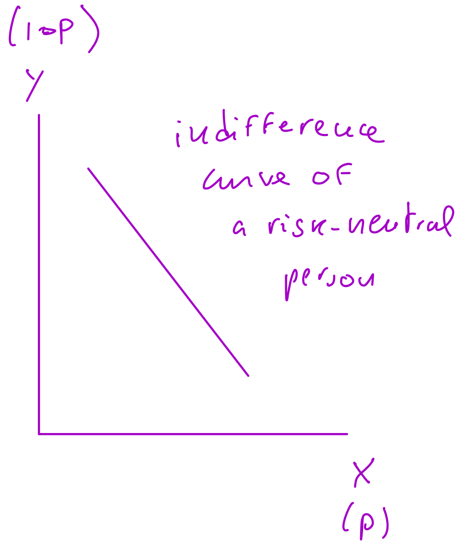
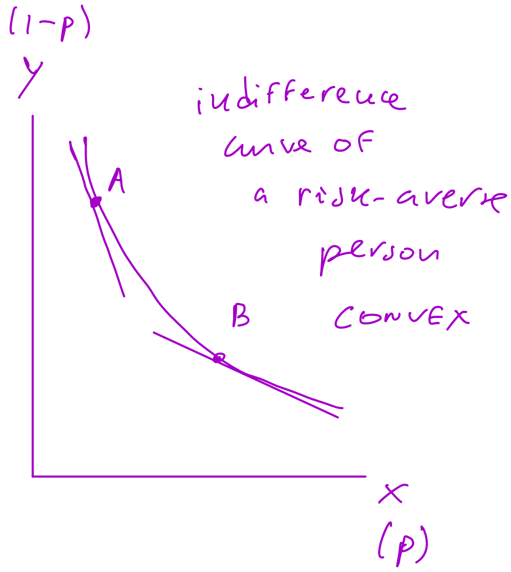




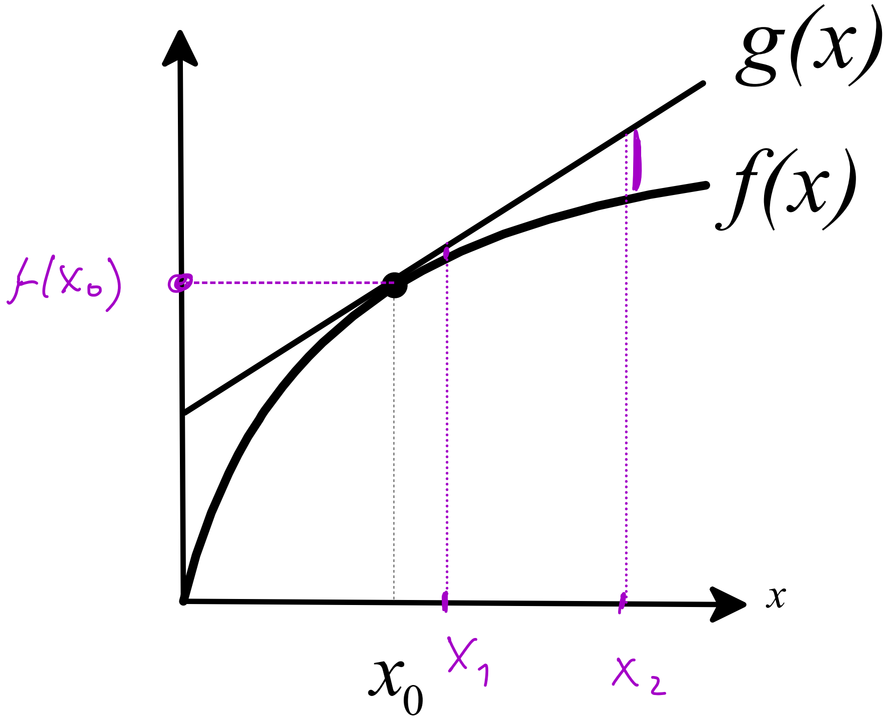
$$\begin{pmatrix} \$x & \$y \\ p & 1-p \end{pmatrix}$$

p is fixed

Slope of an indifference curve

Preliminaries on the meaning of the derivative.

$f(x)=\sqrt{x}+\dfrac{x^2}{3}$. Then $f'(x)=\dfrac{1}{2\sqrt{x}}+\dfrac{2}{3}x$ The derivative is used to construct a linear function to approximate the function $f(x)$ at a point x_0 :



$$g(x) = f(x_0) + f'(x_0)\cdot(x-x_0)$$

$$= \underbrace{f(x_0)} + \underbrace{f'(x_0)\,x} - \underbrace{f'(x_0)\,x_0}$$

constants

how close is $g(x_1)$ to $f(x_1)$?

$f(x) = \sqrt{x} + \frac{x^2}{3}$. Then $f'(x) = \frac{1}{2\sqrt{x}} + \frac{2x}{3}$. Let $x_0 = 9$.

$f(9) =$ 30 and $f'(9) = \frac{37}{6}$

so that $g(x) =$ $30 + \frac{37}{6}x - \frac{37}{6}9$ Let's see how well g approximates f

Take $x = 9.1$. Then $f(9.1) =$ $\sqrt{9.1} + \frac{9.1^2}{3} = 30.61993$ $g(9.1) =$ $30 + \frac{37}{6}(9.1) - \frac{37}{6} \cdot 9 =$ 30.61667

Take $x = 12$. Then $f(12) =$ 51.4641 $g(12) =$ 48.5

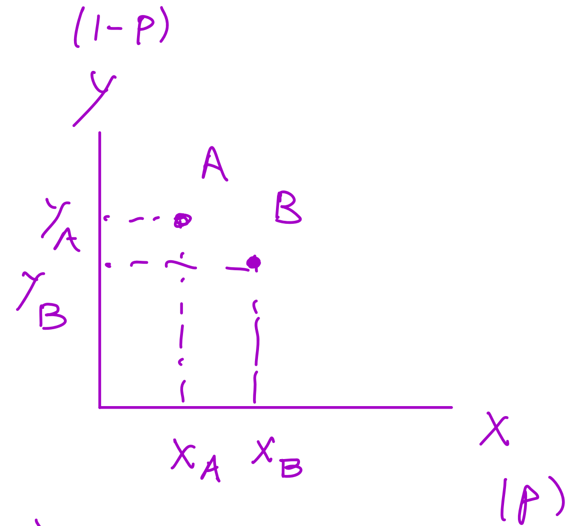
END OF PRELIMINARIES

$pU(x_A)+(1-p)U(y_A)$

$pU(x_B)+(1-p)U(y_B)$

Slope of indifference curve

Let A and B be two points that lie on the same indifference curve: $\mathbb{E}[U(A)] = \mathbb{E}[U(B)]$, (*)



- Since x_B is close to x_A, $U(x_B) \simeq U(x_A) + U'(x_A)(x_B - x_A)$
- Since y_B is close to y_A, $U(y_B) \simeq U(y_A) + U'(y_A)(y_B - y_A)$

Thus the RHS of (*) can be written as

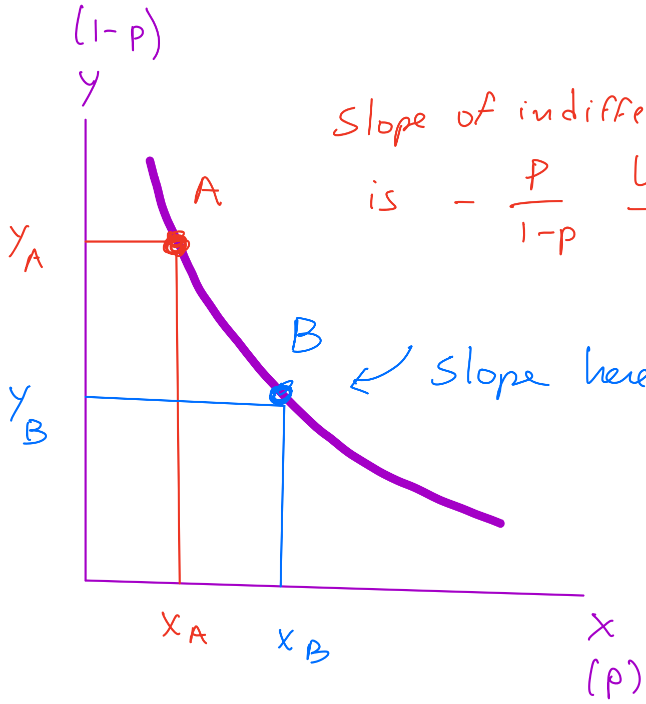
$$E[U(B)] \simeq \underbrace{p\left[U(x_A) + U'(x_A)\cdot(x_B - x_A)\right] + (1-p)\left[U(y_A) + U'(y_A)(y_B - y_A)\right]}$$

So (*) becomes

$$\underbrace{pU(x_A) + (1-p)U(y_A)}_{E[U(A)]} \simeq$$

that is, which can be written as

$$\boxed{\frac{\overbrace{y_A - y_B}^{\text{rise}}}{\underbrace{x_A - x_B}_{\text{run}}} = -\frac{p}{1-p}\,\frac{U'(x_A)}{U'(y_A)}}$$



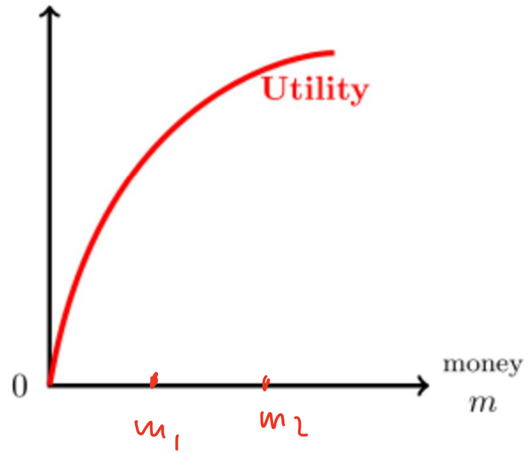
Slope of indifference curve at A is $-\frac{p}{1-p}\frac{U'(x_A)}{U'(y_A)}$

Slope here is $-\frac{p}{1-p}\frac{U'(x_B)}{U'(y_B)}$

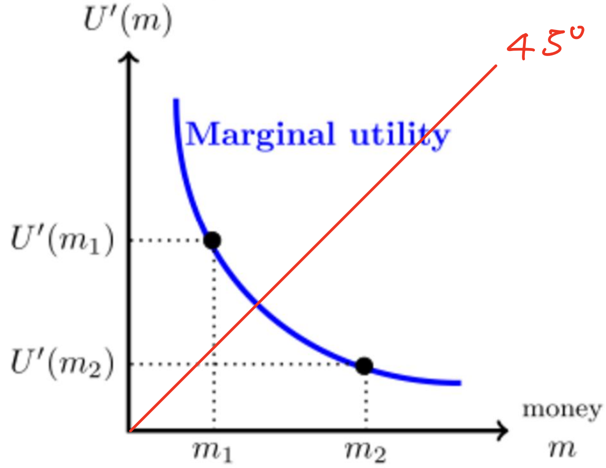
Comparing the slope at a point with the ratio $\frac{p}{1-p}$

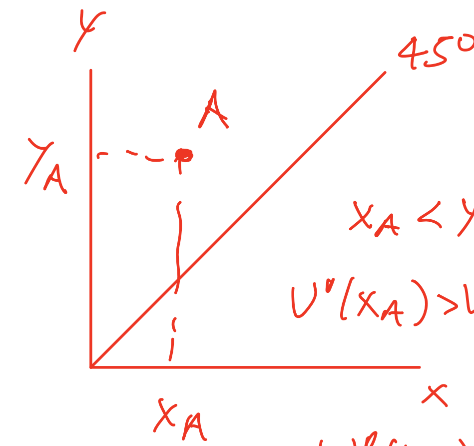
Look at the case of risk aversion but the other cases are similar.

$U(m_1) < U(m_2)$



$U'(m_1) > U'(m_2)$





$x_A < y_A$

$U'(x_A) > U'(y_A)$

$\frac{U'(x_A)}{U'(y_A)} > 1$

- at a point **above** the 45° line, where $x < y$, $\frac{U'(x)}{U'(y)} > 1$

  slope $> \frac{p}{1-p}$ in absolute value

- at a point **on** the 45° line, where $x = y$, $U'(x) = U'(y)$ so $\frac{U'(x)}{U'(y)} = 1$ slope $= \frac{p}{1-p}$

- at a point **below** the 45° line, where $x > y$,

  $U'(x) < U'(y)$ so $\frac{U'(x)}{U'(y)} < 1$

  slope $< \frac{p}{1-p}$

Example. $U(m) = \ln(m)$, $p = \frac{1}{3}$. What is the slope of the indifference curve at points $A = (10,40)$ and $B = (10,10)$?

A is above 45° line

B is on 45° line

The expected utility of lottery $A = \begin{pmatrix} 10 & 40 \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$ is

The slope of the indifference curve at point A is equal to

$$\frac{p}{1-p} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

The expected utility of lottery $B = \begin{pmatrix} 10 & 10 \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$ is

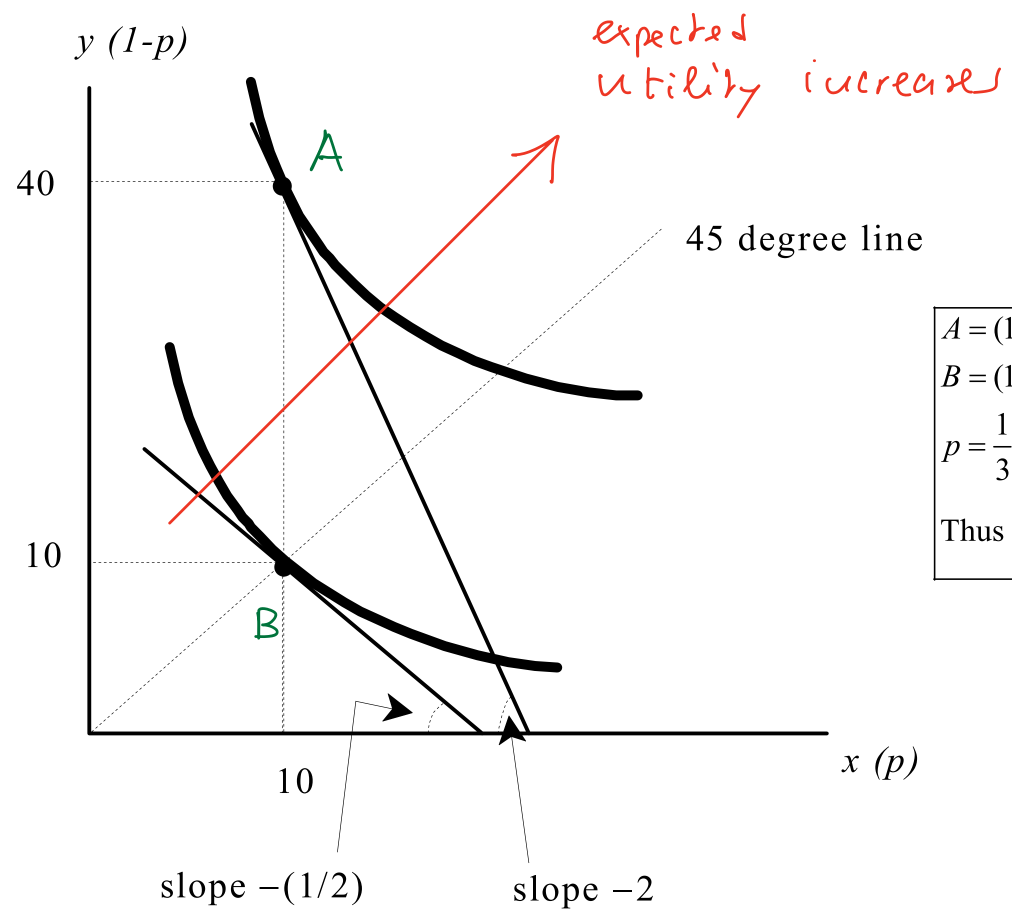
The slope of the indifference curve at point B is equal to .

$$U'(m) = \frac{1}{m}$$

At point A $\frac{p}{1-p} \frac{U'(10)}{U'(40)} = \frac{1}{2} \frac{\frac{1}{10}}{\frac{1}{40}} = \frac{1}{2} \frac{1}{10} 40 = \frac{1}{2} 4 = 2$

At point B slope of ind. curve in absolute value is

$$\frac{p}{1-p} \cdot \frac{U'(10)}{U'(10)} = \frac{1}{2}$$



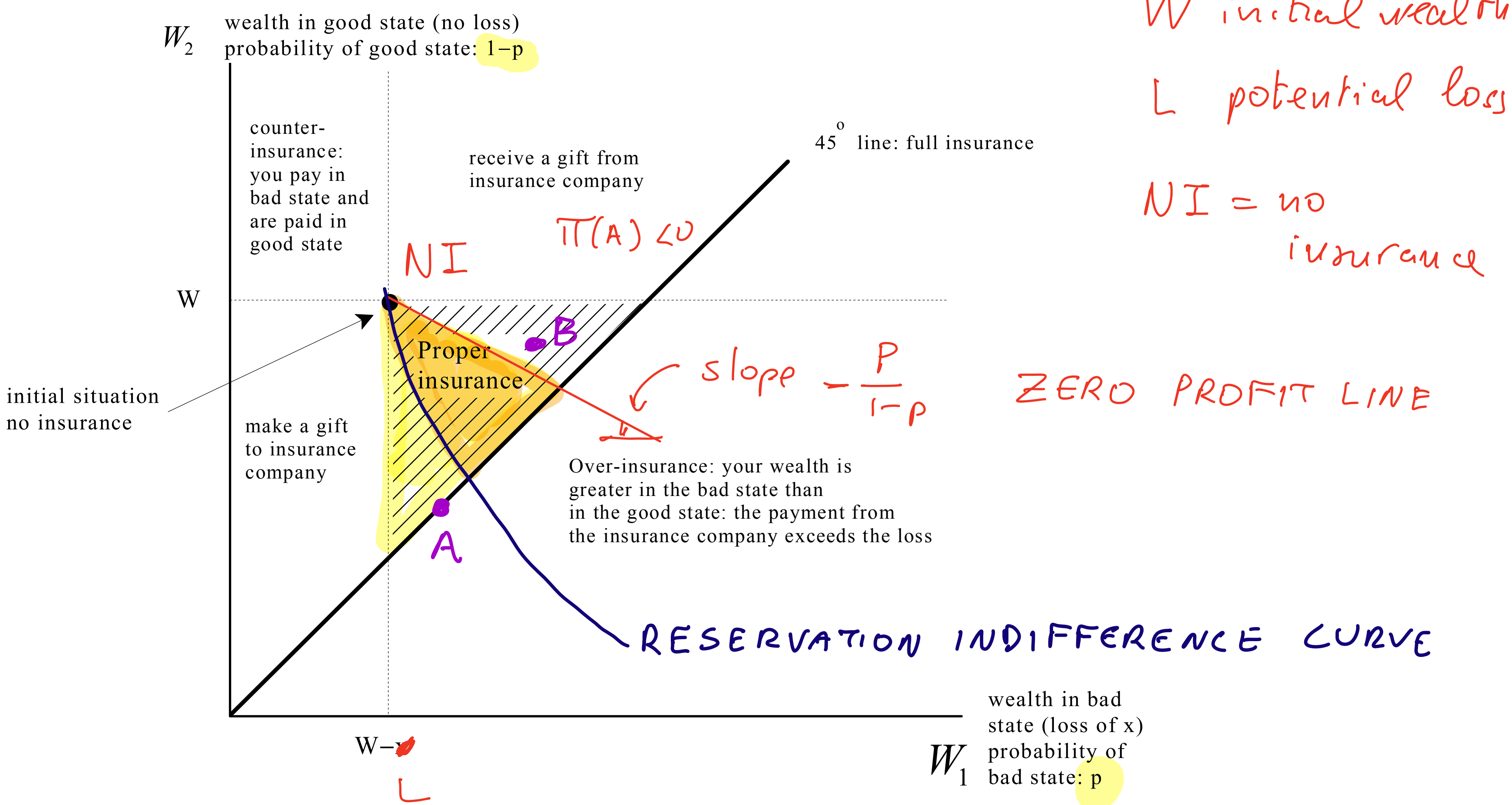
$A = (10, 40)$

$B = (10, 10)$

$p = \dfrac{1}{3}$

Thus $-\dfrac{p}{1-p} = -\dfrac{1}{2}$

DEMAND SIDE OF INSURANCE



$W = 40{,}000$, $L = 5{,}000$, probability of loss $p = \frac{1}{50}$, $U(m) = \ln(m)$

$$NI = \begin{pmatrix} 35{,}000 & 40{,}000 \\ \frac{1}{50} & \frac{49}{50} \end{pmatrix} \qquad \mathbb{E}[U(NI)] = \frac{1}{50}\ln(35{,}000) + \frac{49}{50}\ln(40{,}000) = 10.5940$$

Suppose the consumer is offered $A = (h_A = 200,\ d_A = 0)$ which would yield a profit of

$$\pi_A = 200 - \frac{1}{50}5000 = 100$$

Would she purchase it?

$$A = \begin{pmatrix} 39{,}800 \\ 1 \end{pmatrix} \qquad E[U(A)] = \ln(39{,}800) = 10.5916$$

A is below the reservation indifference curve

Suppose the consumer is offered $B = (h_B = 50,\ d_B = 100)$.

$$\mathbb{E}[U(B)] = \frac{1}{50}\ln(40{,}000 - 50 - 100) + \frac{49}{50}\ln(40{,}000 - 50) = 10.5953$$

$$\pi_B = 50 - \frac{1}{50}(5000 - 100) = -48$$

B is above res. ind. curve

So we must exclude points that are below the indifference curve that goes through NI, called the **reservation indifference curve,** and exclude all those that are above the zero-profit line. The only observable contracts are: