Economics 103

PRACTICE EXAM for the SECOND MIDTERM:

ANSWERS

1. **(a)** $U'(x) = \frac{3}{125} - \frac{x}{500,000}$ and $U''(x) = -\frac{1}{500,000} < 0$. Thus Jennifer is risk averse.

(b) $A(x) = -\frac{U''(x)}{U'(x)} = \frac{1}{12,000 - x}$. Thus $A(4,000) = \frac{1}{8.000} = 0.000125$ and

$A(6,000) = \frac{1}{6.000} = 0.000167$

(c) Jennifer's expected utility if she bets \$2000 is:

$$\frac{3}{4}\left\{200 - \left[12 - \frac{6000+2000}{1000}\right]^2\right\} + \frac{1}{4}\left\{200 - \left[12 - \frac{6000-2000}{1000}\right]^2\right\} = 172.$$

(d) If Jennifer does not bet her utility is:

$$\left\{200 - \left[12 - \frac{6000}{1000}\right]^2\right\} = 164$$

(d) If Jennifer bets \$y her expected utility is:

$$f(y) \equiv \frac{3}{4}\left\{200 - \left[12 - \frac{6000+y}{1000}\right]^2\right\} + \frac{1}{4}\left\{200 - \left[12 - \frac{6000-y}{1000}\right]^2\right\}$$

Jennifer will choose y to maximize f(y). A necessary condition for this is that f ′(y) = 0, i.e.

$$\frac{3}{4}(-2)\left[12 - \frac{6000+y}{1000}\right]\left(-\frac{1}{1000}\right) + \frac{1}{4}(-2)\left[12 - \frac{6000-y}{1000}\right]\left(+\frac{1}{1000}\right) = 0$$

which gives y = 3000, that is, Jennifer will choose to bet \$3000.

(e) Her expected utility if she bets \$3000 is:

$$\frac{3}{4}\left\{200 - \left[12 - \frac{6000+3000}{1000}\right]^2\right\} + \frac{1}{4}\left\{200 - \left[12 - \frac{6000-3000}{1000}\right]^2\right\} = 173.$$

(f) If she bets the optimal amount of \$3000, her utility goes up, compared to not betting, by (173 – 164) = 9.

(g) If the probability is 50%, then her expected wealth (whatever the stake) is her initial wealth. Since Jennifer is risk-averse, she will not want to bet, that is, she will choose y = 0.

2. **(a)** Let W_0 be the initial wealth. Then $h = W_0 - W_2$ and $d = W_2 - W_1$. Replacing in the formula $h = p(\ell - d) + c$ we get $W_0 - W_2 = p\ell - p(W_2 - W_1) + c$. Rearranging we get

$$W_2 = \frac{W_0 - p\ell - c}{1-p} - \frac{p}{1-p} W_1 .$$

(b) The above is the equation of a straight line with slope $-\frac{p}{1-p}$, hence an isoprofit line. Any two contracts on this isoprofit line, expressed as wealth lotteries, have the same expected value. Let F be the full-insurance contract on this line and C any partial insurance contract on this line. Then F guarantees the expected value of C for sure and thus, by risk aversion, the individual will strictly prefer F to C.

[A mathematically sophisticated student might suggest the following alternative proof, which – however – is less general because it assumes that the individual has vNM preferences,. Let U be a vNM utility-of-money function that represents the individual's preferences. If the individual chooses deductible d ≥ 0, her utility is (W denotes initial wealth):

$$U(d) = p\,U(W-d-h) + (1-p)\,U(W-h) = p\,U(W-d-p\ell+pd-c) + (1-p)\,U(W-p\ell+pd-c).$$

The individual will then choose d to maximize U(d). Necessary condition is that $U'(d) = 0$.

$$U'(d) = p\,U'(W-d-p\ell+pd-c)\,(-1+p) + (1-p)\,U'(W-p\ell+pd-c)\,p =$$

$$= p\,(1-p)\left[U'(W-p\ell+pd-c) - U'(W-d-p\ell+pd-c)\right].$$

This is equal to zero if and only if $U'(W-p\ell+pd-c) = U'(W-d-p\ell+pd-c)$. Since the individual is risk-averse, $U'' < 0$, hence U' is decreasing. It follows that the equality is satisfied if and only if $W-p\ell+pd-c = W-d-p\ell+pd-c$ i.e. if and only if d = 0. Hence the individual will choose full insurance (= zero deductible).]

3. Let $X >_{FSD} Y$ mean that X dominates Y in the sense of first-order stochastic dominance. Then the following and nothing else is true: $L >_{FSD} M$, $L >_{FSD} N$ and $M >_{FSD} N$

4. Let $L \rightarrow_{MPS} M$ mean that from L one can obtain M by applying a mean-preserving-spread (MPS) to L (or, equivalently, M is a mean-preserving-spread of L). Then the following and nothing else is true: $B \rightarrow_{MPS} C$.