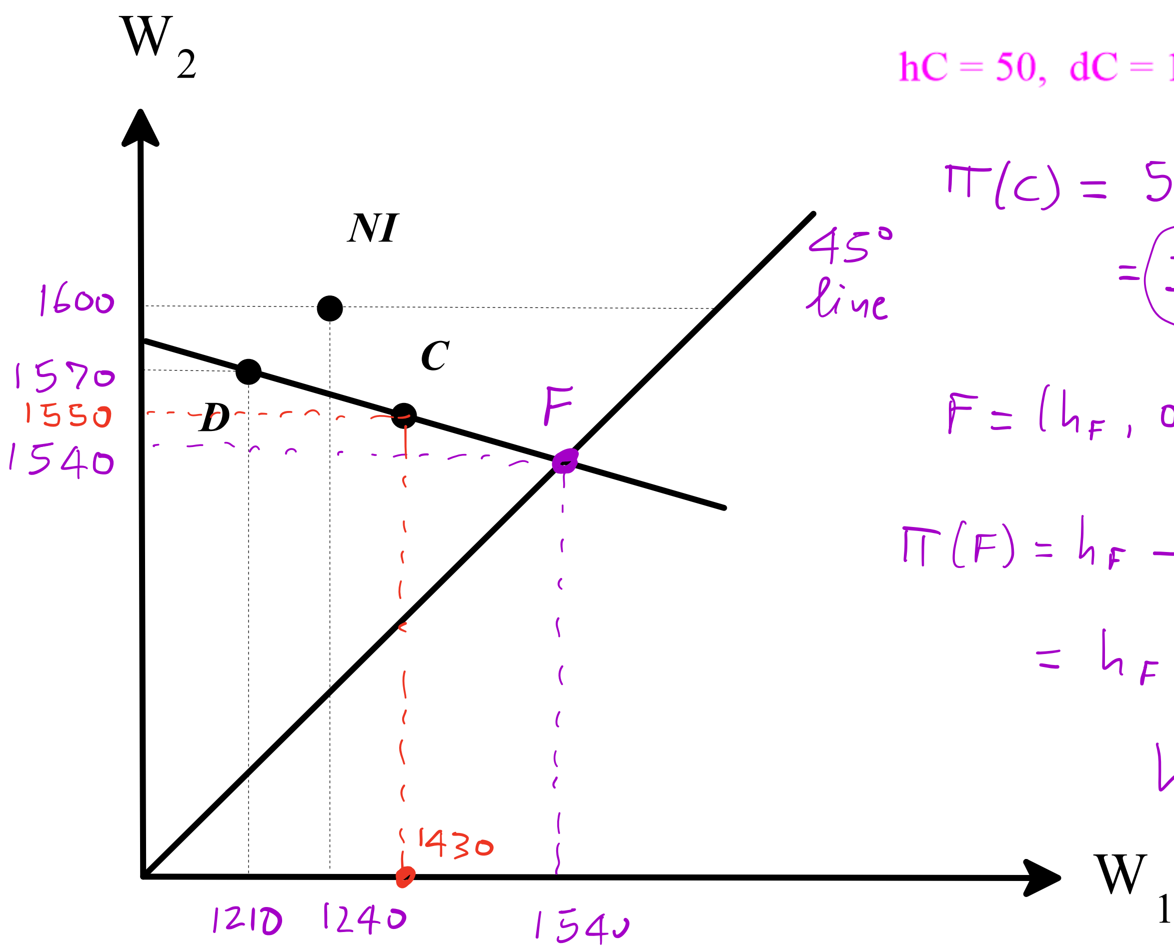
(5) Find the full-insurance contract, call it F, that lies on the isoprofit line that goes through contracts C and D and represent it in the wealth diagram.

W = 1600 L = 360 p = 1/12

hC = 50, dC = 120



$\pi(C) = 50 - \frac{1}{12}(360-120)$

$= 30$

$F = (h_F, 0)$

$\pi(F) = h_F - \frac{1}{12} 360$

$= h_F - 30 = 30$

$h_F = 60$

any isoprofit line

(6) Calculate the slope of the isoprofit line through C in the wealth diagram.

$$-\frac{p}{1-p} = -\frac{\frac{1}{12}}{\frac{11}{12}} = -\frac{1}{11}$$

(7) Calculate the equation of the isoprofit line through C in the wealth diagram.

$$W_2 = a - \frac{1}{11} W_1 \qquad a\,?$$

$$1550 = a - \frac{1}{11} 1430$$

$$a = 1{,}680$$

$$\boxed{W_2 = 1{,}680 - \frac{1}{11} W_1}$$

verify that this line goes through also points D and F

For F check that $1540 = 1680 - \frac{1}{11} 1540$

(8) Next prove that given two contracts $A = (h_A, d_A)$ and $B = (h_B, d_B)$, $\pi(A) = \pi(B)$ if and only if the expected value of the wealth lottery (for the insured) corresponding to contact A is equal to the expected value of the wealth lottery (for the insured) corresponding to contact B

FOR A RISK-NEUTRAL PERSON the indifference curve through A and B coincides with the isoprofit line through A and B

Consumer's point of view:

$$A = \begin{pmatrix} W - h_A & W - h_A - d_A \\ 1-p & p \end{pmatrix} \qquad B = \begin{pmatrix} W - h_B & W - h_B - d_B \\ 1-p & p \end{pmatrix}$$

$$E[A] = (1-p)(W - h_A) + p(W - h_A - d_A) = W - (1-p)h_A - ph_A - pd_A = W - h_A - pd_A$$

$$E[B] = (1-p)(W - h_B) + p(W - h_B - d_B) = W - (1-p)h_B - ph_B - pd_B$$

$$W - h_B - pd_B$$

$$\pi(A) = h_A - p(L - d_A) = h_A - pL + pd_A \quad = \quad \pi(B) = h_B - pL + pd_B$$

if and only if

$$h_A + pd_A = h_B + pd_B \qquad (1)$$

subtract pL from both sides

$$h_A - pL + pd_A = h_B - pL + pd_B \qquad (2)$$

$\Pi(A)$ $\Pi(B)$

$$\text{operation } O = \begin{pmatrix} \text{cured} & \text{permanent disability} \\ 90\% & 10\% \end{pmatrix}$$

$$\text{drug treatment } D = \begin{pmatrix} \text{cured} & \text{no benefit} & \text{adverse reaction} \\ 75\% & 10\% & 15\% \end{pmatrix}$$

What is the expected value of lottery O?

meaningless question !

What is the expected value of lottery D?

meaningless question !

Which of the two lotteries is better?

$m = 4$

$Z = \{ \underset{z_1}{\text{cured}}, \underset{z_2}{\text{permanent disability}}, \underset{z_3}{\text{adverse reaction to drug}}, \underset{z_4}{\text{status quo}} \}$

EXPECTED UTILITY THEORY

degenerate lottery

$\begin{pmatrix} z_1 & z_2 & z_3 & \dots & z_m \\ 0 & 1 & 0 & & 0 \end{pmatrix} \sim z_2$

$Z = \{z_1, z_2, \dots, z_m\}$ set of basic outcomes.

For every $i \in \{1, 2, \dots, m\}$

A lottery is a probability distribution over Z: $L = \begin{pmatrix} z_1 & z_2 & \dots & z_m \\ p_1 & p_2 & \dots & p_m \end{pmatrix}$ $0 \le p_i \le 1$

$p_1 + p_2 + \dots + p_m = 1$

Let $\mathcal{L}$ be the set of lotteries. Suppose that the agent has a ranking $\succsim$ of the elements of $\mathcal{L}$:

if $L = \begin{pmatrix} z_1 & z_2 & \dots & z_m \\ p_1 & p_2 & \dots & p_m \end{pmatrix}$ and $M = \begin{pmatrix} z_1 & z_2 & \dots & z_m \\ q_1 & q_2 & \dots & q_m \end{pmatrix}$ then

$\begin{pmatrix} z_1 & z_2 & z_3 & \dots & z_m \\ \frac{1}{3} & \frac{2}{3} & 0 & & 0 \end{pmatrix}$

preference

$L \succ M$ means that L is considered to be better than M

$\sim \begin{pmatrix} z_1 & z_2 \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$

$L \sim M$ means that L is considered to be just as good as M

indifference

Rationality constraints on $\succsim$ (von Neumann-Morgenstern axioms):

…

$$Z = \{ z_1, z_2, z_3, z_4, z_5 \}$$

$$z_4 \succ z_1 \succ z_2 \sim z_5 \succ z_3$$

best	z_4	z_{best}
	z_1	
	z_2, z_5	
worst	z_3	z_{worst}

Axiom : if $L = \begin{pmatrix} z_{best} & z_{worst} \\ p & 1-p \end{pmatrix}$

$M = \begin{pmatrix} z_{best} & z_{worst} \\ q & 1-q \end{pmatrix}$

$L \succ M$ if and only if $p > q$

$L \succsim M$ L is at least as good as M

Theorem 1 Let $Z = \{z_1, z_2, ..., z_m\}$ be a set of basic outcomes and $\mathcal{L}$ the set of lotteries over Z. If $\succsim$ satisfies the von Neumann-Morgenstern axioms then there exists a function $U: Z \to \mathbb{R}$, called a *von Neumann-Morgenstern utility function*, that assigns a number to every basic outcome and is such that, for any two lotteries $L = \begin{pmatrix} z_1 & z_2 & \dots & z_m \\ p_1 & p_2 & \dots & p_m \end{pmatrix}$ and $M = \begin{pmatrix} z_1 & z_2 & \dots & z_m \\ q_1 & q_2 & \dots & q_m \end{pmatrix}$, $\to \begin{pmatrix} U(z_1) & U(z_2) & \dots & U(z_m) \\ q_1 & q_2 & & q_m \end{pmatrix}$ $\to \begin{pmatrix} U(z_1) & U(z_2) & \dots & U(z_m) \\ p_1 & p_2 & & p_m \end{pmatrix}$

$$L \succ M \quad \text{if and only if} \quad \underbrace{p_1U(z_1) + p_2U(z_2) + ... + p_mU(z_m)}_{\text{expected utility of lottery } L} > \underbrace{q_1U(z_1) + q_2U(z_2) + ... + q_mU(z_m)}_{\text{expected utility of lottery } M}$$

and

$$L \sim M \quad \text{if and only if} \quad \underbrace{p_1U(z_1) + p_2U(z_2) + ... + p_mU(z_m)}_{\text{expected utility of lottery } L} = \underbrace{q_1U(z_1) + q_2U(z_2) + ... + q_mU(z_m)}_{\text{expected utility of lottery } M}$$

2.25 3.33

EXAMPLE 1. $Z=\{z_1,z_2,z_3,z_4\}$ $L=\begin{pmatrix} z_1 & z_2 & z_3 & z_4 \\ \frac{1}{8} & \frac{5}{8} & 0 & \frac{2}{8} \end{pmatrix}$ $M=\begin{pmatrix} z_1 & z_2 & z_3 & z_4 \\ \frac{1}{6} & \frac{2}{6} & \frac{1}{6} & \frac{2}{6} \end{pmatrix}$

Suppose we know that $U=\begin{cases} z_1 & z_2 & z_3 & z_4 \\ 6 & 2 & 8 & 1 \end{cases}$

Then

$$\mathbb{E}[U(L)] = \frac{1}{8}\cdot 6 + \frac{5}{8}\cdot 2 + 0\cdot 8 + \frac{2}{8}1 = 2.25$$

$\wedge$ so $M \succ L$

$$\mathbb{E}[U(M)] = \frac{1}{6}\cdot 6 + \frac{2}{6}\cdot 2 + \frac{1}{6}\cdot 8 + \frac{2}{6}1 = 3.33$$

EXAMPLE 2.

$$A = \begin{pmatrix} \text{paid 3-week vacation} & \text{no vacation} \\ 50\% & 50\% \end{pmatrix} \qquad B = \begin{pmatrix} \text{paid 1-week vacation} \\ 100\% \end{pmatrix}$$

Suppose Ann says $\boxed{B \succ A}$ How would she rank

$$C = \begin{pmatrix} \text{paid 3-week vacation} & \text{no vacation} \\ 5\% & 95\% \end{pmatrix} \text{ and } D = \begin{pmatrix} \text{paid 1-week vacation} & \text{no vacation} \\ 10\% & 90\% \end{pmatrix} ?$$

Is it rational to say $B > A$

and $C > D$?