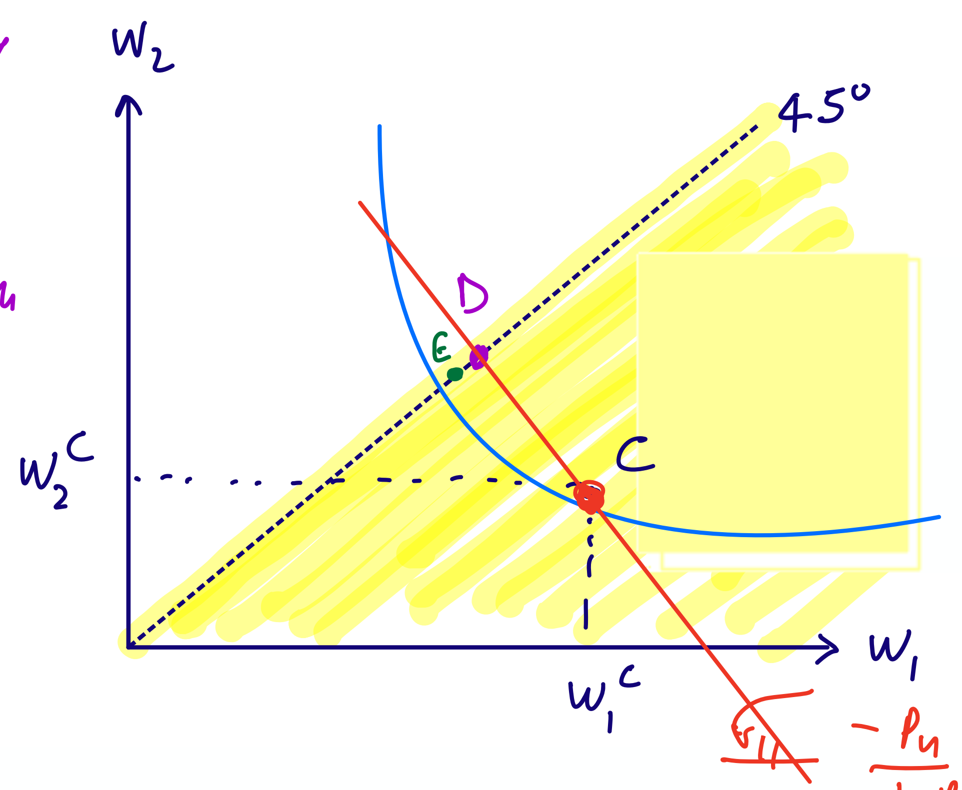
P_n = probability of X_1 with no effort

P_e = " with effort

$P_n > P_e$



At any contract on or below 45° line the Agent chooses low effort

$$u_A(m,e) = \begin{cases} U_A(m) & \text{if } e = e_L \\ U_A(m) - c & \text{if } e = e_H \end{cases}$$

A contract C <u>below</u> the 45° line is Pareto inefficient.

$C = \begin{pmatrix} w_1^c & w_2^c \\ P_n & 1-P_n \end{pmatrix}$

Let $\hat{w} = P_n w_1^c + (1-P_n) w_2^c$

$D = (\hat{w}, \hat{w}) \rightsquigarrow \begin{pmatrix} \hat{w} \\ 1 \end{pmatrix}$ by risk aversion

$D \succ_A C$

P's $\quad EU(C) = p_u(X_1 - w_1^c) + (1-p_u)(X_2 - w_2^c)$

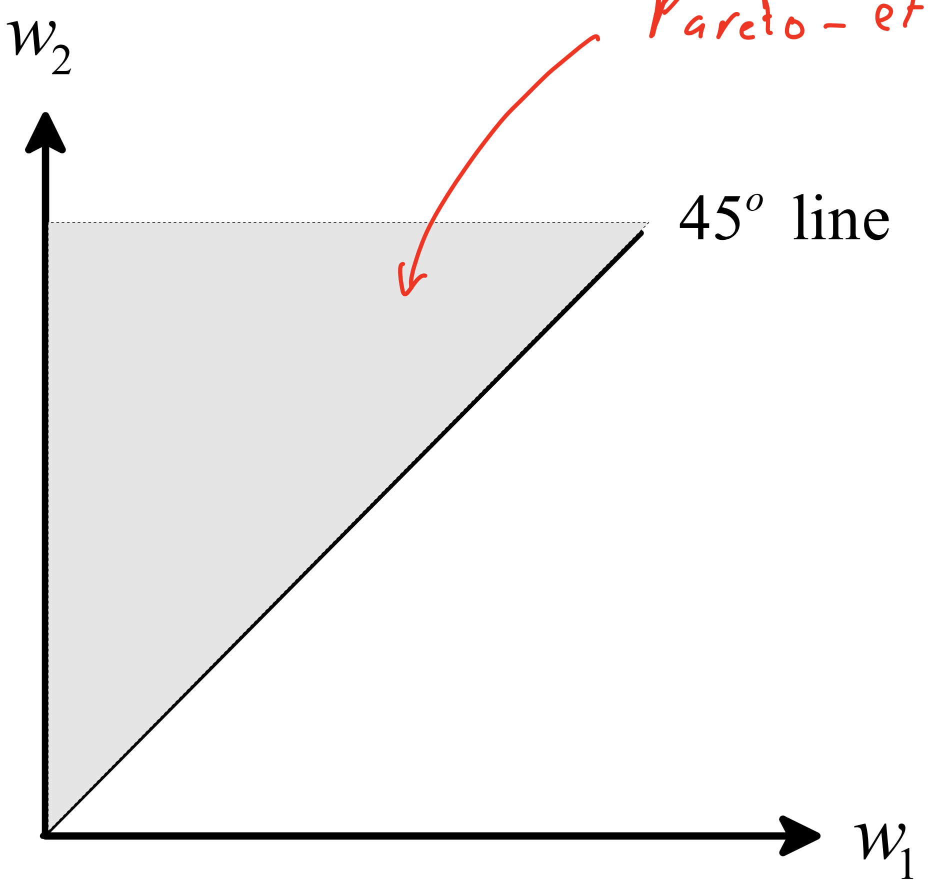
$$= \underbrace{p_u X_1 + (1-p_u) X_2} - [\underbrace{p_u w_1^c + (1-p_u) w_2^c}_{\hat{w}}]$$

$$EU(D) = p_u(X_1 - \hat{w}) + (1-p_u)(X_2 - \hat{w})$$

$$= \underbrace{p_u X_1 + (1-p_u) X_2} - \hat{w}$$

$D \sim_P C$

Pareto-efficient contracts must be in this area



Identify those contracts that are

1. individually rational and

2. Pareto efficient.

A contract C is **individually rational** if, for each party, signing the contract is at least as good as not signing it.

$\hat{r}_P$ = reservation utility of the Principal

$\hat{r}_A$ = reservation utility of the Agent.

C is individually rational if

(1) $EU_P(C) \geq \hat{r}_P$

(2) $\max\{EU_n^A(C), EU_e^A(C)\} \geq \hat{r}_A$

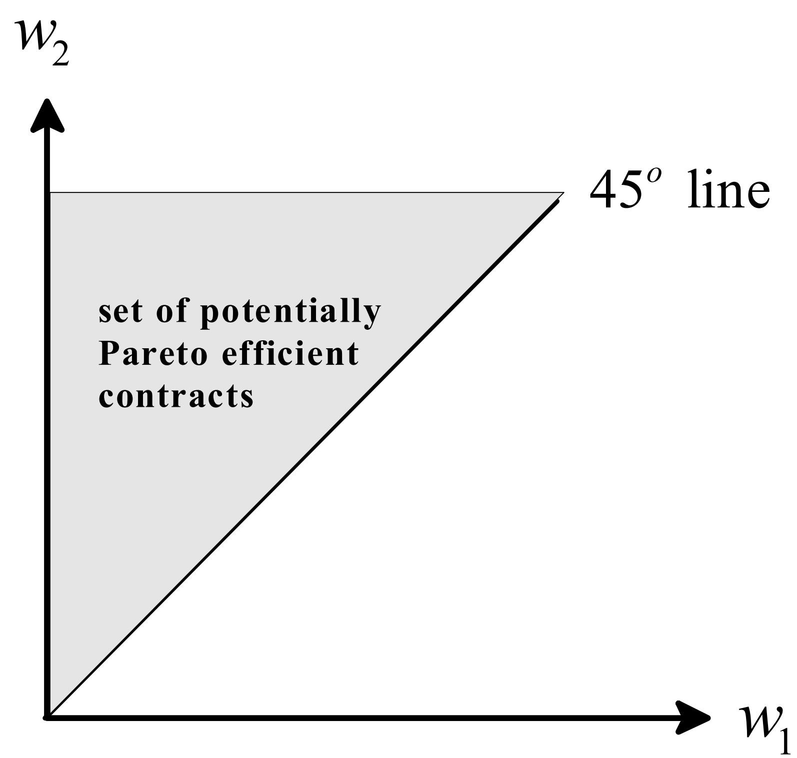
Contract C is **Pareto efficient** if, for every other contract D,

- if $EU_P(D) > EU_P(C)$ then $\max\{EU_n^A(C), EU_e^A(C)\} >$ $\{EU_n^A(D), EU_e^A(D)\}$

  $D \succ_P C$ $\quad$ $C \succ_A D$

- $D \succ_A C$ then $C \succ_P D$

To simplify, assume that $\hat{r}_P = \hat{r}_A = 0$ so that every contract (w_1, w_2) with $0 \le w_1 \le X_1$ and $0 \le w_2 \le X_2$ is individually rational. This assumption allows us to concentrate on the issue of Pareto efficiency.



Fix any contract C in the shaded area. Then, for each individual, there are two indifference curves that go through point C: one corresponding to the case where the Agent chooses e_L and the other corresponding to the case where the Agent chooses e_H.

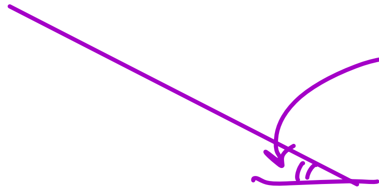
e_L = low effort or no effort

e_H = high effort or effort

$p_1^L > p_1^H$

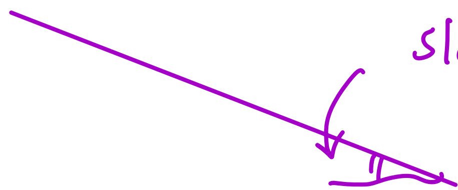
Let us begin with the risk-neutral Principal. Let $C = \left(w_1^C, w_2^C\right)$ and $D = \left(w_1^D, w_2^D\right)$ be two contracts. Let $\overline{X}_L = p_1^L X_1 + \left(1 - p_1^L\right) X_2$ and $\overline{X}_H = p_1^H X_1 + \left(1 - p_1^H\right) X_2$

- **Conditional on the Agent choosing** e_L, the Principal is indifferent between C and D if and only if



slope $- \dfrac{p_1^L}{1-p_1^L}$

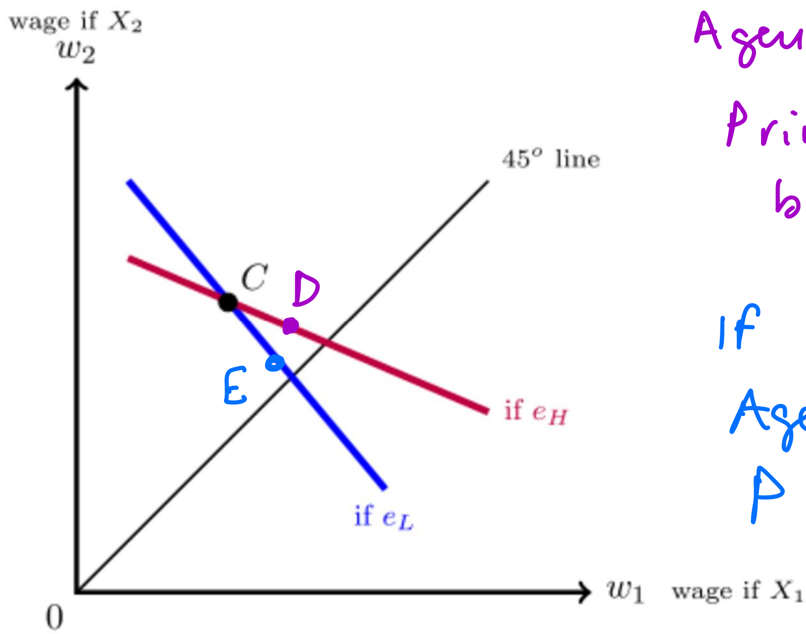
- **Conditional on the Agent choosing** e_H, the Principal is indifferent between C and D if and only if



slope $- \dfrac{p_1^H}{1-p_1^H}$

$p_1^L > p_1^H$

$$\frac{p_1^L}{1-p_1^L} > \frac{p_1^H}{1-p_1^H}$$



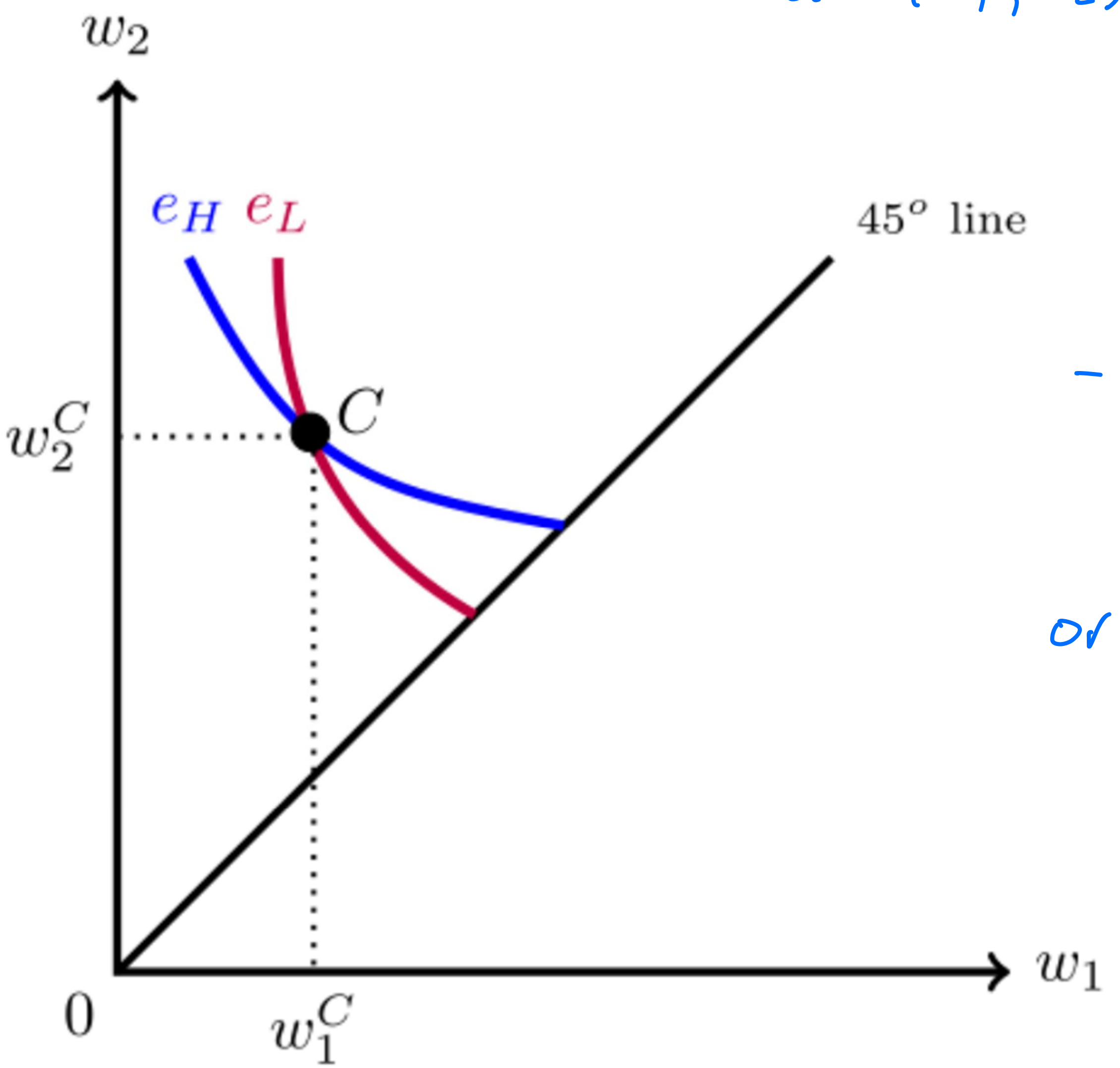
If at both C and D Agent chooses e_H then Principal is indifferent between C and D

If at both C and E Agent chooses e_L then P is indifferent between C and E

Now the Agent, who is risk averse with utility-of-money function $u_A(m,e) = \begin{cases} U_A(m) & \text{if } e = e_L \\ U_A(m) - c & \text{if } e = e_H \end{cases}$ with $c > 0$. Through any contract $C = \left(w_1^C, w_2^C\right)$ there are two indifference curves:

- a steeper one, corresponding to the case where the Agent exerts low effort e_L, whose slope at C is

- a less steep one, corresponding to the case where the Agent exerts high effort e_H, whose slope at C is



In general slope of ind. curve at (w_1, w_2) is $-\frac{p_1}{1-p_1}\frac{U'(w_1)}{U'(w_2)}$

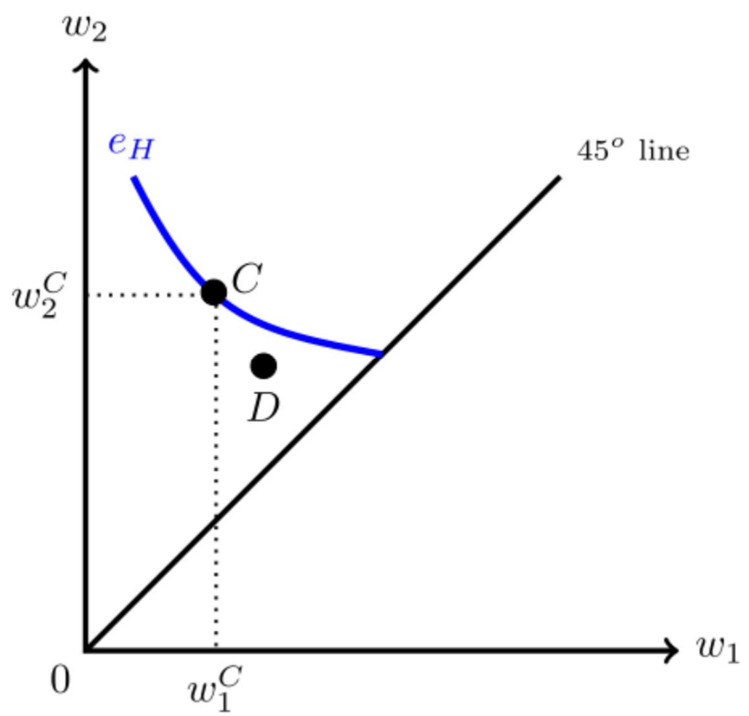
So either

$$-\frac{p_1^L}{1-p_1^L}\frac{U'(w_1)}{U'(w_2)}$$

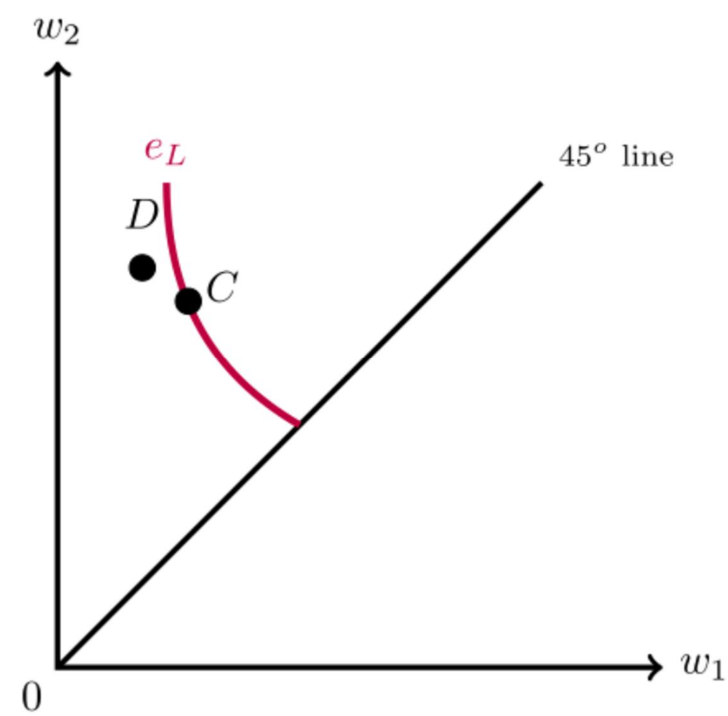
or $-\frac{p_1^H}{1-p_1^H}\frac{U'(w_1)}{U'(w_2)}$

How can we tell which of two contracts, C and D, gives higher utility?

For the Agent the direction of increasing utility is the North-East direction.

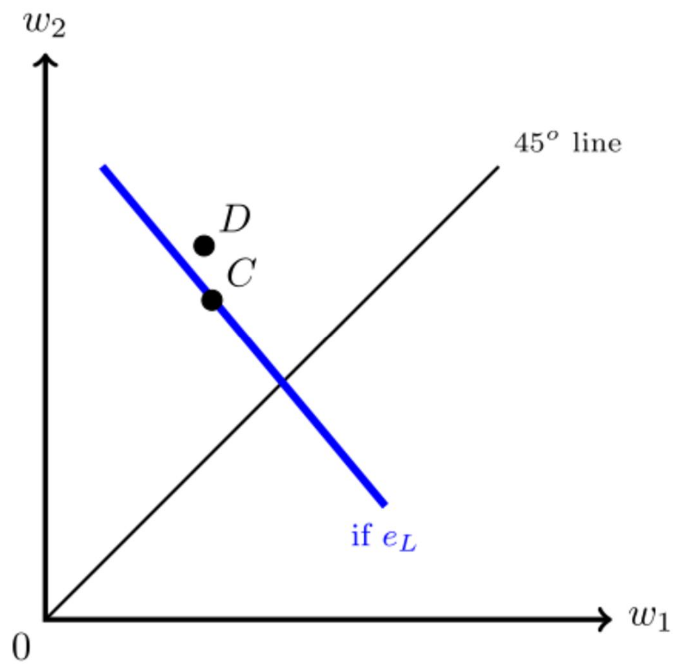


Contract D gives lower utility than contract C, *if the Agent exerts high effort under both contracts*. But it could be that contract D under low effort gives higher utility than contract C.

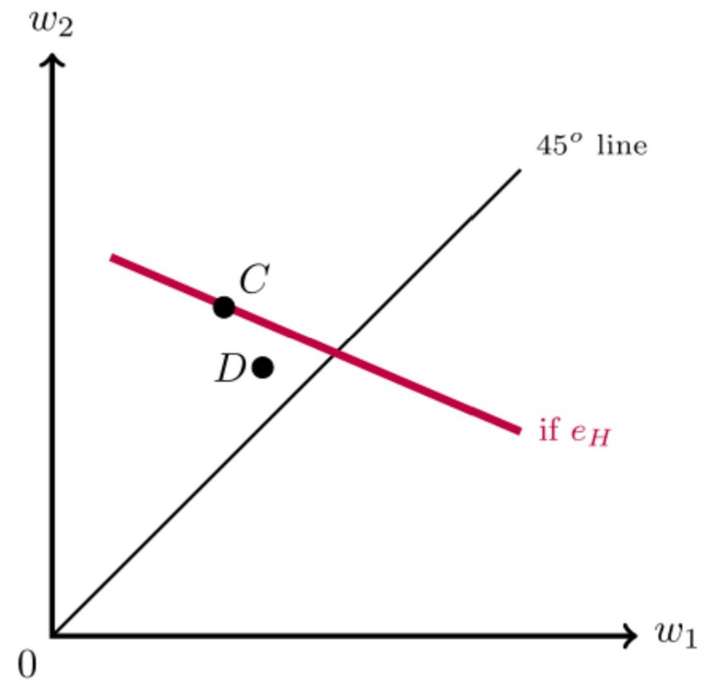


Contract D gives lower utility than contract C, *if the Agent exerts low effort under both contracts*. But it could be that contract D under high effort gives higher utility than contract C.

For the Principal the direction of increasing utility is the South-West direction.



Contract D gives **lower** utility to the Principal than contract C, *if the Agent exerts low effort under both contracts*. But it could be that contract D induces the Agent to choose high effort and gives the Principal higher utility than contract C.



Contract D gives **higher** utility to the Principal than contract C, *if the Agent exerts high effort under both contracts*. But it could be that contract D induces the Agent to choose low effort and gives the Principal lower utility than contract C.

How do we determine which contracts are Pareto efficient?

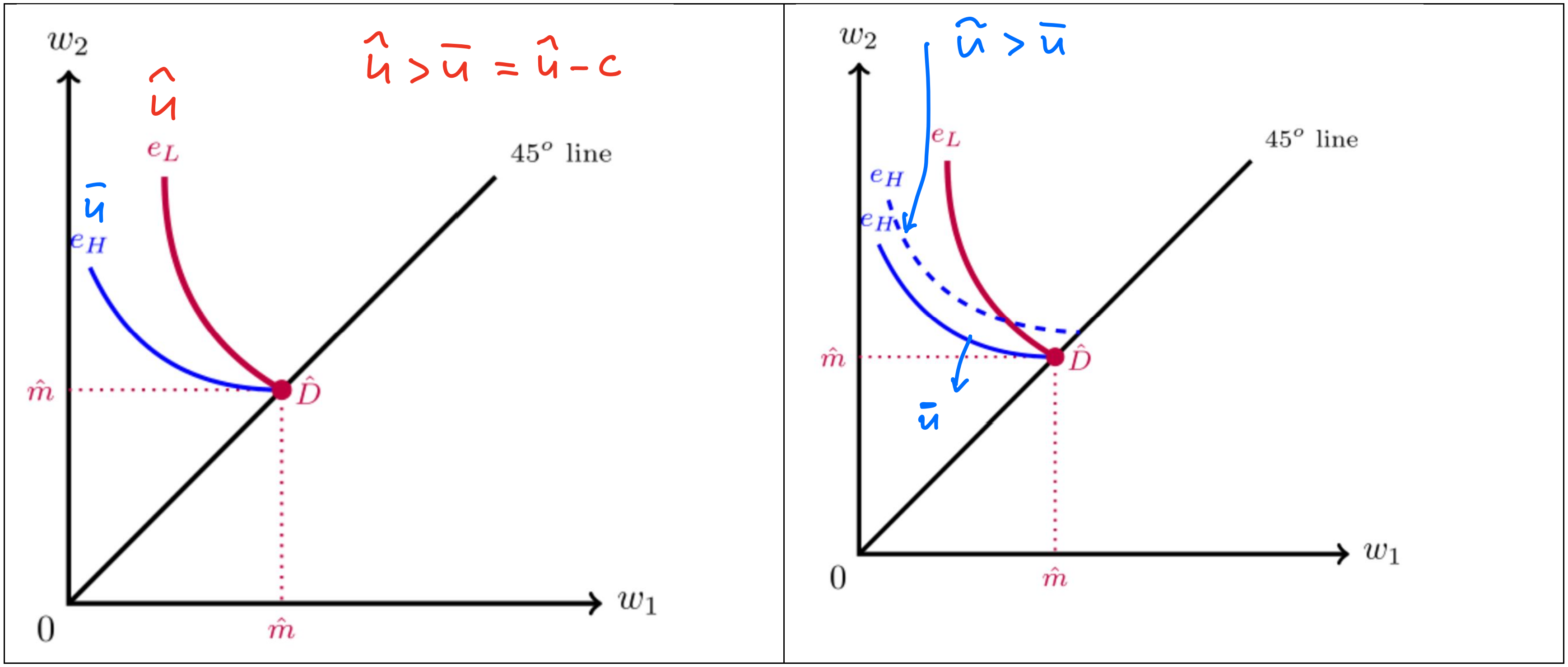
so Agent chooses low effort

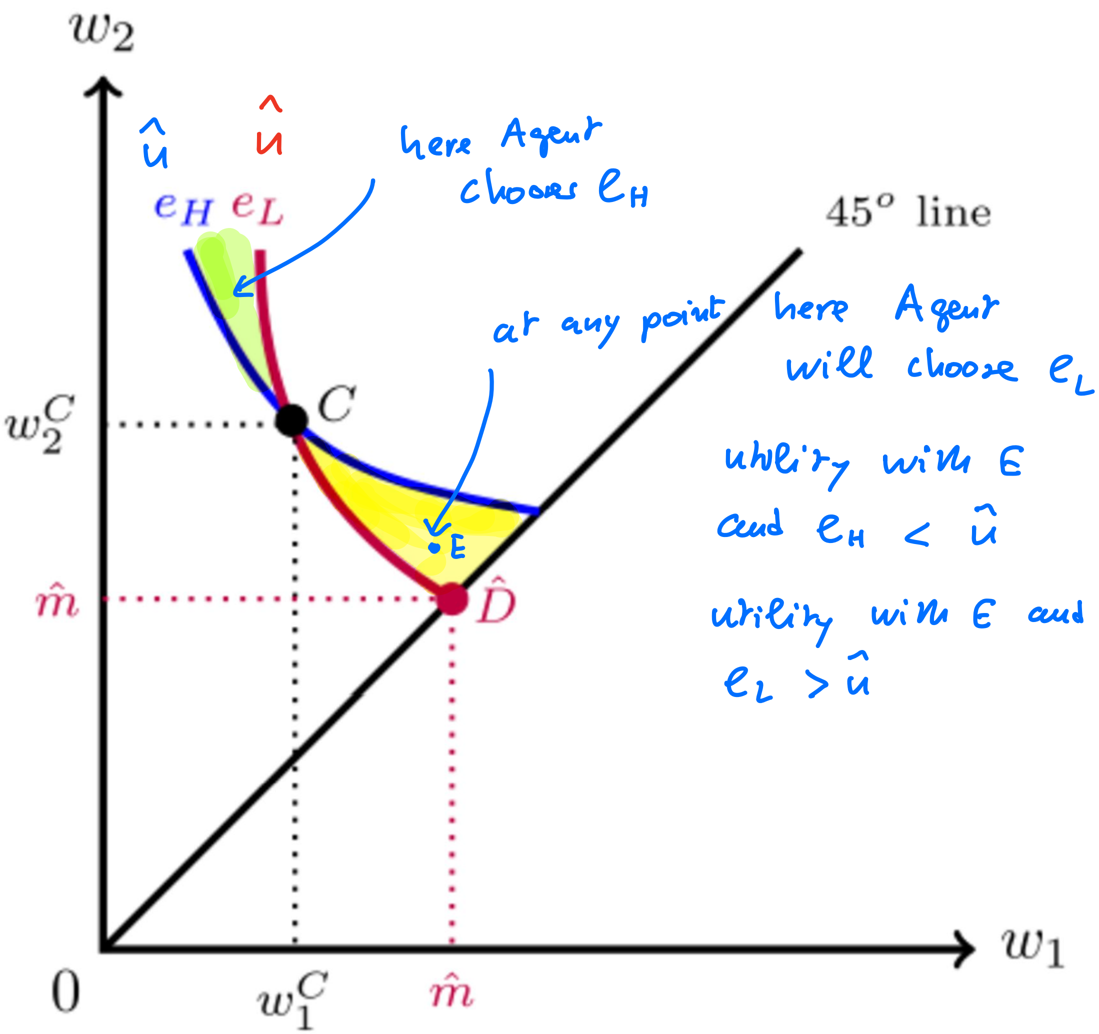
Step 1. Pick an arbitrary contract $\hat{D} = (\hat{m}, \hat{m})$ on the 45° line and let $\hat{u}$ be the Agent's utility from this contract. Then we know that A choose e_L

Step 2. Determine the set of contracts that give the Agent utility $\hat{u}$ when she chooses the best level of effort for each contract. Call this set the $\hat{u}$ -*utility locus* for the Agent.

Step 3. Find which contracts on the $\hat{u}$ -*utility locus* are Pareto efficient.

The indifference curve corresponding to e_H that goes through contract $\hat{D}$ corresponds to a level of utility less than $\hat{u}$ (in fact, equal to $\hat{u}-c$).





w_2
$\hat{u}$
$\hat{u}$
e_H e_L
here Agent chooses e_H
45^o line
at any point
here Agent will choose e_L
w_2^C
C
utility with E and $e_H < \hat{u}$
E
$\hat{m}$
$\hat{D}$
utility with E and $e_L > \hat{u}$
0
w_1^C
$\hat{m}$
w_1