ISOPROFIT LINES

Assume that the **insurance company** is **risk neutral** so that it considers selling an insurance contract $C = (h,d)$, corresponding to the lottery

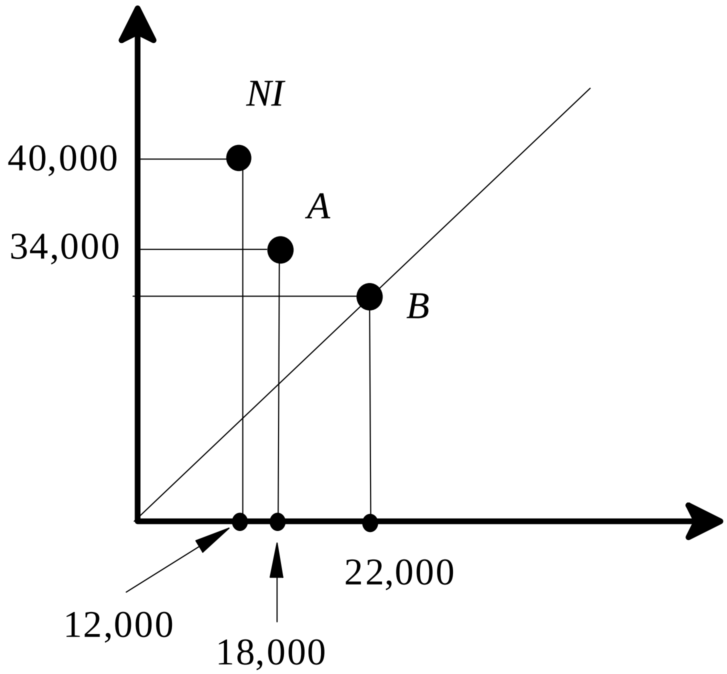
$$C = \begin{pmatrix} h-(L-d) & h \\ p & 1-p \end{pmatrix},$$ as equivalent to getting its expected value

for sure: $\mathbb{E}[C] = p[h-(L-d)] + (1-p)h = h - p(L-d) = h - pL + pd$

We denote the expected profit from contract (h,d) by $\pi(h,d)$. Thus

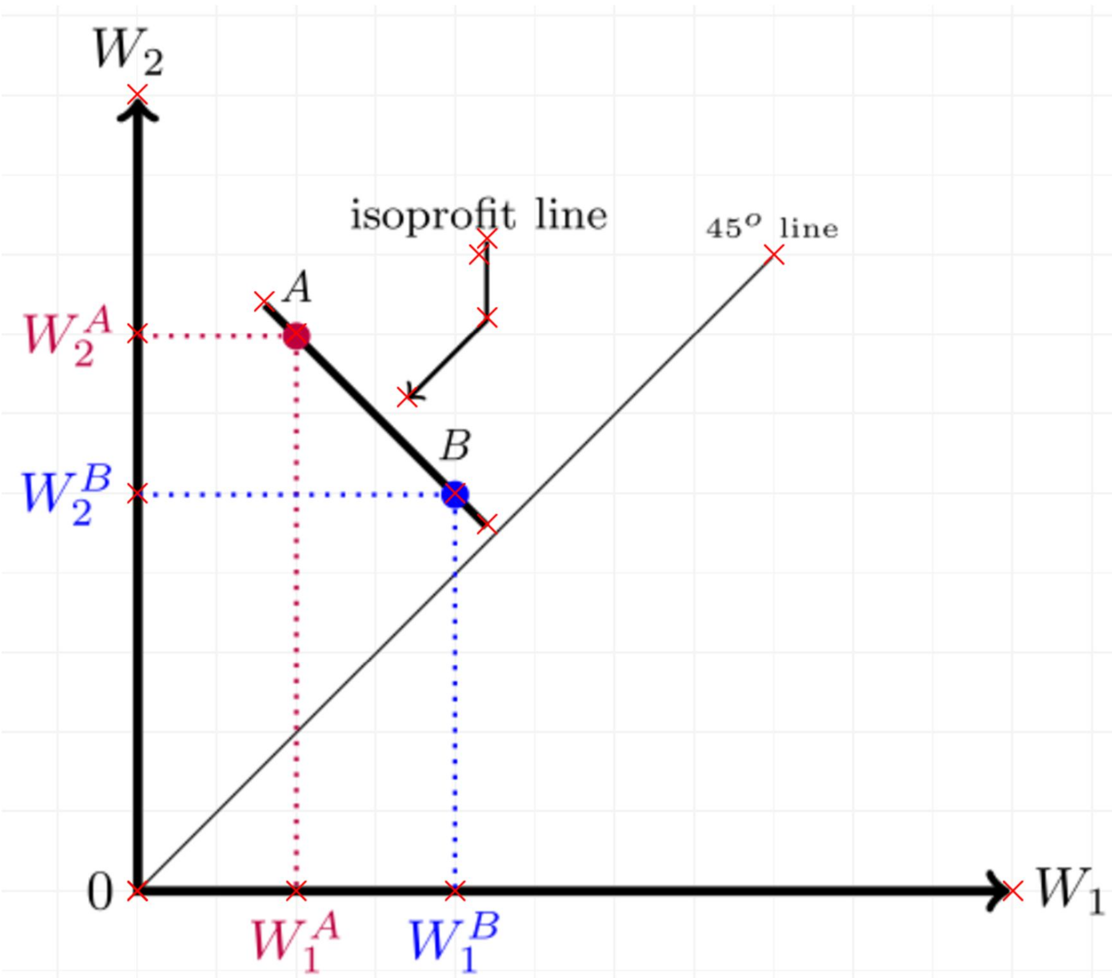
$$\pi(h,d) = h - p(L-d) = h - pL + pd$$

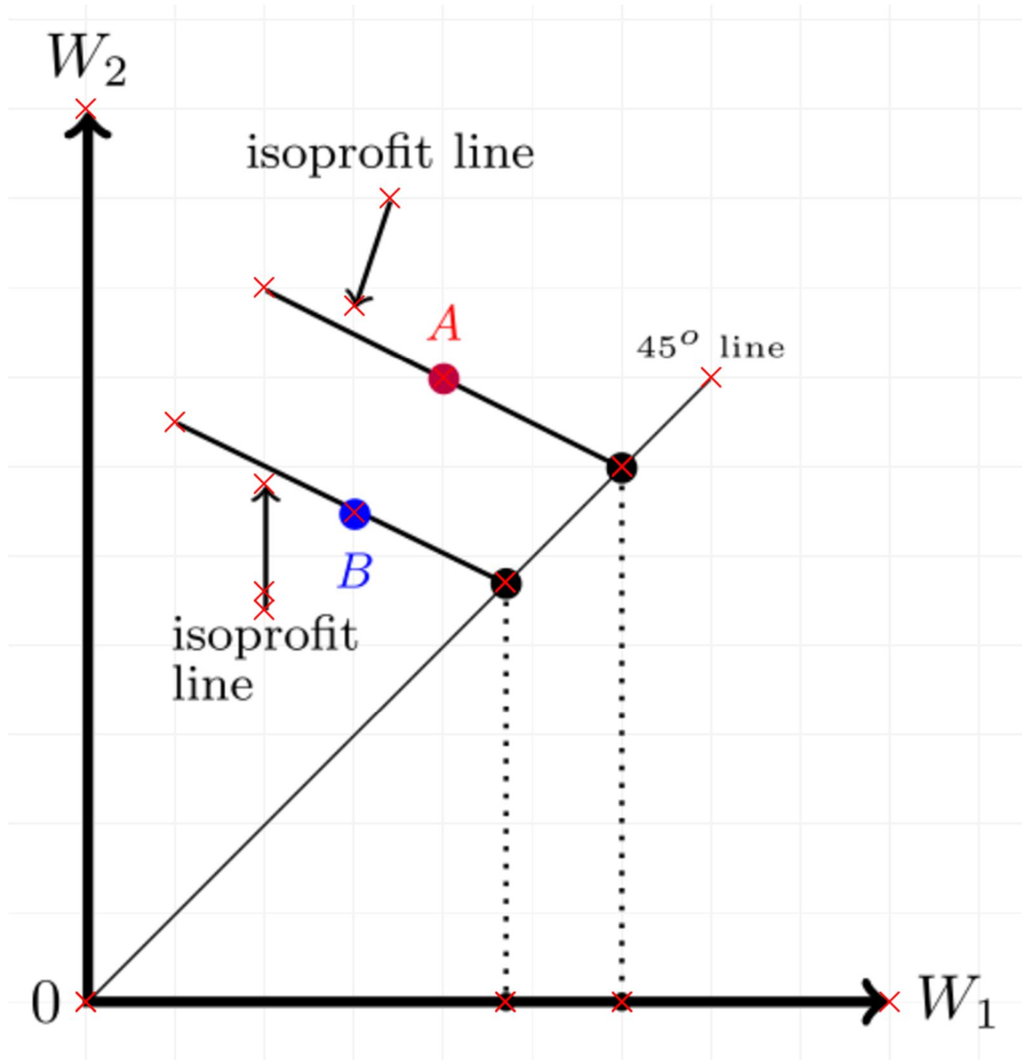
If the contract is expressed as a point (W_1, W_2) in wealth space then

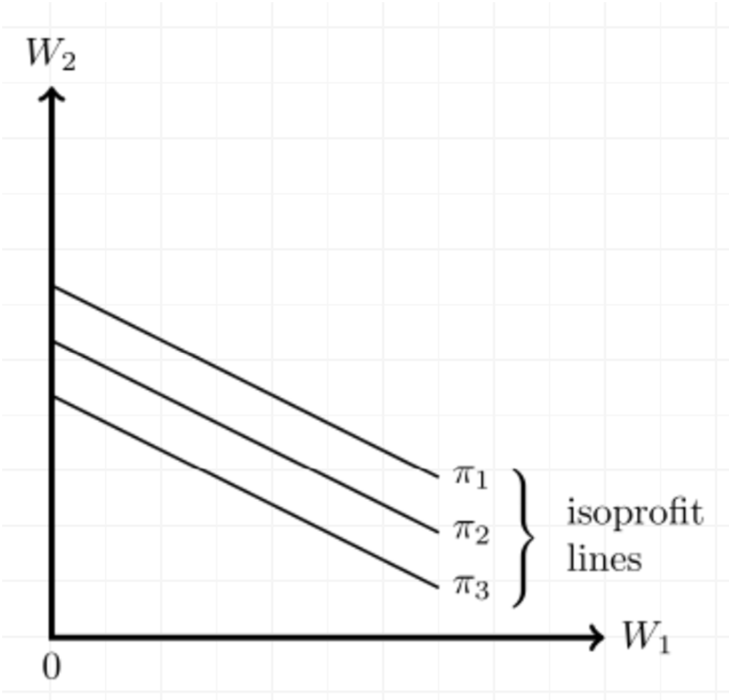


Suppose that $p = \frac{1}{10}$. What is $\pi(A)$? What is $\pi(B)$?

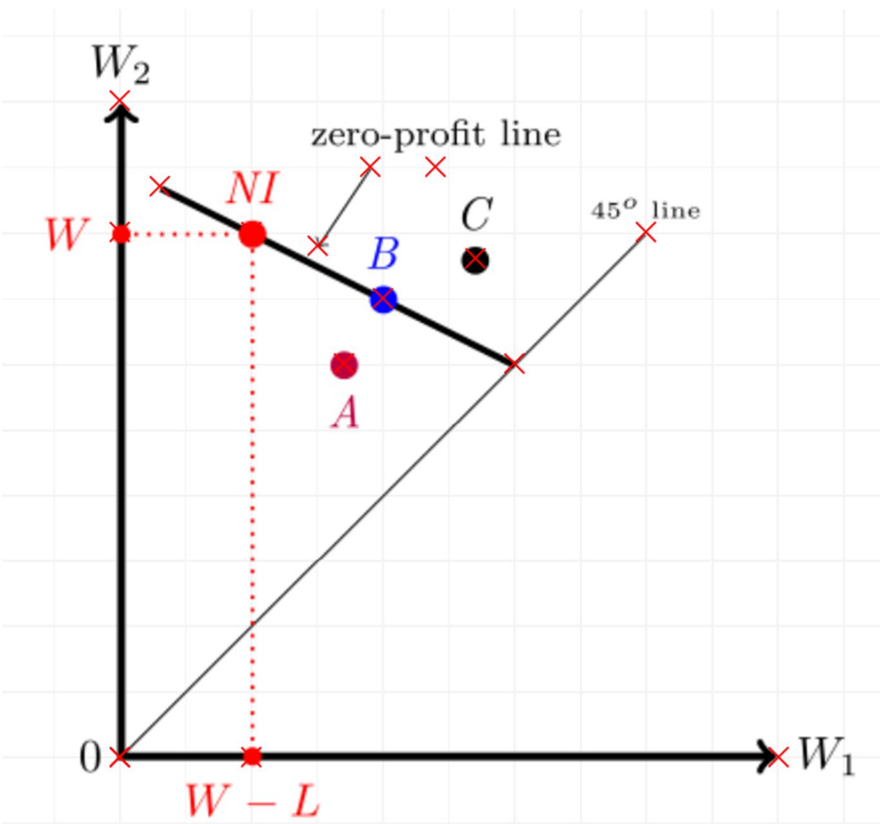
An **isoprofit line** is defined as a line joining contracts that give the same expected profit. Let $A = (W_1^A, W_2^A)$ and $B = (W_1^B, W_2^B)$ be such that $\pi(A) = \pi(B)$




W_2
isoprofit line
A
45^o line
B
isoprofit
line
0
W_1



Since No Insurance can be thought of as the trivial contract h = 0 and d = L, which gives zero profits, the isoprofit line going through the NI point is the zero-profit line:



EXAMPLE

Let $W = 1{,}600,\ L = 360,\ \ p = \frac{1}{12}$. Consider contract $C = (h_C = 50,\ d_C = 120)$.

(1) Represent NI and C in a wealth diagram.

(2) Calculate $\pi(C)$.

(3) Let D be a contract obtained from C by **reducing** the premium by 20 and increasing the deductible in such a way that $\pi(D) = \pi(C)$. Find the premium and deductible of contract D.

(4) Represent contract D in the wealth diagram.

(5) Find the full-insurance contract, call it *F*, that lies on the isoprofit line that goes through contracts C and D and represent it in the wealth diagram.

(6) Calculate the slope of the isoprofit line through C in the wealth diagram.

(7) Calculate the equation of the isoprofit line through C in the wealth diagram.

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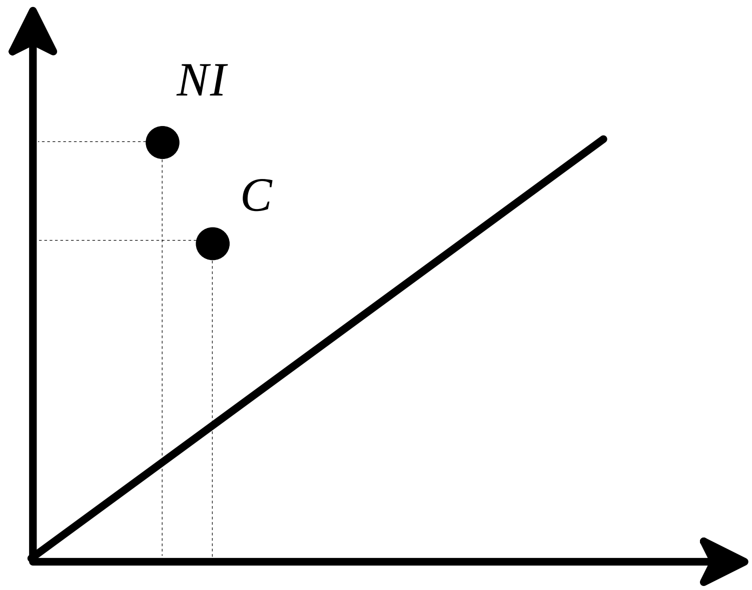
(8) Next prove that given two contracts $A = (h_A, d_A)$ and $B = (h_B, d_B)$, $\pi(A) = \pi(B)$ if and only if the expected value of the wealth lottery (for the insured) corresponding to contact *A* is equal to the expected value of the wealth lottery (for the insured) corresponding to contact *B*

$W = 1{,}600,\ L = 360,\ p = \frac{1}{12}$ $C = \left(h_C = 50,\ d_C = 120\right)$

(1) Represent NI and C in a wealth diagram.



(2) Calculate $\pi(C)$

$W = 1{,}600,\ L = 360,\ \ p = \tfrac{1}{12}$ $\qquad C = \left(h_C = 50,\ d_C = 120\right)$

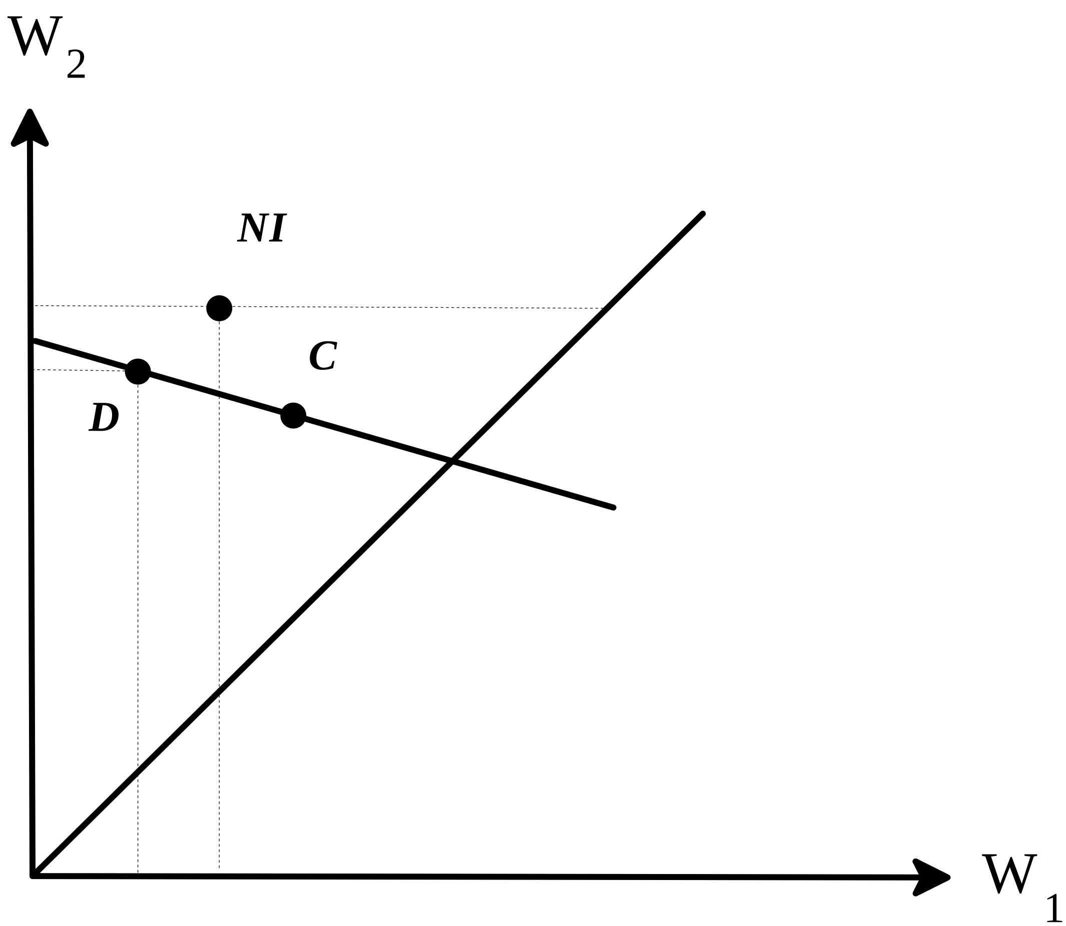
$\pi(C) = 30$

(3) Let D be a contract obtained from C by **reducing** the premium by 20 and increasing the deductible in such a way that $\pi(D) = \pi(C)$. Find the premium and deductible of contract D.

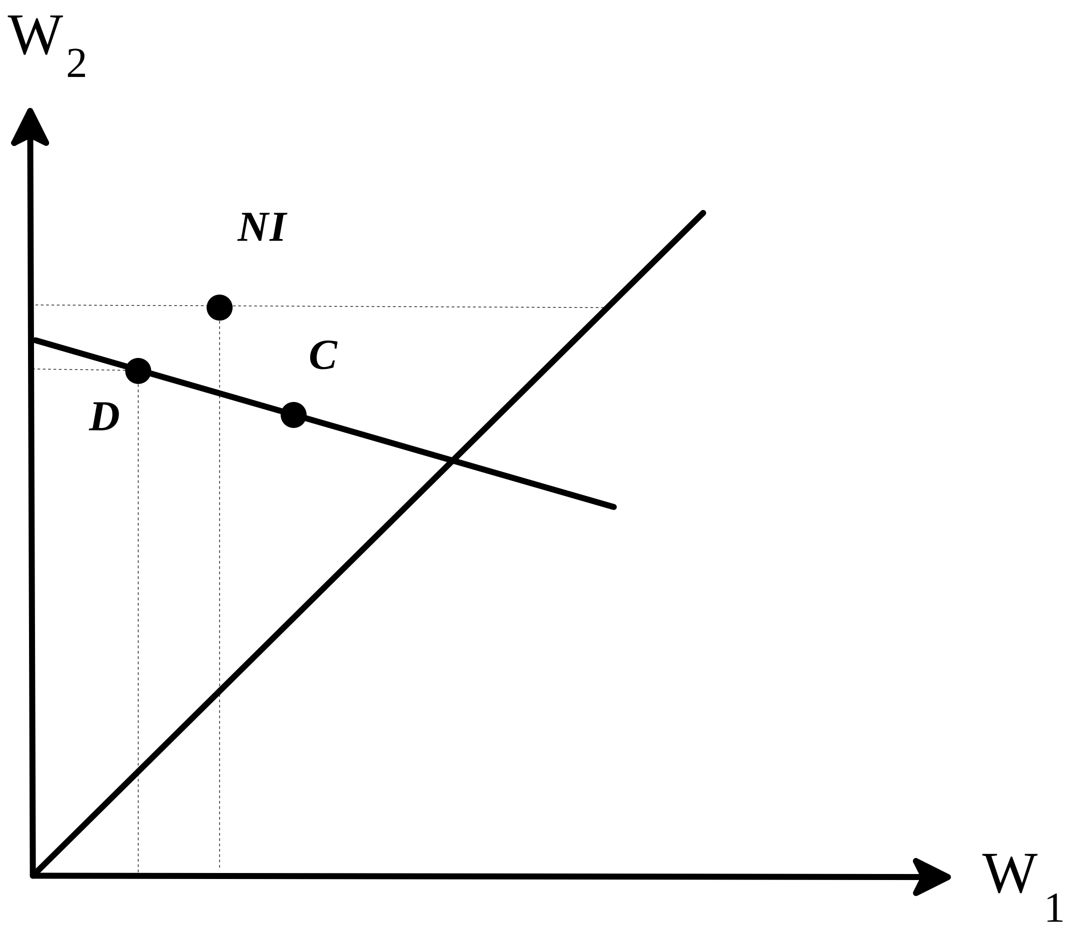
$W = 1{,}600, \; L = 360, \; p = \frac{1}{12}$ $\qquad C = \left(h_C = 50, \; d_C = 120\right)$

$\pi(C) = 30$ $\qquad D = (h_D = 30, \; d_D = 360)$

(4) Represent contract D in the wealth diagram.



(5) Find the full-insurance contract, call it *F*, that lies on the isoprofit line that goes through contracts C and D and represent it in the wealth diagram.



(6) Calculate the slope of the isoprofit line through C in the wealth diagram.

(7) Calculate the equation of the isoprofit line through C in the wealth diagram.

(8) Next prove that given two contracts $A = (h_A, d_A)$ and $B = (h_B, d_B)$, $\pi(A) = \pi(B)$ if and only if the expected value of the wealth lottery (for the insured) corresponding to contact *A* is equal to the expected value of the wealth lottery (for the insured) corresponding to contact *B*