ECN 103 Final Exam

Thursday, March 21, 6:00-8:00 pm in this room (Storer 1322)

- Four questions. Emphasis (at least two questions) on the material after the third Midterm (Chapters 9, 10 and 11).

- **What you can skip:**
- ▸ Chapter 3: No need to memorize the axioms of expected utility (Section 3.2)
- ▸ Chapter 5: Choosing from a continuum of options (Section 5.3.2) and Mutual insurance (Section 5.4)
- ▸ Chapter 6: Edgeworth box when the parties have positive initialwealth (Section 6.5)
- ▸ Chapter 8: Perfectly competitive industry (Section 8.4)
- ▸ Chapter 9: A more general analysis (Section 9.5) and Signaling in other markets (Section 9.6)
- ▸ Chapter 11: The case with more than two outcomes (Section11.4).

1. Expected utility / expected value

- What is the expected value of the following lottery? $L = \begin{pmatrix} \$100 & \$200 & \$400 \\ \frac{1}{10} & \frac{3}{10} & \frac{6}{10} \end{pmatrix}$

$EV(L) = \frac{1}{10} 100 + \frac{3}{10} 200 + \frac{6}{100} 400$

$= 310$

$EV(L) = 310$ $\qquad$ $EV(L') = 450$

- Between lottery $L = \begin{pmatrix} \$100 & \$200 & \$400 \\ \frac{1}{10} & \frac{3}{10} & \frac{6}{10} \end{pmatrix}$ and lottery $L' = \begin{pmatrix} \$0 & \$900 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ what would a rational individual choose?

$\rightarrow EU(L') = \frac{1}{2}0 + \frac{1}{2}1$

$= 0.5$

normalized	\$0	\$100	\$200	\$400	\$900
vNM U:	0	0.4	0.6	0.8	1

$EU(L) = \frac{1}{10}(0.4) + \frac{3}{10}(0.6) + \frac{6}{10}(0.8) = 0.7$

normalized vNM

best	z_3	1
	z_1	a
worst	z_2	0

$a > 0.5$

- Consider the following outcomes:

  z_1 = Congress introduces a new immigration bill,

  z_2 = no new bill is introduced

  z_3 = Congress introduces an improved health care bill

  Suppose that the President prefers z_1 to the lottery $\begin{pmatrix} z_2 & z_3 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$. Is he risk averse or risk neutral?

  meaningless question

  $EV = \frac{1}{2}0 + \frac{1}{2}1 = 0.5$

- Suppose that the President's ranking of the three outcomes is $\begin{pmatrix} \text{best} & z_3 \\ & z_1 \\ \text{worst} & z_2 \end{pmatrix}$ and he is indifferent between z_1 and the lottery $\begin{pmatrix} z_2 & z_3 \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$. What is the President's normalized von Neumann-Morgenstern utility function?

z_3	1
z_1	$\frac{3}{4} = 0.75$
z_2	0

$$\begin{pmatrix} \$0 & \$100 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \quad \text{versus} \quad \begin{pmatrix} \$49 \\ 1 \end{pmatrix}$$

- Is it possible for a risk-averse individual to be indifferent between two lotteries L and L' despite the fact that the expected value of L' is greater than the expected value of L?

- What is the Arrow-Pratt measure of risk aversion $R_A(m)$?

  Suppose that $U(m) = \ln(m)$. What is $R_A(10)$?

$$R_A(m) = -\frac{\overbrace{U''(m)}^{<0}}{\underbrace{U'(m)}_{>0}}$$

$$R_A(10) = -\frac{-\frac{1}{10^2}}{\frac{1}{10}} = \frac{1}{10}$$

$U(m) = \ln(m)$

$U'(m) = \frac{1}{m} = m^{-1}$

$U''(m) = -\frac{1}{m^2}$

$EV(L) = 17$ $\quad$ $EV(L') = 20$

$$\begin{pmatrix} \$17 \\ 1 \end{pmatrix} \succ \begin{pmatrix} \$9 & \$25 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

- Risk premium

  Suppose that $U(m) = \sqrt{m}$, $L = \begin{pmatrix} 9 & 25 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ and $L' = \begin{pmatrix} 4 & 36 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$. What is the risk premium of lottery L? and the premium of lottery L'? [Assume zero initial wealth.]

R_L = that value of R such that $\begin{pmatrix} \$(17-R) \\ 1 \end{pmatrix} \sim L$

solution to $\sqrt{17-R} = \frac{1}{2}\sqrt{9} + \frac{1}{2}\sqrt{25} = 4$

$R = 1$

$R_{L'}$ solution to $\sqrt{20-R} = \frac{1}{2}\sqrt{4} + \frac{1}{2}\sqrt{36} = 4$

$R = 4$

- Certainty equivalent. Suppose that $U(m) = \sqrt{m}$, $L = \begin{pmatrix} 9 & 25 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ and $L' = \begin{pmatrix} 4 & 36 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$. What is the certainty equivalent of lottery L? and the certainty equivalent of lottery L'?

$C_L = EV(L) - R_L = 17 - 1 = \$16$

$C_{L'} = EV(L') - R_{L'} = 20 - 4 = \$16$

2. Stochastic dominance

- Having to choose between $L = \begin{pmatrix} \$9 & \$16 & \$25 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$ and $M = \begin{pmatrix} \$10 & \$16 & \$25 \\ \frac{1}{4} & \frac{5}{12} & \frac{1}{3} \end{pmatrix}$ what would a rational individual choose?

$$L = \begin{pmatrix} \$9 & \$10 & \$16 & \$25 \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$$

cdf $\quad \frac{1}{3} \quad \frac{1}{3} \quad \frac{2}{3} \quad 1$

$$M = \begin{pmatrix} \$9 & \$10 & \$16 & \$25 \\ 0 & \frac{1}{4} & \frac{5}{12} & \frac{1}{3} \end{pmatrix}$$

cdf $\quad 0 \quad \frac{1}{4} \quad \frac{2}{3} \quad 1$

M dominates L in the sense of FOSD

$EU(M) > EU(L)$ for any increasing function $U(m)$

- Let $A = \begin{pmatrix} \boxed{\begin{smallmatrix} \$9 \\ \frac{1}{5} \end{smallmatrix}} & \begin{smallmatrix} \$16 \\ \frac{3}{5} \end{smallmatrix} & \begin{smallmatrix} \$25 \\ \frac{1}{5} \end{smallmatrix} \end{pmatrix}$ and $B = \begin{pmatrix} \boxed{\begin{smallmatrix} \$9 \\ \frac{1}{5} \end{smallmatrix}} & \begin{smallmatrix} \$12 \\ p \end{smallmatrix} & \begin{smallmatrix} \$16 \\ \frac{1}{5} \end{smallmatrix} & \begin{smallmatrix} \$25 \\ q \end{smallmatrix} \end{pmatrix}$.

For what values of p and q would a risk-averse individual prefer A to B, while a risk-neutral individual would be indifferent between A and B?

Mean-preserving spread

(1) $EV(A) = EV(B)$ $\qquad \frac{3}{5}16 + \frac{1}{5}25 = 12p + \frac{1}{5}16 + 25q$

(2) $p + q = \frac{3}{5}$ $\qquad$ Solution: $p = \frac{18}{65} \quad q = \frac{21}{65}$

3. Insurance

- An individual has an initial wealth of \$360,000, faces a potential loss of \$90,000 with probability $\frac{1}{100}$ and her von Neumann-Morgenstern utility-of-money function is $U(m) = \sqrt{m}$. Suppose that she is offered a contract, call it contract A, with premium \$5,975 and deductible \$17,625.

  (a) Represent contract A in terms of wealth levels. $A = (360000 - 5975 - 17625,\ 360000 - 5975)$

  (b) Calculate the slope of the indifference curve through A at point A.

  (c) Calculate the expected profit from contract A.

$A = (W_1^A, W_2^A)$

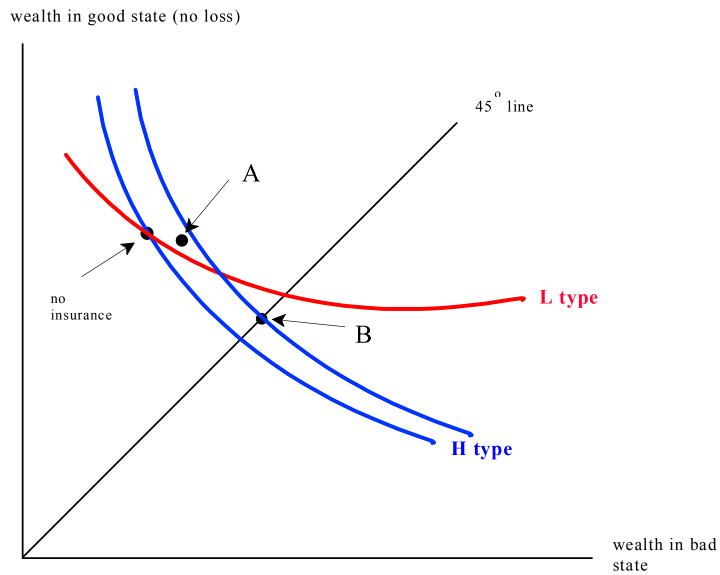
$-\frac{p}{1-p}\frac{U'(W_1^A)}{U'(W_2^A)}$

$\pi = h - p(L-d)$

$= 5,975 - \frac{1}{100}(90,000 - 17,625)$

- Two types of customers, *H* and *L*. Same initial wealth of \$360,000, Same potential loss of \$90,000 and Same utility function $U(m)$ with $U''(m) < 0$. The probability of loss is $\frac{5}{100}$ for *H* people and $\frac{1}{100}$ for *L* people.

P_L P_H



For L people

$A \succ_L NI \succ_L B$

they choose A

For H people

$B \succ_H A \succ_H NI$

they choose B

(a) The insurance industry is a monopoly and it offers contracts *A* and *B* shown above. Contract *A* has a deductible of \$15,000 and a premium of \$800. Contract *B* has a premium of \$4,550. Calculate expected profits if there are **1,500 *H* people and 1,000 *L* people**.

total $\Pi = 1500[h_B - P_H(L - d_B)] + 1000[h_A - P_L(L - d_A)]$

(b) What would the monopolist's profits be if it offered only contract *B*?

Only H people buy

$\Pi = 1500 - [h_B - P_H L]$

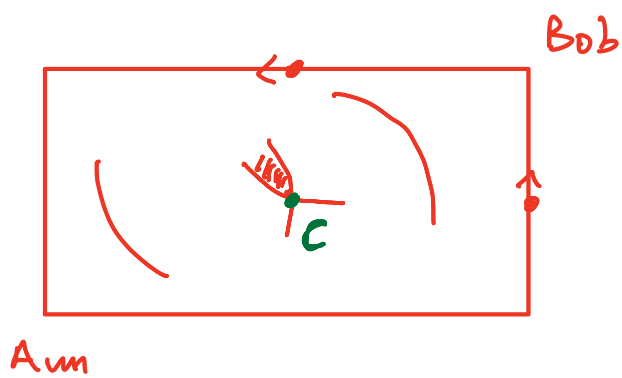
4. Pareto efficient risk-sharing

- Ann and Bob have started a business together. With probability $\cancel{\frac{2}{5}}$ the profits will be \$8,000 while with probability $\cancel{\frac{3}{5}}$ the profits will be \$5,000. They have agreed that they will split the profits equally (each will get 50%). Ann's von Neumann-Morgenstern utility-of-money function is $U(m) = \sqrt{m}$. Bob's von Neumann-Morgenstern utility-of-money function is $V(m) = \ln(m)$.

  (a) Represent their agreement as a point in an Edgeworth box.

  (b) Show that their agreement is not Pareto efficient

  (c) If you were to propose an alternative agreement that Pareto dominated their initial agreement, how would you modify the initial agreement?



5. Signaling

- Group I: productivity **$20+2y$**, Group II: productivity **$25 + 3y$**, Group III: **$40 + y$**.

  The cost of y units of education is **$12y$** for Group I, **$5y$** for Group II and **$4y$** for Group III.

  The potential employer believes that those applicants with education **less than *a*** belong to Group I, those with education **at least *a* but less than *b*** belong to Group II and those with education **at least *b*** belong to Group III and offers each applicant a wage equal to the applicant's estimated productivity, given the applicant's level of education. **Nobody can choose a level of education below $\hat{y}$.**

  Inequalities that are necessary and sufficient for the existence of a signaling equilibrium.

For Group I: $20+2\hat{y}-12\hat{y} \geq 25+3a-12a$

$$20+2\hat{y}-12\hat{y} \geq 40+b-12b$$

For Group II: $25+3a-5a \geq 20+2\hat{y}-5\hat{y}$

$$25+3a-5a \geq 40+b-5b$$

For Group III: $40+b-4b \geq 20+2\hat{y}-4\hat{y}$

$$40+b-4b \geq 25+3a-4a$$

7. Principal-Agent with moral hazard

$X_1 = 400$ and $X_2 = 900$ $\quad$ $e_L = 0$ and $e_H = 3$

$U_P(\$m) = m$ $\quad$ $U_A(m,e) = \sqrt{m} - e$

$$\text{probability of } X_1 = \begin{cases} \frac{3}{5} & \text{if } e = 0 \\ \frac{1}{5} & \text{if } e = 3 \end{cases}$$

Find a Pareto efficient contract that gives utility 12 to the Agent.

One candidate is the contract $\hat{D} = (144,144)$ (the Agent chooses $e = 0$ and her utility is $\sqrt{144} - 0 = 12$.

The other candidate is the contract $C = \left(w_1^C, w_2^C\right)$ that lies on the two indifference curves of the Agent corresponding to a utility level of 12.

- To be on the LOW-effort indifference curve, contract C must satisfy:

$$\frac{3}{5}\sqrt{w_1} + \frac{2}{5}\sqrt{w_2} = 12$$

- To be on the HIGH-effort indifference curve, contract C must satisfy:
  - $$\frac{1}{5}\left(\sqrt{w_1} - 3\right) + \frac{4}{5}\left(\sqrt{w_2} - 3\right) = 12$$

The solution is: $\boxed{w_1^C = \quad \text{and} \quad w_2^C =}$

$w_1^C = 81 \quad w_2^C = 272.25$

Which of the two contracts does the Principal prefer?

- With contract $\hat{D} = (144,144)$ the Principal's expected utility is

$$\frac{3}{5}(400 - 144) + \frac{2}{5}(900 - 144) = 456$$

- With contract $C=(\ ,\ )$ the Principal's expected utility is

$$\frac{1}{5}(400-81)+\frac{4}{5}(900-272.25)=566$$

Thus the Principal prefers C to $\hat{D}$ and C is the Pareto efficient contract that gives utility 12 to the Agent.