EXAMPLE 2.

$$A = \begin{pmatrix} \text{paid 3-week vacation} & \text{no vacation} \\ 50\% & 50\% \end{pmatrix} \qquad B = \begin{pmatrix} \text{paid 1-week vacation} \\ 100\% \end{pmatrix}$$

Suppose Ann says $\boxed{B \succ A}$ How would she rank

$$C = \begin{pmatrix} \text{paid 3-week vacation} & \text{no vacation} \\ 5\% & 95\% \end{pmatrix} \text{ and } D = \begin{pmatrix} \text{paid 1-week vacation} & \text{no vacation} \\ 10\% & 90\% \end{pmatrix} ?$$

Is it rational to say $B > A$

and $C > D$?

EXAMPLE 2.

$$A = \begin{pmatrix} \text{paid 3-week vacation} & \text{no vacation} \\ 50\% & 50\% \end{pmatrix} \qquad B = \begin{pmatrix} \text{paid 1-week vacation} \\ 100\% \end{pmatrix}$$

Suspect Ann says $\boxed{B \succ A}$. How would she rank

$$C = \begin{pmatrix} \text{paid 3-week vacation} & \text{no vacation} \\ 5\% & 95\% \end{pmatrix} \text{ and } D = \begin{pmatrix} \text{paid 1-week vacation} & \text{no vacation} \\ 10\% & 90\% \end{pmatrix} ?$$

$b > \frac{1}{2} a + \frac{1}{2} c$

$\boxed{2b > a + c}$

Basic outcomes

$a > b > c$

best z_1 3-week a

z_2 1-week b

worst z_3 no vacation c

contradiction

$E[U(B)] = 1 \cdot U(z_2) = b$

$E[U(A)] = \frac{1}{2} U(z_1) + \frac{1}{2} U(z_3) = \frac{1}{2} a + \frac{1}{2} c$

If she says $C > D$ it must be that $E[U(C)] > E[U(D)]$

$E[U(C)] = \frac{5}{100} \cdot U(z_1) + \frac{95}{100} \cdot U(z_3) > \frac{10}{100} U(z_2) + \frac{90}{100} U(z_3) = E[U(D)]$

$\frac{5}{100} a + \frac{95}{100} c > \frac{10}{100} b + \frac{90}{100} c$

$\frac{5}{100} a + \frac{5}{100} c > \frac{10}{100} b$ multiply by 100

$5a + 5c > 10b$ divide by 5 $\boxed{a + c > 2b}$

Money lotteries

$$L = \begin{pmatrix} \$17 \\ 1 \end{pmatrix} \qquad M = \begin{pmatrix} \$9 & \$25 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$\mathbb{E}[L] = 17 \qquad \mathbb{E}[M] = \frac{1}{2}9 + \frac{1}{2}25 = 17$$

Suppose Bob's vNM utility function is: $U(\$x) = \sqrt{x}$

$$\mathbb{E}[U(L)] = 1 \cdot \sqrt{17} \quad > \quad \mathbb{E}[U(M)] = \frac{1}{2}\sqrt{9} + \frac{1}{2}\sqrt{25} =$$

$$= \frac{1}{2}3 + \frac{1}{2}5 = 4$$

risk averse

a risk-neutral person : $L \sim M$

averse $L \succ M$

$$A = \begin{pmatrix} \$0 & \$100 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \qquad B = \begin{pmatrix} \$40 & \$60 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$\mathbb{E}[A] = \frac{1}{2}0 + \frac{1}{2}100 = 50 \qquad \mathbb{E}[B] = \frac{1}{2}40 + \frac{1}{2}60 = 50$$

Suppose Bob's vNM utility function is: $\boxed{U(\$x) = \sqrt{x}}$

$$\mathbb{E}[U(A)] = \frac{1}{2}\cdot\sqrt{0} + \frac{1}{2}\sqrt{100} = \frac{1}{2}0 + \frac{1}{2}10 = 5$$

$$\mathbb{E}[U(B)] = \frac{1}{2}\sqrt{40} + \frac{1}{2}\sqrt{60} = 7.03 \qquad B \succ A$$

RISK-NEUTRAL $\quad U(\$x) = x \quad$ identity function

$$\underbrace{E[U(A)]} = \frac{1}{2}U(0) + \frac{1}{2}U(100) = \underbrace{\frac{1}{2}0 + \frac{1}{2}100}_{E[A]} = 50$$

$E[A] = \frac{1}{2}4 + \frac{1}{2}6 = 5$

$$A = \begin{pmatrix} \$4 & \$6 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \succ \quad B = \begin{pmatrix} \$5 \\ 1 \end{pmatrix}$$

$E[B] = 5$

U($x) = x²

RISK LOVING

$E[U(A)] = \frac{1}{2}4^2 + \frac{1}{2}6^2 = \frac{1}{2}16 + \frac{1}{2}36 = \frac{52}{2} =$

$= \underbrace{26}$

$E[U(B)] = 1 \cdot 5^2 = \underbrace{25}$

Money lottery L $E[L]$ Compare L to $\begin{pmatrix} \$E[L] \\ 1 \end{pmatrix}$

Re-~~define~~ state attitudes to risk in terms of utility:

Risk-averse if $E[L] \succ L$ $\quad U(E(L)) > E[U(L)]$

$E[U(\cdot)] = 1 \cdot U(E[L])$

Risk-neutral if $U(E[L]) = E[U(L)]$

Risk-loving if $U(E[L]) < E[U(L)]$

Theorem 2. Let $\succsim$ be a von Neumann-Morgenstern ranking of the set of basic lotteries $\mathcal{L}$. Then the following are true.

(A) If $U : Z \to \mathbb{R}$ is a von Neumann-Morgenstern utility function that represents $\succsim$, then, for any two real numbers a and b with $a > 0$, the function $V : Z \to \mathbb{R}$ defined by $V(z_i) = aU(z_i) + b$ ($i = 1,2,\ldots,m$) is also a von Neumann-Morgenstern utility function that represents $\succsim$.

(B) If $U : Z \to \mathbb{R}$ and $V : Z \to \mathbb{R}$ are two von Neumann-Morgenstern utility functions that represent $\succsim$, then there exist two real numbers a and b with $a > 0$ such that $V(z_i) = aU(z_i) + b$ ($i = 1,2,\ldots,m$).

$$U = \begin{cases} z_1 & z_2 & z_3 & z_4 & z_5 & z_6 \\ 10 & 6 & 16 & 8 & 6 & 14 \end{cases}$$

$a = 1$

$b = -6$

$V = 4 \quad 0 \quad 10 \quad 2 \quad 0 \quad 8$

$a = \frac{1}{10} \quad b = 0$

$W = \frac{4}{10} \quad 0 \quad 1 \quad \frac{2}{10} \quad 0 \quad \frac{8}{10}$

normalized utility function

$Z = \{z_1, z_2, z_3, z_4\}$

Q.1 How do you rank the basic outcomes?

		U
best	z_3	1
	z_1, z_4	
worst	z_2	0