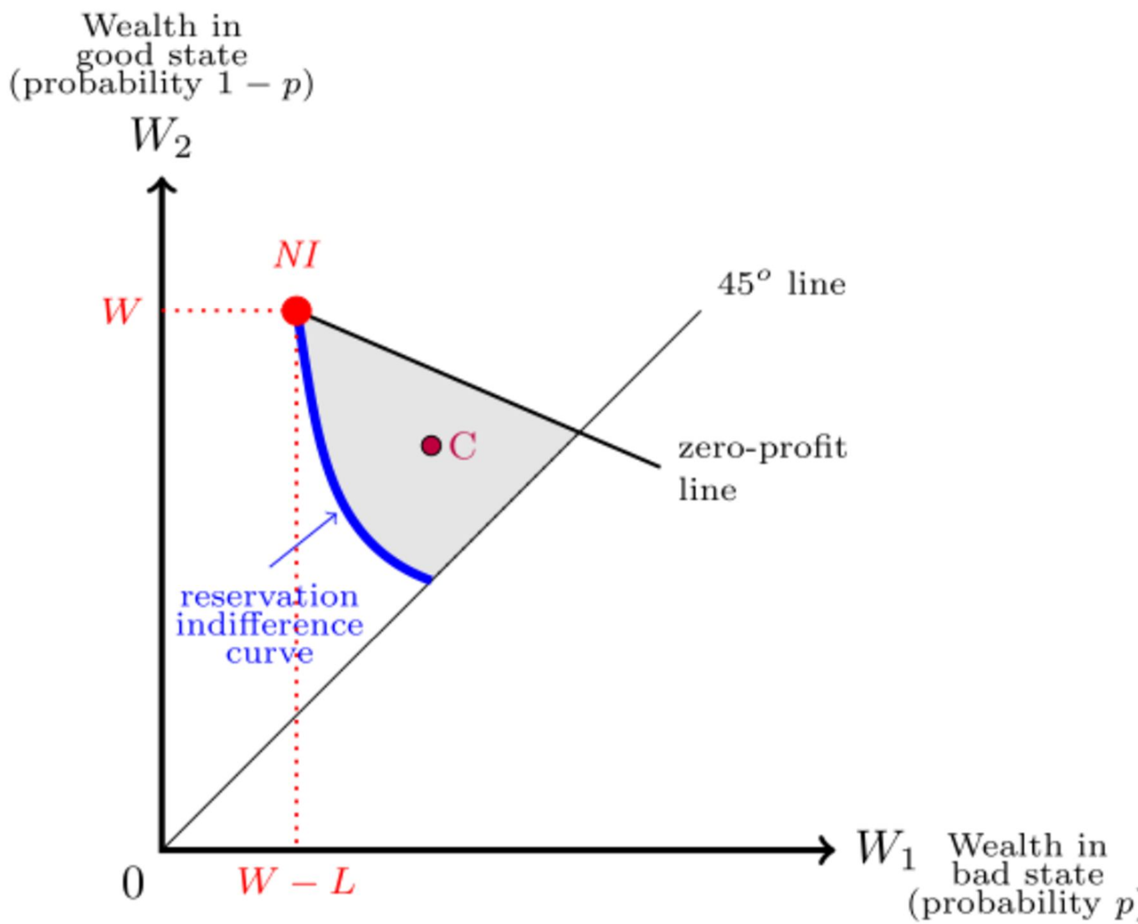
$\pi_A =$ Would she purchase it?

Suppose the consumer is offered $B = (h_B = 50,\ d_B = 100)$.

$\mathbb{E}[U(B)] =$

$\pi_B =$

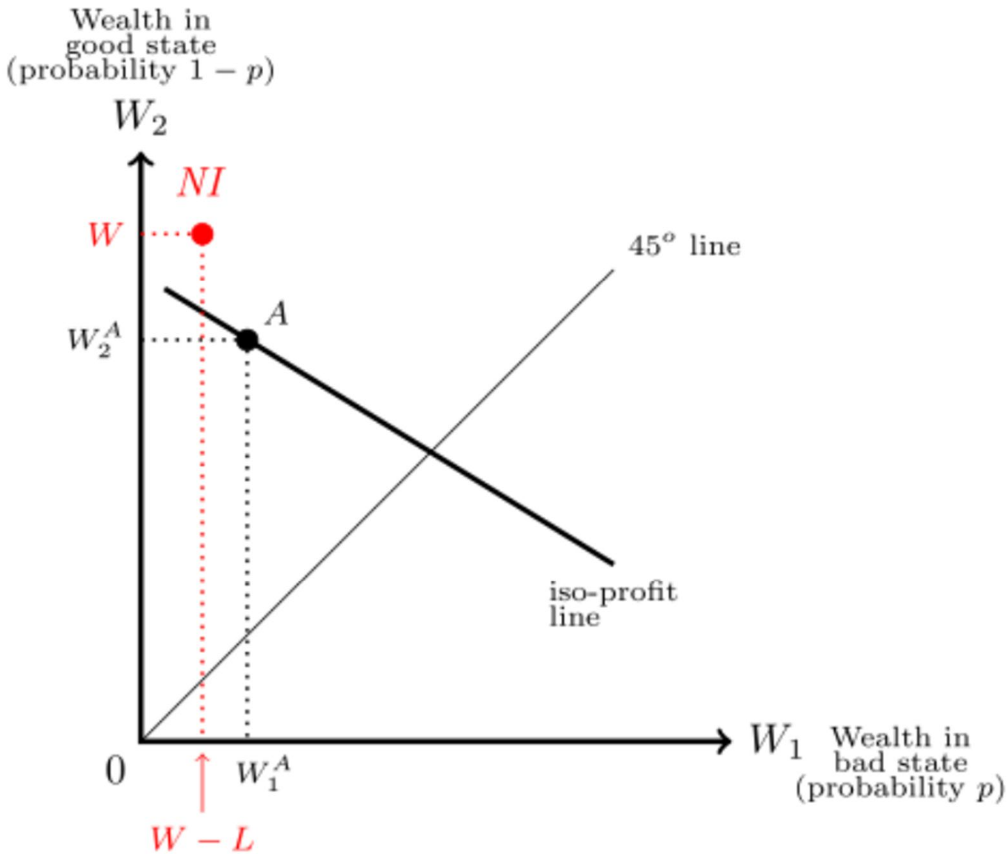
So we must exclude points that are below the indifference curve that goes through NI, called the **reservation indifference curve,** and exclude all those that are above the zero-profit line. The only observable contracts are:



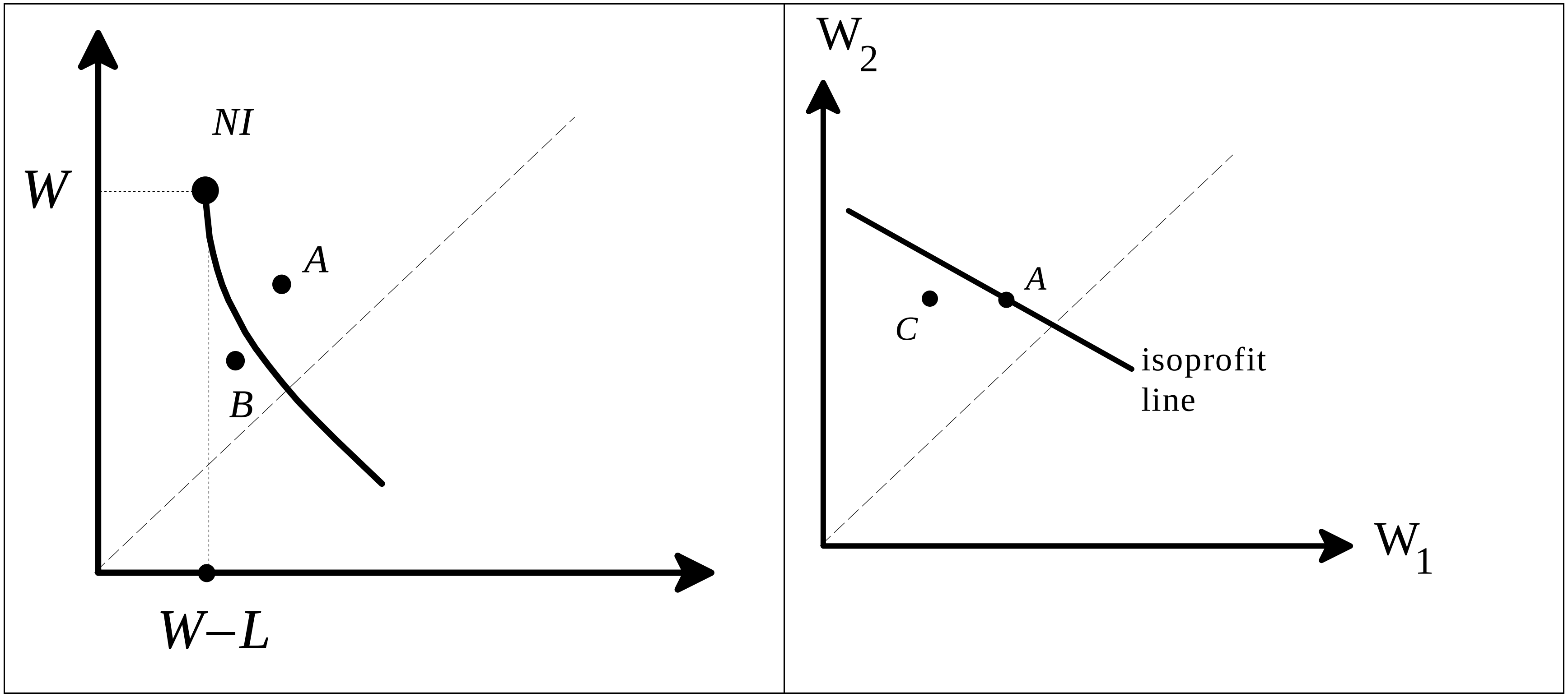
Reminder:

The absolute value of the slope of the indifference curve that goes through point $A = \left(W_1^A, W_2^A\right)$ is

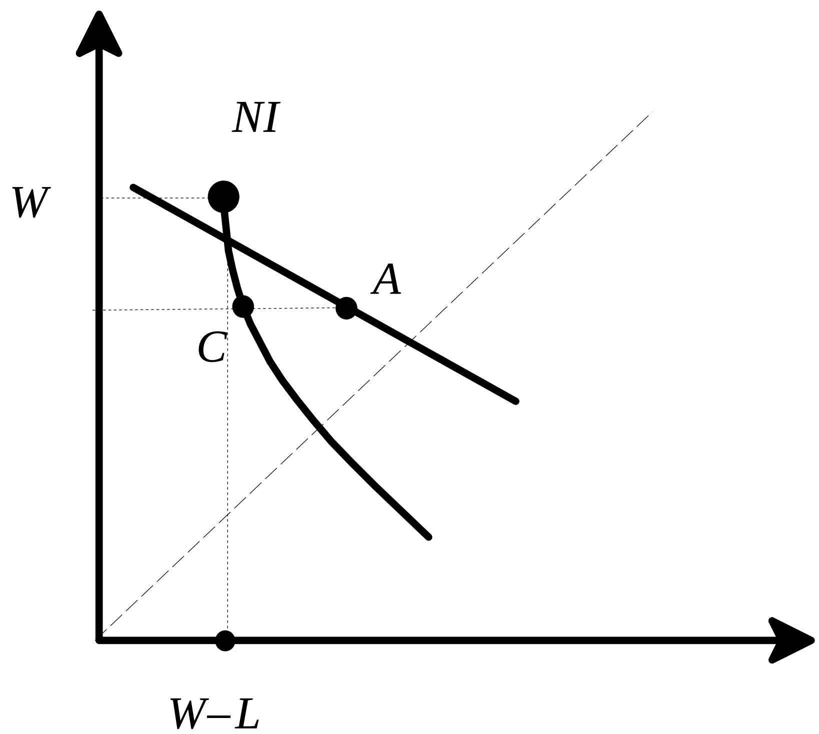
$$\frac{p}{1-p}\frac{U'(W_1^A)}{U'(W_2^A)}$$



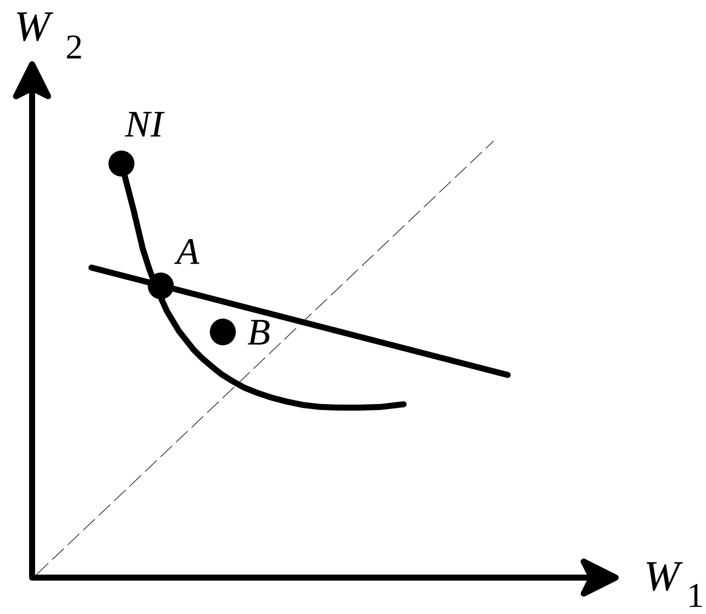
1. Suppose the insurance industry is a monopoly



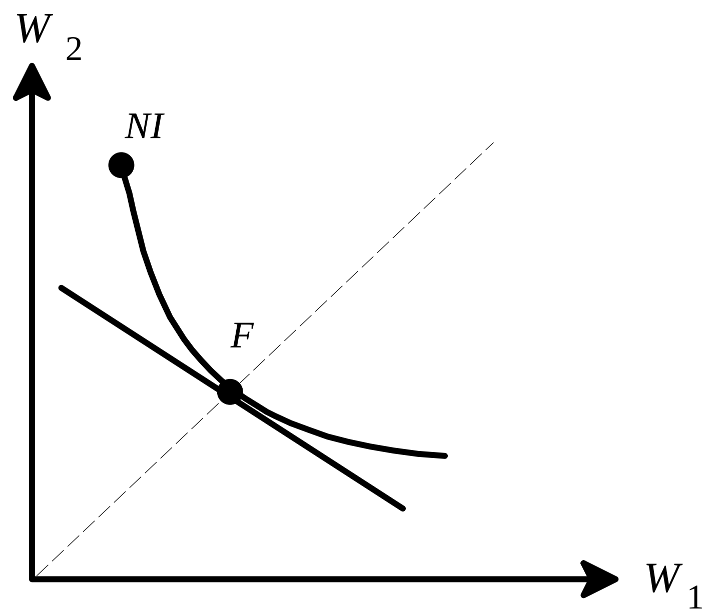
A monopolist will try to make the consumer pay as much as possible and thus will offer a contract which is **on** the reservation indifference curve and not above it.

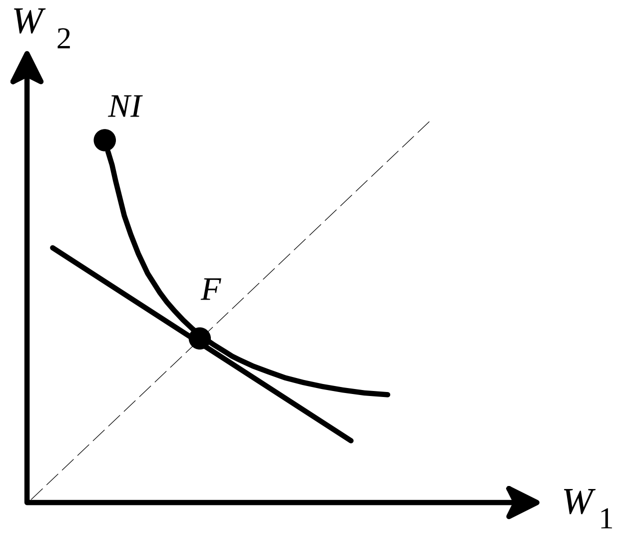


Contract *A* on the reservation indifference curve cannot be profit-maximizing because …



The only contract on the reservation indifference curve where this cannot happen is the contract at the intersection of the reservation indifference curve and the 45° line: contract *F* below:





Let W_F be the horizontal (and vertical) coordinate of point *F*.

$$W_F = \qquad (1)$$

$$U(W_F) = \qquad (2)$$

$NI = \left(\qquad \right)$ Expected **value:**

$\mathbb{E}[NI] =$ Then from the definition of risk premium,

(3)

Thus from (1)-(3) we get that

, that is, $\boxed{h_F = pL + R_{NI}}$

Thus the monopolist will offer a full-insurance contract with premium equal to expected loss + risk premium of *NI*.

For example, if $W = 1{,}600$, $L = 700$, $p = \frac{1}{10}$ and $U(\$m) = \sqrt{m}$ then h_F is given by the solution to

which is $h_F =$ Since $pL =$

it follows that $R_{NI} =$

2. Suppose the insurance industry is perfectly competitive

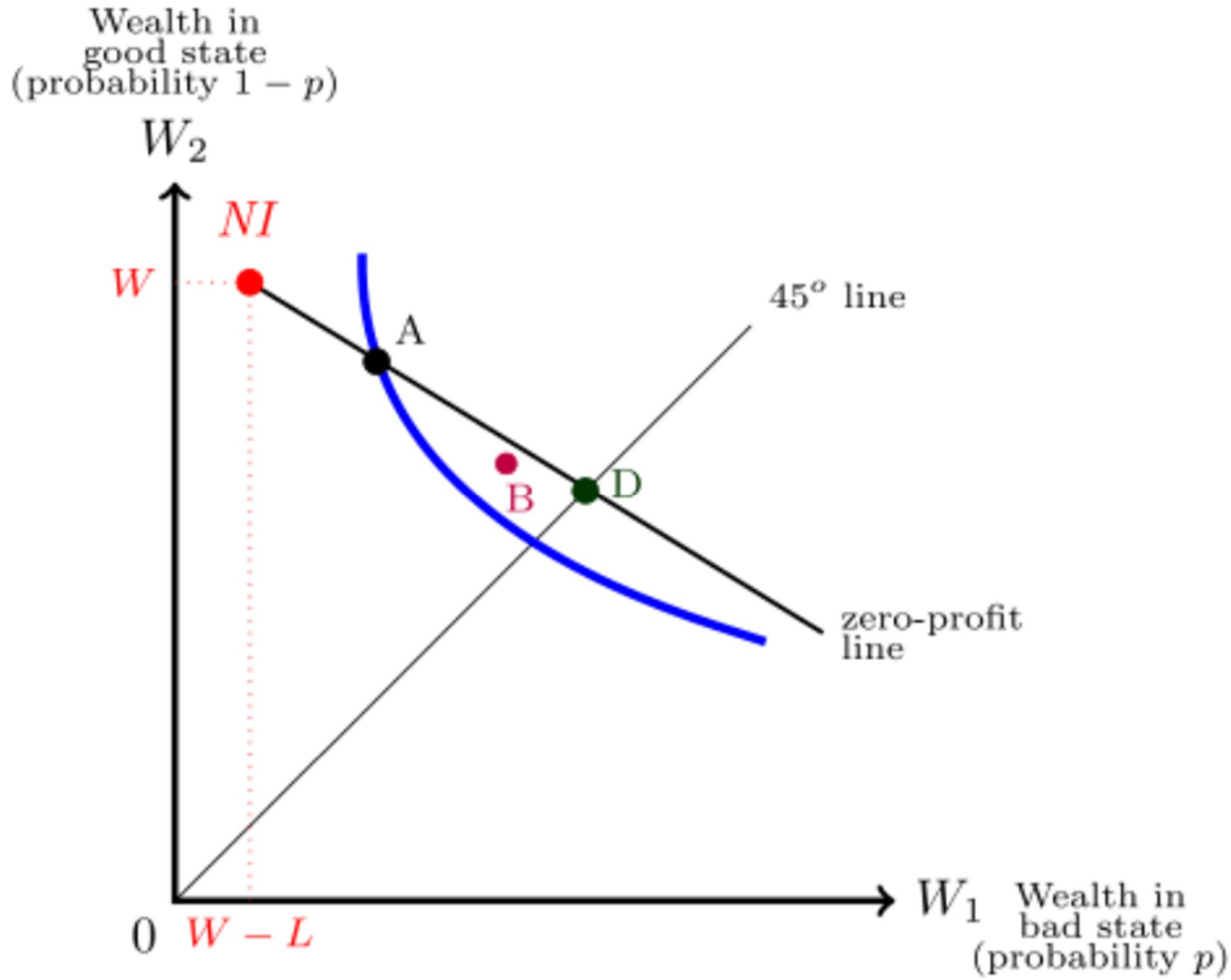
A contract that yields zero profit is called a **fair contract** and the zero profit line is called the **fair odds line**. Recall that the zero profit line is the straight line that goes through the No Insurance point and has slope $-\frac{p}{1-p}$.

Define an equilibrium in a competitive insurance industry as a situation where

(1) every firm makes zero profits and

(2) no firm (existing or new) can make positive profits by offering a new contract.

By the zero-profit condition (1), any equilibrium contract must be on the zero-profit line.


Wealth in
good state
(probability $1 - p$)
W_2
NI
W
A
45^o line
B
D
zero-profit
line
0
$W - L$
W_1
Wealth in
bad state
(probability p)

$d_D = 0$ and $h_D =$

CHOOSING FROM A MENU OF CONTRACTS

1. Finite menu of contracts

$W = 900$, $L = 700$, $p = \frac{1}{50}$, $U(m) = \sqrt{m}$

	premium	*deductible*
A	90	0
B	60	100
C	55	500

$\mathbb{E}[U(A)] =$

$\mathbb{E}[U(B)] =$

$\mathbb{E}[U(C)] =$

$\mathbb{E}[U(NI)] =$